An elasto-plastic approach to estimate lifetime of notched components under variable amplitude fatigue loading: a preliminary investigation

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ABSTRACT. The present paper is concerned with the formulation of an elasto-plastic strain based approach suitable for assessing fatigue strength of notched components subjected to in-service variable amplitude cyclic loading. The hypothesis is formed that the crack initiation plane is closely aligned with the plane of maximum shear strain amplitude, its orientation and the associated stress/strain quantities being determined using the Maximum Variance Method. Fatigue damage is estimated by applying the Modified Manson-Coffin Curve Method (MMCCM) along with the Point Method (PM). In the proposed approach, the required critical distance is treated as a material property whose value is not affected either by the sharpness of the notch being assessed or by the profile of the load spectrum being applied. The detrimental effect of non-zero mean stresses and degree of multiaxiality of the local stress/strain histories is also considered. The accuracy and reliability of the proposed design methodology was checked against several experimental data taken from the literature and generated under different uniaxial variable amplitude load histories. In order to determine the required local stress/strain states, refined elasto-plastic finite element models were solved using commercial software ANSYS®. This preliminary validation exercise allowed us to prove that the proposed approach is capable of estimates laying within an error factor of about 2. These preliminary results are certainly promising, strongly supporting the idea that the proposed design strategy can successfully be used to assess the fatigue lifetime of notched metallic components subjected to in-service multiaxial variable amplitude loading sequences.

KEYWORDS. Notched Components; Variable Amplitude; Critical Plane; Manson-Coffin Curve; non-zero mean stress.

INTRODUCTION

In situations of practical interest, real engineering components are characterised by complex geometries resulting in local stress/strain concentration phenomena. They normally contain either notches or complex features that favour the initiation of fatigue cracks. The presence of stress/strain raiser results in multiaxial stress/strain states in the critical regions even if the nominal load history being applied is uniaxial. Furthermore, real mechanical components are often exposed to variable amplitude load histories. Accordingly, in the recent past, a tremendous effort has been made by the international scientific community to device specific design techniques suitable for accurately assessing the durability of notched components subjected to in-service variable amplitude load histories [1]. In this complex scenario, this paper summarises the results from a preliminary investigation aiming at developing an elasto-plastic strain based approach capable of predicting fatigue lifetime of notched components subjected to variable amplitude load histories.

Stress or strain based approaches are used to assess fatigue damage in mechanical components. Stress based approaches are recommended to be used to perform the high-cycle fatigue assessment [2] since, under these circumstances, cyclic plastic deformations can be neglected with little loss of accuracy [3]. The accuracy and reliability of this design strategy has been validated by performing several experimental investigations [4, 5]. However, when cyclic plasticity cannot be disregarded, it is commonly accepted that strain based approaches are more accurate in predicting lifetime of components, with this holding true especially in the low -cycle fatigue regime [1, 6]. This explains why, nowadays, the strain based approach is considered as an irreplaceable tool that is daily used by structural engineers to assess fatigue damage in real engineering components [7, 8].

FUNDAMENTALS OF THE MODIFIED MANSON-COFFIN CURVE METHOD

The Modified Manson-Coffin Curve Method (MMCCM) is a strain-based fatigue criterion that allows uniaxial/multiaxial fatigue damage in real mechanical components subjected to in-service time-variable load histories to be estimated accurately [9, 10]. According to the classical strain-based criterion, Manson-Coffin curve is defined by slopes *c* and *b* that links the maximum shear strain amplitude with the number of reversals to failure. From a physical point of view, the formalisation of the MCCM takes as its starting point the assumption that the material plane experiencing the maximum shear strain amplitude coincides with the Stage I plane [1] (Fig. 1).

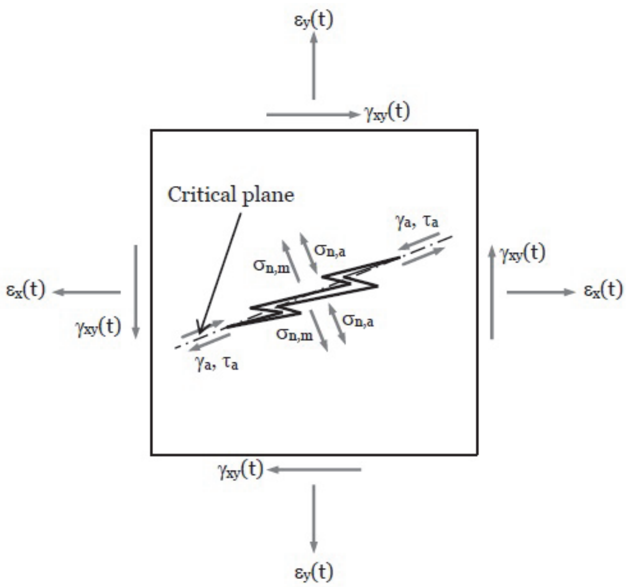


Figure 1: Fatigue damage model [1].

According to the fatigue model depicted in Fig. 1, the following relationship can be defined [11]:

$$\gamma_a = \frac{\tau'_f}{G}\left(2N_f\right)^b + \gamma'_f\left(2N_f\right)^c \tag{1}$$

where, γ_a is the shear strain amplitude relative to the critical plane; τ'_f and γ'_f are the multiaxial fatigue strength coefficient and the multiaxial fatigue ductility coefficient, respectively; *b* and *c* are the multiaxial fatigue strength exponent and the multiaxial fatigue ductility exponent, respectively; N_f is the number of cycles to failure. All these fatigue constants can be evaluated by running an appropriate experiment.

The classic Manson and Coffin curve, as shown in Fig. 2a and defined by Eq. 1, is reformulated to deal with multiaxial fatigue stress/strain tensors by calibrating all functions in Eq. 1, as expressed in Eq. 2. By following a systematic validation exercise, the MMCCM is seen to be capable of accurately modelling not only the detrimental effect of non-zero mean stresses, but also the degree of multiaxiality and non-proportionality of the load history being assessed as shown in Fig.2b [11-13].

$$\gamma_a = \frac{\tau'_f(\rho)}{G}\left(2N_f\right)^{b(\rho)} + \gamma'_f(\rho)\left(2N_f\right)^{c(\rho)} \quad (2)$$

In Eq. (2) the functions $\tau'_f(\rho)$, $\gamma'_f(\rho)$, $b(\rho)$, $c(\rho)$ are fatigue constants that described above but need to be calibrated in terms of rho. ρ is the critical plane stress ratio and it is defined according to Eq. 3, [10].

$$\rho = \frac{\sigma_{n,m} + \sigma_{n,a}}{\tau_a} = \frac{\sigma_{n,\max}}{\tau_a} \quad (3)$$

In Eqs. (3) $\sigma_{n,m}$ and $\sigma_{n,a}$ are the mean value and amplitude of stress normal to the critical plane; τ_a is shear stress amplitude relative to the same plane [10].

As to the MMCCM's *modus operandi*, the modified Manson-Coffin diagram depicted in Fig. 2b shows how this multiaxial fatigue criterion estimates fatigue lifetime, with the modified Manson-Coffin curves, Eq. (2), moving the curve downwards as ratio ρ increases, resulting an increase in fatigue damage. To conclude, it is worth recalling here that, in the absence of stress concentration phenomena, critical plane stress ratio ρ equals unity under uniaxial fully-reversed loading, whereas it is equal to zero under pure torsional loading [11].

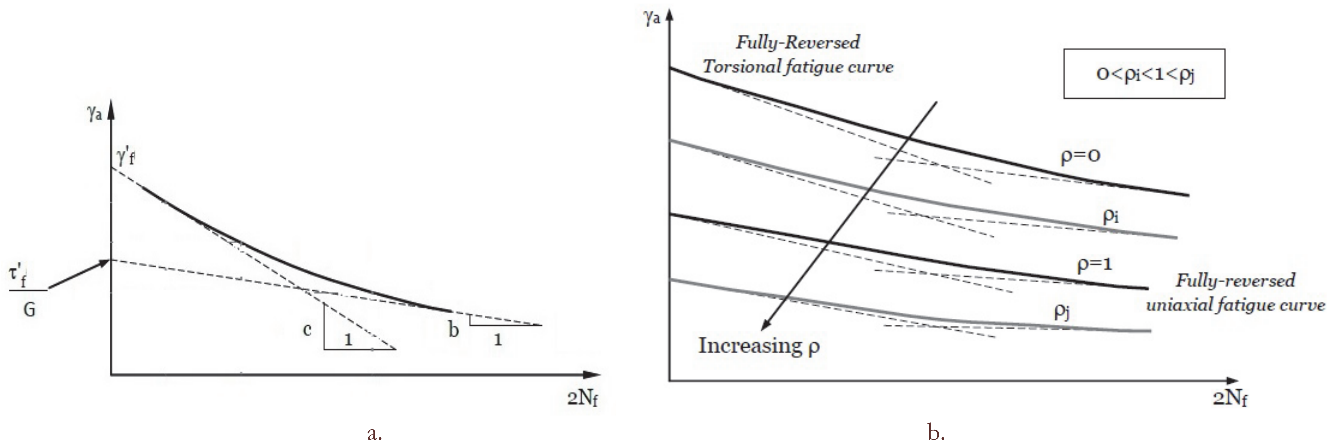


Figure 2: a. Classic Manson-Coffin Curve. b. Modified Manson-Coffin Curve [11].

THEORY OF CRITICAL DISTANCE TO QUANTIFY THE EFFECTIVE LOCAL STRESS/STRAIN STATE

As far as notched components are concerned in this study, a specific methodology is required in order to accurately take into account the presence of stress/strain concentration phenomena and determine the effective local stress/strain histories. The engineering aim of this section is to summarise a fundamental Theory of Critical distance and using the theory to estimate the effective local stress/strain states at the vicinity of notch apex different than notch tip, to predict fatigue lifetime of notched components.

Generally, TCD is formalised in different forms that include a point, line, area, and volume method [4]. The point method is the simplest form that commonly used [6] and postulated that the elastoplastic stress/strain state to be used to assess the damaging effect of stress/strain concentrators and has to be determined at a distance (equal to $L_{PM}/2$) from the notch apex (see Fig. 3b). The hypothesis is formed that the required critical distance is a material property, changed in different materials. However, its value remains constant in the same material regardless of notch geometry and notch sharpness [4]. According to the previous findings, that validity is fully supported by the experimental evidence and proved that the TCD is successful not only in predicting fatigue lifetime under constant amplitude loading but also under variable amplitude loading condition [6, 14].

From a practical point of view, to indicate the critical distance for a specific material, the best way is running an appropriate experimental investigation by testing specimens containing with known notched geometry under fully reversed constant amplitude axial force.

In the present paper, typical notched samples were considered from a pre-investigated literature [6] as shown in Fig. 3a. The samples were tested under fully reversed nominal tension-compression CA cyclic force *F(t)*, resulting in a fatigue failure at N_f number of cycles to failure. In the meantime, by post processing the elastoplastic FE model, a stabilized stress/distance and strain/distance curve were plotted along the notch bisector as illustrated in Fig. 3d. Furthermore, it is worth mentioning here that the stress/strain states at the vicinity of notch tip are experiencing a triaxial history even if the external applied load is uniaxial. The behaviour of these multiaxial stress/strain states are varying proportionally. Under these particular circumstances, the level of multiaxiality of the local stress/strain and effect of nonzero mean stress are considered to modify Manson-Coffin curve. Then, by using the experimental number of cycles to failure, average value of a critical distance was computed for specimens subjected to different values of nominal loads.
To sum up, according to the theory of critical distance, for a given material, the hypothesis is formed that such a distance is always the same in the same material regardless of notch geometry and notch sharpness.

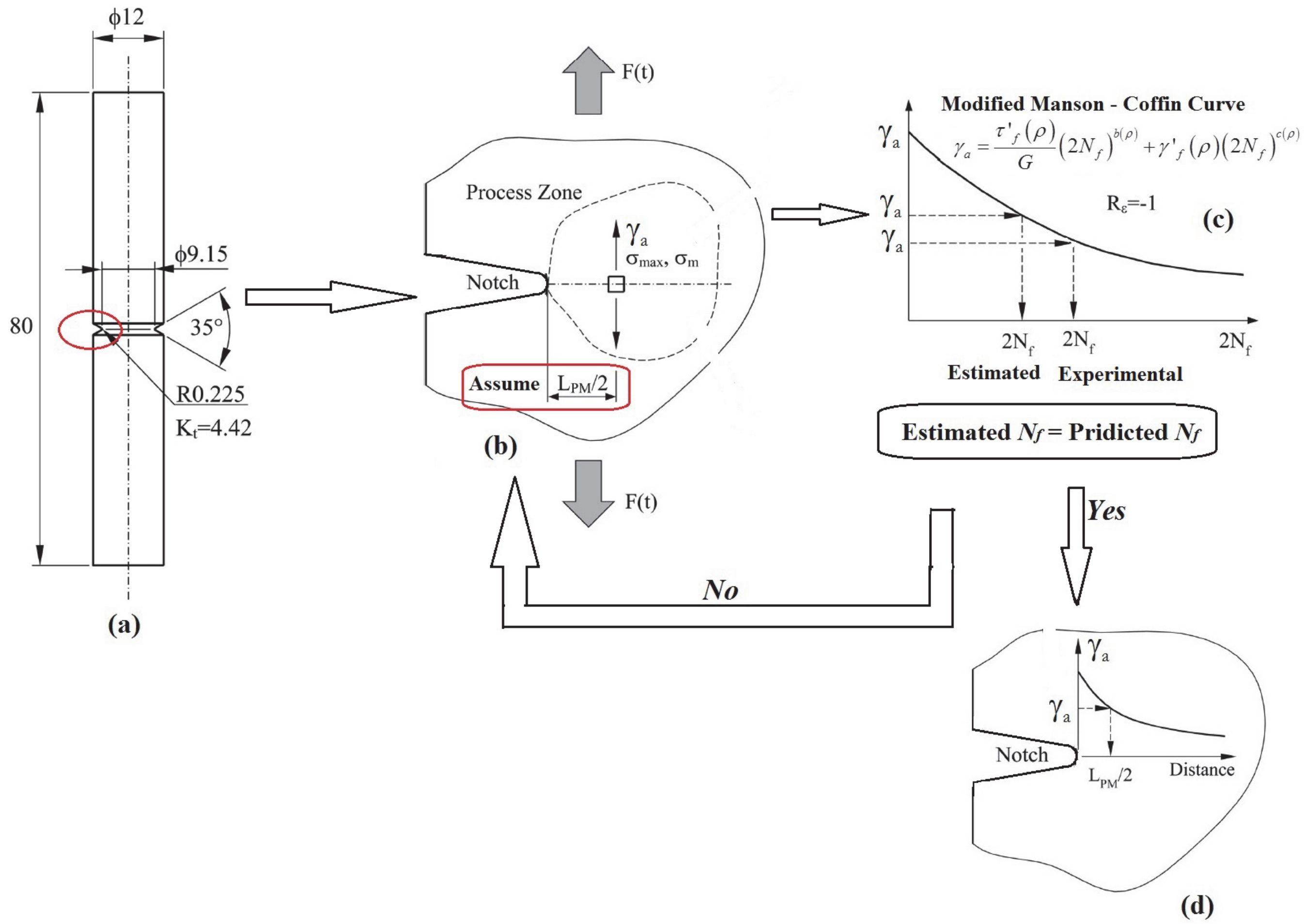
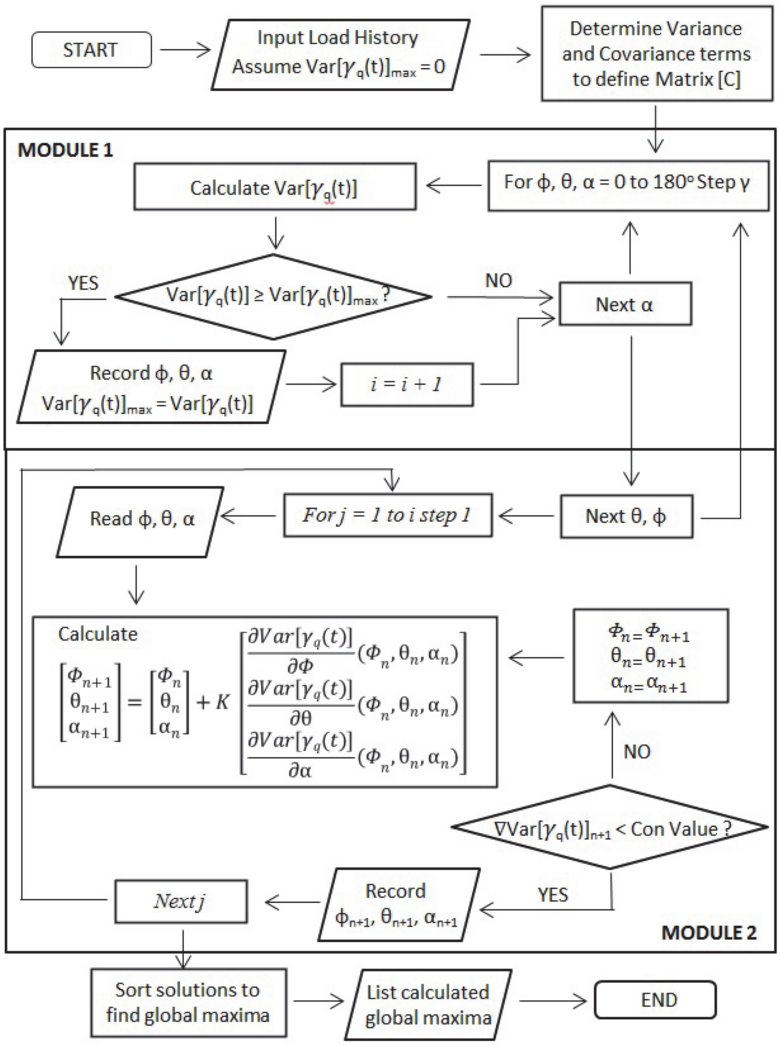


Figure 3: Summary of a methodology proposed to determine the critical distance – Point Method L_{PM} [6].

ORIENTATION OF THE CRITICAL PLANE AND MAXIMUM VARIANCE METHOD

Predicting fatigue lifetime of a component mainly depends on the accuracy in determining the orientation of the critical plane as well as the stress/strain components relative to that plane. The hypothesis is formed that the critical plane, which is defined as the plane experiencing maximum shear strain amplitude [1], coincides with the crack initiation plane.
Recently, Susmel [15] has formalised a numerical algorithm to explore the orientation of the critical plane been applied along with the stress based approach. The algorithm is then reformulated in terms of cyclic strain that summarised in

Fig.4. This algorithm is developed based on the multi-variable optimisation method known as a Gradient Ascent Method [15].



$$\begin{bmatrix}\Phi_{n+1}\\ \theta_{n+1}\\ \alpha_{n+1}\end{bmatrix} = \begin{bmatrix}\Phi_{n}\\ \theta_{n}\\ \alpha_{n}\end{bmatrix} + K \begin{bmatrix}\dfrac{\partial Var[\gamma_q(t)]}{\partial \Phi}(\Phi_n, \theta_n, \alpha_n)\\ \dfrac{\partial Var[\gamma_q(t)]}{\partial \theta}(\Phi_n, \theta_n, \alpha_n)\\ \dfrac{\partial Var[\gamma_q(t)]}{\partial \alpha}(\Phi_n, \theta_n, \alpha_n)\end{bmatrix}$$

Figure 4: Flowchart to explore the orientation of the critical plane [15].

In order to explore the critical plane, a notched component is considered subjected to in-service cyclic load as shown in Fig. 5a. Then, by taking full advantage of the theory of critical distance, multiaxial local strain history is determined at a specific distance from the notch apex equal to critical distance. The local strain history is described with the following strain tensor (see Fig. 5b):

$$[\varepsilon(t)] = \begin{bmatrix} \varepsilon_x(t) & \dfrac{\gamma_{xy}(t)}{2} & \dfrac{\gamma_{xz}(t)}{2} \\ \dfrac{\gamma_{xy}(t)}{2} & \varepsilon_y(t) & \dfrac{\gamma_{yz}(t)}{2} \\ \dfrac{\gamma_{xz}(t)}{2} & \dfrac{\gamma_{xz}(t)}{2} & \varepsilon_z(t) \end{bmatrix} \tag{4}$$

In the above equation, $\varepsilon_x(t)$, $\varepsilon_y(t)$ *and* $\varepsilon_z(t)$ are normal strain components, whereas $\gamma_{xy}(t), \gamma_{yz}(t),$ *and* $\gamma_{xz}(t)$ are shear strain history. According to the maximum variance method, shear strain amplitude is described by the following Eq. 5:

$$\gamma_a = \sqrt{2.Var\left[\gamma_q(t)\right]} \tag{5}$$

The variance of shear strain can be determined directly by using the simple form of Eq. 6, [15]:

$$Var\left[\gamma_q(t)\right] = d^T\,[C]\,d \tag{6}$$

According to Fig 5c, for specific values of angles ϕ, θ and α, the terms *d* and *[C]* in Eq. 6 can be simply defined with the following definition:

$$d = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left[\sin(\theta)\sin(2\phi)\cos(\alpha)+\sin(\alpha)\sin(2\theta)\cos(\phi)^2\right] \\ \frac{1}{2}\left[-\sin(\theta)\sin(2\phi)\cos(\alpha)+\sin(\alpha)\sin(2\theta)\sin(\phi)^2\right] \\ -\frac{1}{2}\sin(\alpha)\sin(2\theta) \\ \frac{1}{2}\left[\sin(\alpha)\sin(2\phi)\sin(2\theta)-\cos(\alpha)\cos(2\phi)\sin(\theta)\right] \\ \sin(\alpha)\cos(\phi)\cos(2\theta)+\cos(\alpha)\sin(\phi)\cos(\theta) \\ \sin(\alpha)\sin(\phi)\cos(2\theta)-\cos(\alpha)\cos(\phi)\cos(\theta) \end{bmatrix}$$

$$[C] = \begin{bmatrix} V_x & C_{x,y} & C_{x,z} & C_{x,xy} & C_{x,xz} & C_{x,yz} \\ C_{x,y} & V_y & C_{y,z} & C_{y,xy} & C_{y,xz} & C_{y,yz} \\ C_{x,z} & C_{y,z} & V_z & C_{z,xy} & C_{z,xz} & C_{z,yz} \\ C_{x,xy} & C_{y,xy} & C_{z,xy} & V_{xy} & C_{xy,xz} & C_{xy,yz} \\ C_{x,xz} & C_{y,xz} & C_{z,xz} & C_{xy,xz} & V_{xz} & C_{xz,yz} \\ C_{x,yz} & C_{y,yz} & C_{z,yz} & C_{xy,yz} & C_{xy,yz} & V_{yz} \end{bmatrix} \tag{7}$$

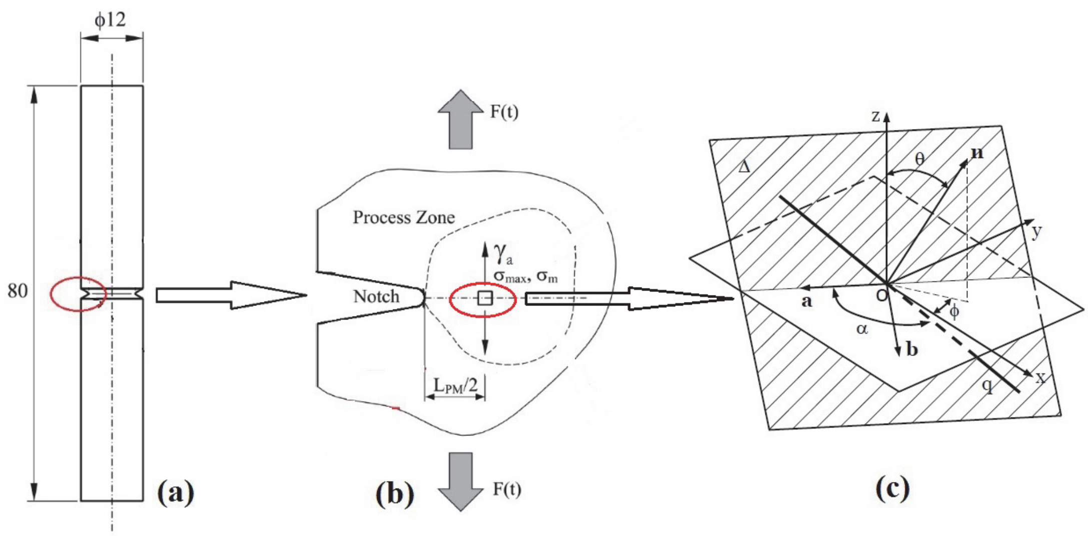


Figure 5: Orientation of the Critical Plane [1].

Now, to estimate the variance and covariance terms in matrix [C], consider a time variable strain components $\varepsilon_i(t)$ and $\varepsilon_j(t)$ that described over time period [0,T]. $\varepsilon_{i,m}$ and $\varepsilon_{j,m}$ are the mean values of strain histories, the variance and covariance of $\varepsilon_i(t)$ and $\varepsilon_j(t)$ can be determined by using the following definitions:

$$Var[\varepsilon_i(t)] = \frac{1}{T}\int_0^T [\varepsilon_i(t) - \varepsilon_{i,m}]^2\, dt \tag{8}$$

$$CoVar[\varepsilon_i(t), \varepsilon_j(t)] = \frac{1}{T}\int_0^T [\varepsilon_i(t) - \varepsilon_{i,m}].\left[\varepsilon_j(t) - \varepsilon_{j,m}\right] dt \tag{9}$$

After the orientation of the maximum critical plane is indicated, then, by taking full advantage of the maximum variance method [1], all those stress amplitudes relative to the critical plane is calculated. The hypothesis is postulated that fatigue failure is proportional to the variance of cyclic strain at a critical point. From a statistical viewpoint, the variance of variable amplitude cyclic stress/strain is the squared deviation from the mean value. According to the well-documented evidence [1], the above mentioned approach has given satisfactory results when applied in terms of long-life fatigue. In the light of the reliable solution obtained in the stress based critical plane, the maximum variance concept was reformulated for being applied in strain based strategy.

After exploring the orientation of the critical plane, maximum variance and normal unit vectors on the critical plane are used to determine the required mean stress/strain values and amplitudes. Strictly speaking, for components under constant amplitude CA fatigue load, the stress/strain values of interest related to the critical plane can directly be found using Eqs. 10-11 [1]:

$$\tau_a = \frac{1}{2}\left(\tau_{MV,\max} - \tau_{MV,\min}\right) \qquad \tau_m = \frac{1}{2}\left(\tau_{MV,\max} + \tau_{MV,\min}\right) \tag{10}$$

$$\sigma_{n,a} = \frac{1}{2}\left(\sigma_{n,\max} - \sigma_{n,\min}\right) \qquad \sigma_{n,m} = \frac{1}{2}\left(\sigma_{n,\max} + \sigma_{n,\min}\right) \tag{11}$$

where: γ_a and τ_a are the shear strain and stress amplitudes relative to the critical plane. γ_m and τ_m are the mean value of shear strain and stress. $\sigma_{n,a}$ and $\sigma_{n,m}$ are the normal stress amplitude and normal mean value. $\gamma_{MV,max}$ and $\gamma_{MV,min}$ are the maximum and minimum variance of shear strain history respectively. $\tau_{MV,max}$ and $\tau_{MV,min}$ are used to denote the maximum and minimum variance of shear stress history. $\sigma_{n,max}$ and $\sigma_{n,min}$ are the maximum and minimum normal stress history respectively. All the above described variables are relative to the critical plane.

However, in those situations involving variable amplitude cyclic load, the corresponding stress/strain state on the critical plane that damage the component are also variable. The mean value and stress/strain amplitudes of interest related to the critical plane can directly be calculated by the following definitions 12-14 [1 & 15]:

$$\tau_m = \frac{1}{T}\int_0^T \tau_{MV}(t).dt \qquad Var.[\tau_{MV}(t)] = \frac{1}{T}\int_0^T [\tau_{MV}(t) - \tau_m]^2\, dt \tag{12}$$

$$\sigma_{n,m} = \frac{1}{T}\int_0^T \sigma_n(t).dt \qquad Var.[\sigma_n(t)] = \frac{1}{T}\int_0^T [\sigma_n(t) - \sigma_{n,m}]^2\, dt \tag{13}$$

$$\tau_a = \sqrt{2.Var\left[\tau_{MV}(t)\right]} \qquad \sigma_{n,a} = \sqrt{2.Var\left[\sigma_n(t)\right]} \tag{14}$$

CLASSIS RAIN FLOW COUNTING METHOD

Real engineering components are often subjected to a complex cyclic load that either constant or variable amplitude. For the constant amplitude applied loads, the calculated stress/strain amplitude can be used straightforward to estimate number of cycles to failure. However, if the applied nominal loads are changed with time, the local stress/strain history are variable amplitude and the solution may be further complicated. One of the main objectives of this study is to investigate variable amplitude fatigue lifetimes of a component. Therefore, the most complex issue that needs to be addressed properly is the cycle counting strategy. Examination of the state of the art found that the classic Rain-Flow Method [16] is the best accurate methodology that gives a satisfactory prediction to account the cycles in variable amplitude loading [3, 17-19] and then, leading to better fatigue lifetime predictions.

The classic Rain-Flow method is a cycle counting rule that is used to define whether cycles is formed in every three consecutive points of the time history stress/strain amplitudes. A typical variable amplitude stress/strain history is presented in Fig.6. The differences between absolute value of first two consecutive points $|\Delta S_1|$ need to be compared with the difference between second and third points $|\Delta S_2|$.

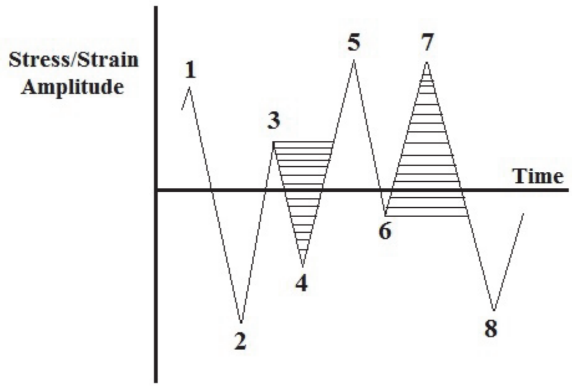


Figure 6: Rain Flow Cycle Counting.

If $|\Delta S_1|$ is greater than $|\Delta S_2|$, no cycle is considered, otherwise cycle is counted. The same process should be followed until all cycles are identified.

$\Delta S_1 = \|S_1 - S_2\|$ and $\Delta S_2 = \|S_2 - S_3\|$	$\Delta S_1 > \Delta S_2$	No cycle is considered
$\Delta S_3 = \|S_3 - S_4\|$ and $\Delta S_4 = \|S_4 - S_5\|$	$\Delta S_3 \le \Delta S_4$	Cycle 3-4 is counted
$\Delta S_6 = \|S_6 - S_7\|$ and $\Delta S_7 = \|S_7 - S_8\|$	$\Delta S_6 \le \Delta S_7$	Cycle 6-7 is counted

It is worth mentioning here that according to the Rain Flow rules, before start cycle counting, rearrangement is required in the stress/strain history so that it starts either in the highest peak or the lowest valley whichever is greater in absolute value [20], and a new stress/strain-time history is arranged. Then, three-point rain flow cycle counting method is applied on every three consecutive points in the new generated stress or strain history. As illustrated in Fig. 6, two data points is extracted to form the first cycle, and a new state history is generated by connecting the points before and after the cycle. The subsequent step is repeating the above mentioned cycle extraction technique on every three consecutive points to identify another cycle and generating a new stress/strain history. This process is continued until all cycles are formed.

LIFETIME ESTIMATION BY USING THE DEVELOPED APPROACH:

From the application point of view, this chapter summarised the procedure being followed to validate the developed approach by integrating with the pre-experimentally investigated notched samples from other literature [6]. Generally, the developed approach methodology is briefly illustrated in Fig.8 and presented in the last sections with a great detail. For the validation view point, pre-investigated cylindrical notched samples of three different notch root radius were considered: 0.225mm, 1.2mm, and 3.0mm as shown in Fig 9e. The specimens were tested under fully reversed constant/variable amplitude uniaxial nominal loads with load ratio (R) equals -1. The samples were made of C40 material. All mechanical and fatigue properties were summarized in Tab. 1. The stabilized stress-strain relationship and Manson-Coffin curve were generated under fully reversed axial load. The corresponding local elasto-plastic triaxial stress/strain history was obtained by post-processing the FE model using ANSYS® software. The solved model allows the corresponding stress/strain sequences to be determined at any nodes of interest on the sample. From the accuracy point of view, the element size of FE model at notched bisector was gradually refined and solved under simple linear-elastic behaviour until convergence level. The meshing size at notched region was described by elements with 0.005mm dimension. Then, by taking full advantage of the TCD being applied in terms of Point Method [4], the effective stress/strain history was determined at a given distance from the notch apex (see Fig. 7a). In the present investigation, the critical distance was estimated by considering a number of experimental results generated by testing notched specimens under constant amplitude uniaxial fatigue loading [9], the procedure was described in section 3 (*Theory of critical distance to quantify the effective local stress/strain state*).

σ_{UTS} (MPa)	σ_y (MPa)	E (MPa)	K' (MPa)	n'	σ'_f (MPa)	ε'_f	b	c	b_o	c_o
852	672	209000	773.3	0.0951	710.6	0.3641	-0.0568	-0.5794	-0.023	-0.98

Table 1: Mechanical and Fatigue properties of C40 steel [9].

The next significant steps is exploring orientation of the critical plane based on the Gradient Ascent Method [1, 15] and estimate the mean and stress/strain amplitudes on the critical plane. The MVM are used to determine the stress/strains amplitudes as shown in Fig. 7b. From a computational view point, MATLAB computer software was used to perform the analysis by exploring orientation of the critical plane and quantify all stress/strain values on that plane. The procedure of finding a critical plane and relative stress/strain amplitudes were summarised clearly in section 4 (*Orientation of the critical plane and maximum variance method*).

Under constant amplitude load history, after indicating the orientation of the critical plane, all relative shear stress/strain amplitudes and normal stresses can directly be calculated according to the Eqs. 3-5. Those stress/strain values allow the ratio ρ in Eq.2 to be determined. Then by using the modified Manson-Coffin curve, number of cycles to failure can be estimated. However, under variable amplitude fatigue loading, the direction of maximum variance of the resolved shear strain is used to perform the cycle counting based on the classic Rain-Flow method [1, 16] (see Fig. 5i-j). The estimated shear stress amplitude and maximum normal stress can be used to determine a stress ration ρ to modify Manson-Coffin curve. Finally, number of cycles to failure can be estimated by using Eq. 15 [1]:

$$N_{f,e} = \frac{D_{cr}}{D_{tot}} \sum_{i=1}^{j} n_i \tag{15}$$

where:

D_{tot} is the total amount of fatigue damage that can be defined by Eq. 16,

D_{cr} is the critical value of the damage sum,

n_i is the number of cycles at the i-th strain level.

The classical theory formalised by Palmgren and Miner [21] suggests that fatigue failures take place as soon as the critical value of the damage sum equals unity. However, according to several experimental investigations, Sonsino [22] has shown that the average value of D_{cr} is 0.27 for steel and 0.37 for aluminium.

$$D_{tot} = \sum_{i=1}^{j} \frac{n_i}{N_{f,i}} \tag{16}$$

where:

$N_{f,i}$ is the number of cycles to failure for each strain amplitude being considered.

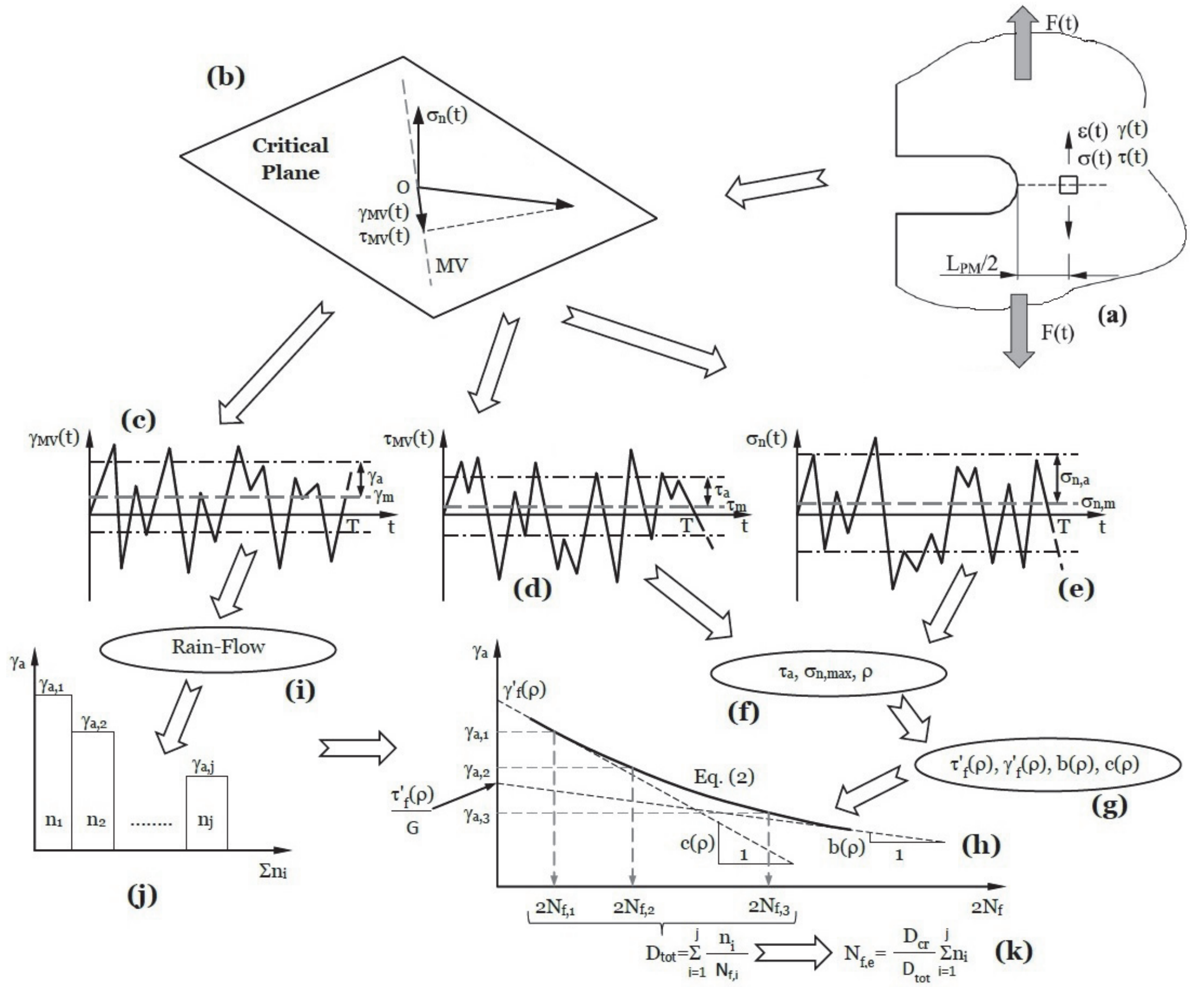


Figure 7: In-field use of the developed approach: Methodology [1].

VALIDATION BY EXPERIMENTAL DATA

To check the accuracy and reliability of the devised approach, the model was validated against a number of experimental results reported in Ref [9]. This validation exercise involved a systematic study of various elastic/elasto-plastic loading conditions in relation to the fatigue life, particularly when the stress/strain amplitude varies between loading sequences in multiple step loading. As a preliminary stage, experimental results from 70 previously-tested cylindrical notched specimens were taken directly from Refs [9], the notch root radius are 0.225mm, 1.2mm, and 3.0mm as shown in Fig 8e. The samples were tested under uniaxial sinusoidal loading waves as shown in Figs 8a-d, where $\Sigma_{a\text{-}max}$ is the amplitude of the most damaging cycle in the spectrum, and $\Sigma_{a\text{-}i}$ is the amplitude of the i^{th} cycle (both expressed in terms of nominal net stresses). Three types of load spectra were considered, which are a simple overloading case (OL), a concave downwards spectrum (CDS), and a concave upwards spectrum (CUS). Those cases represent the potential variable amplitude applied loads on real engineering components. The load ratio R was invariably equal to -1. The material being tested was C40 carbon steel. The required material properties were taken from the aforementioned published work [9]. Fig. 8e shows the geometries of the investigated specimens.

As far as the numerical stress analysis is concerned, under constant amplitude loading, ten virtual cycles were considered in the theoretical analysis to confirm the stress-strain response reaches a stabilized configuration level [23]. Kinematic hardening was used for the elasto-plastic deformation [24].

Due to assumptions made while choosing the experimental data in the validation process, particularly while identifying material's fatigue data that had not been found within the original paper, a narrower error band was defined for the comparison chart. The estimated $N_{f,e}$ versus the experimental N_f were arranged in Fig. 9. It can be observed that most of the estimated points are located within the designated error interval. Subsequently, from the validation view point, it can be pointed out that the elasto-plastic model was in a position to adequately predict the number of cycles to failure. Moreover, Fig. 9 offers evidence that the devised technique is capable of successfully estimate fatigue lifetime. To sum up, the experimental results are close to the estimated values that are generated using the devised methodology and gave a similar pattern. This demonstrates applicability of the devised approach.

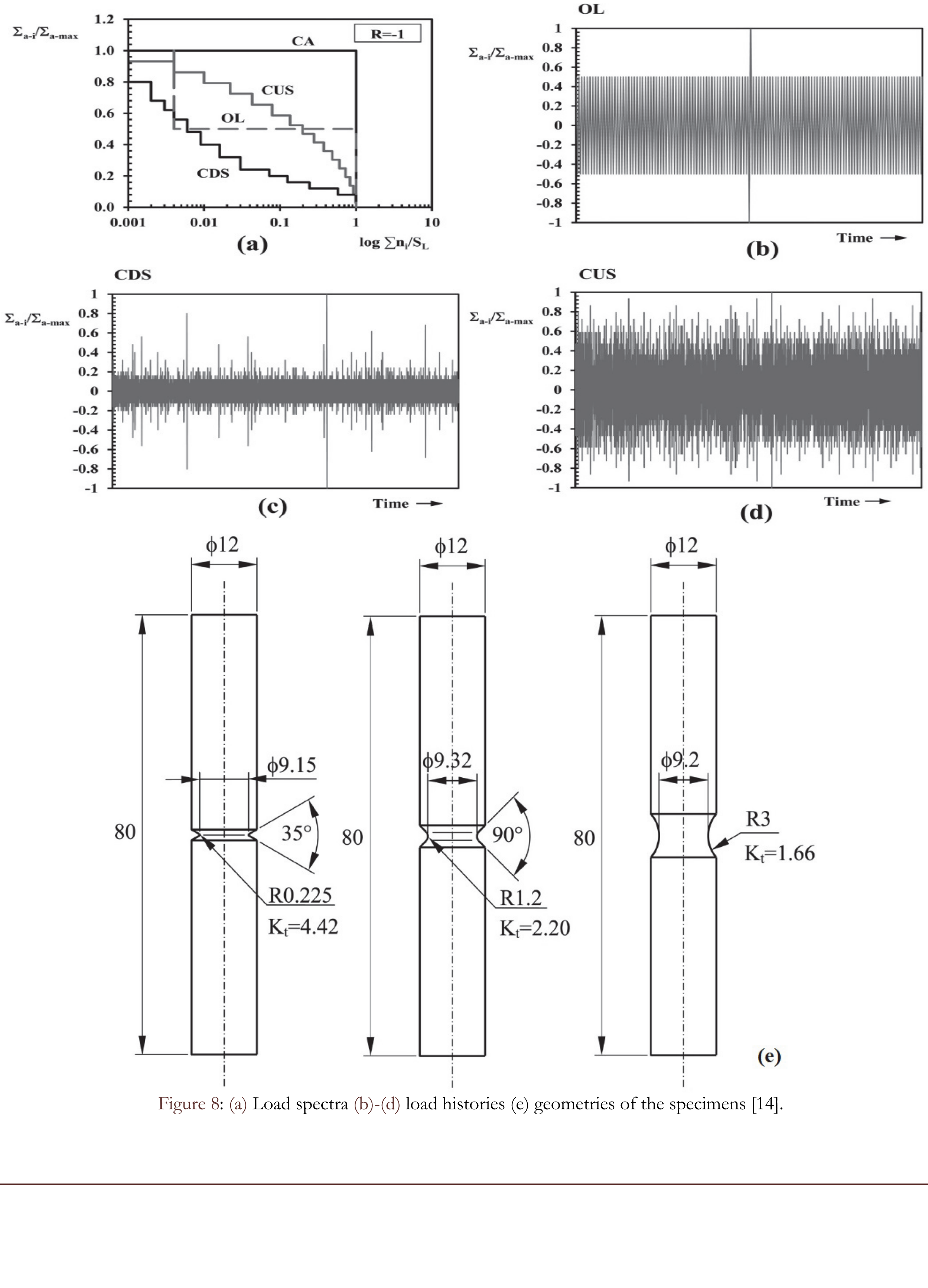


Figure 8: (a) Load spectra (b)-(d) load histories (e) geometries of the specimens [14].

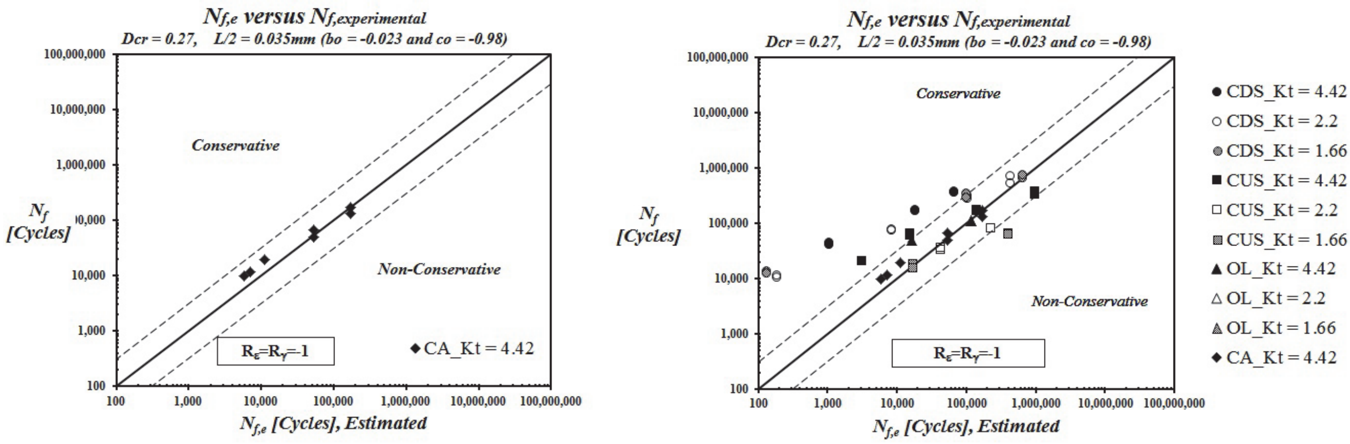


Figure 9: Accuracy of the Modified Manson-Coffin Curve versus experimental results.

CONCLUSION

1. It can be concluded that the entire range of the developed approach, produced by either constant amplitude or multiple stepwise loading is satisfactory suitable for predicting fatigue lifetime of a notched metallic materials by taking full advantage of the MMCCM applied along with the critical distance approach. This demonstrates that the formalised approach can be successful in estimating longevity of the notched components.
2. Strain-based approach MMCCM, can offer a reliable solution to the local triaxial stress/strain based state, being applied using the critical distance theory.
3. The formalised approach can consider the detrimental effect of non-zero mean stress and degree of multiaxiality, different from other techniques that rationally account this effect.

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