Effect of flexural crack on plain concrete beam failure mechanism A numerical simulation

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ABSTRACT. The flexural failure of plain concrete beam occurs along with development of flexural crack on beam. In this paper by using ABAQUS, mechanism failure of plain concrete beam under three steps have been simulated. The cracking moment has been analytically calculated and applied on the both sides of the fixed beam, and flexural crack has been simulated on beam. Displacement, von Mises, load reaction, displacement-crack length, von Mises-crack length and von Mises-displacement of beams have been graphical depicted. Results indicated that, the flexural crack governs beam mechanism failure and its effects on beam resistance failure. It has been found that the flexural crack in initial stage it developed slowly and changes to be fast at the final stage of collapsing beam due to reduction of the flexural resistance of beam. Increasing mechanical properties of concrete, collapse displacement is reduced.

KEY WORDS: Stress; Crack; Length; Concrete interaction; FEM.

INTRODUCTION

The fracture mechanics concerns the crack development in materials during load applying in any directions. Brittle fracture material cracks occur in excess tensile stress more than tensile strength. For brittle solids material, such as plain concrete, it leads to rapid crack development if applied load is higher than fracture resistance and toughness, subsequently resulting in a failure of the structural elements and/or whole structure. Therefore, linear elastic fracture mechanics is supportive to understand mechanism of elastic stress distributions around cracks and their development; in this regard the numerical simulation is used in study of such problems.

The fracture mechanics has been started by Griffith, it is based on the energy-based theory of failure [1]. The flexural crack on plain concrete beam has been experimentally investigated [2]. The state-of-the-art, prediction of three-dimensional crack propagation paths has been numerically simulated to observe the damage [3]. It has been reported

ATENA finite element (FE) as software used for concrete crack simulation, and the fracture energy values calculated [4]. There are several studies on predict crack growth in concrete [5-7]. A link between the averaged strain energy density (SED) approach and the peak stress method in the case of cracks subjected to mixed mode (I+II) loading, has been investigated [8]. There are several software such as ABAQUS, ADAPCRACK3D, ANSYS and ATENA finite element (FE) used for crack modeling and design of different materials [3-7, 9-14]. In this paper, the ABAQUS has been used to characterize flexural failure mechanism of beam subjected to the cracking moment and to understand stress transfer in plain concrete during crack development. A numerical analysis has been made in order to realize fracture resistance and toughness mechanism of plain concrete beam in crack extension, considering displacement, von Mises, load reaction, displacement-crack length, von Mises-crack length and von Mises-displacement of beams.

THEORETICAL CONCEPT

Linear Elastic Fracture Mechanics (LEFM) considers the fundamentals of linear elasticity theory. The flexural crack is simulated at the center of the beam. It initiates from bottom and it terminates at the top. The plain concrete is an isotropic material. It exhibits the same behavior in all directions, while subjected to the load.

In isotropic linear elastic material, the stress and strain, in three dimensional case are defined by $[\sigma]$ and $[\varepsilon]$ [15].

$$[\sigma]=\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{bmatrix} \text{ and } [\varepsilon]=\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{bmatrix}$$

where matrix $[C]$ is called the elastic elastic moduli matrix then $[\sigma]=[C][\varepsilon]$.

$$[C]=\left[\frac{E}{(1+\upsilon)(1-\upsilon)}\right]\begin{bmatrix}1-\upsilon & \upsilon & \upsilon & 0 & 0 & 0\\ \upsilon & 1-\upsilon & \upsilon & 0 & 0 & 0\\ \upsilon & \upsilon & 1-\upsilon & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{(1-2\upsilon)}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{(1-2\upsilon)}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\upsilon)}{2}\end{bmatrix}$$

For υ and E in terms of K and G, then C will be;

$$[C]=\begin{bmatrix}K+\frac{4}{3}G & K-\frac{2}{3}G & K-\frac{2}{3}G & 0 & 0 & 0\\ K-\frac{2}{3}G & K+\frac{4}{3}G & K-\frac{2}{3}G & 0 & 0 & 0\\ K-\frac{2}{3}G & K-\frac{2}{3}G & K+\frac{4}{3}G & 0 & 0 & 0\\ 0 & 0 & 0 & G & 0 & 0\\ 0 & 0 & 0 & 0 & G & 0\\ 0 & 0 & 0 & 0 & 0 & G\end{bmatrix}$$

Subsequently can write:

$$[\varepsilon]=[C]^{-1}[\sigma]=[D][\sigma]$$

where the elastic compliance matrix, $[D]$, is given by inverse of matrix $[C]$

$$[D]=\frac{1}{E}\begin{bmatrix} 1 & -\upsilon & -\upsilon & 0 & 0 & 0 \\ -\upsilon & 1 & -\upsilon & 0 & 0 & 0 \\ -\upsilon & -\upsilon & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1+\upsilon}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1+\upsilon}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1+\upsilon}{2} \end{bmatrix}$$

According to [16], the strain can be categorized under three types which are;

Natural or true strain; $[\varepsilon]=\int_{l_0}^{l}\frac{dl}{l}=\ln\left(\frac{l}{l_0}\right)$

Engineering strain; $[\varepsilon]=\frac{\Delta l}{l_0}$

Hooke's law strain; $[\varepsilon]=\frac{\sigma}{E}$

The Hooke's law strain has been used in numerical simulation giving a linear stress-strain relationship. This strain energy density is the required energy to deforming the material. Fracture toughness is the elastic strain energy density known as resilience and it is defined as $W=\int_0^{\varepsilon}\sigma d\varepsilon$, this expression represents an elastic behavior up to the yield strain [16].

MODELING SETUP FOR NUMERICAL SIMULATION

The crack mode is an important issue in fracture mechanics. The crack can be categorized in three modes; mode 1: opening or tensile mode, mode 2: sliding or in-plane shear mode, mode 3: out of plane tearing. In this paper tensile mode (mode 1) has been simulated. The tensile mode is the most common mode where crack occurs on the plane characterized by maximum tensile stress. The cases of failure are typical crack under tensile mode with different length, start at center of calibrated length beam, initiating at the bottom and ending at the top. The fracture toughness of beam, with resist progressive tensile crack extension has been modeled considering linear elastic fracture mechanics concept. The stress intensity modification in developing crack was numerically simulated, and crack development and stress intensity are studied. The Material properties have been referred to literature, and cracking moment design calculated according to ACI code. The cracking moment is applied on the beam. The mechanical property of ultra-high strength concrete (UHPC) and normal strength concrete (NSC) have been indicated in Tab. 1. The SI units have been used in numerical simulation (Tab. 2). Three steps have been made in crack development on body of plain concrete beam. The crack started in 25 % of beam length from bottom and subsequently up to 50 % and finally fully cracked and collapsing. At each stage crack remain stable in order to understand of the beam mechanism failure. The geometry and boundary condition are shown in Figs. 1-2.

Specimen	Young's modulus (MPa)	Poisson's ratio	Ultimate strength (MPa)	Ultimate strain	Density (kg/m^3)
NSC	49,195	0.24	56.6	0.0014	2440.5
UHPC	51,503	0.20	128.9	0.0025	2424.9

Table 1: Concrete material properties for elastic analyses [17].

Length	Force	Mass	Time	Stress	Energy	Density
mm	N	t	s	MPa (N/mm^2)	mJ	t/m^3

Table 2: The following dimensions were used in numerical analysis.

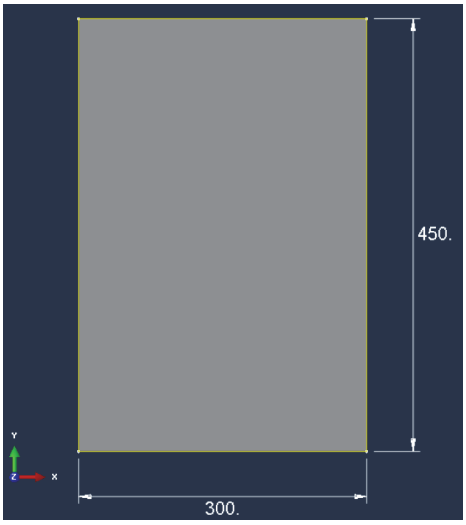


Figure 1: The plain concrete beam cross section with length of 4000 (mm) used in analytical and numerical analysis, all dimension in (mm).

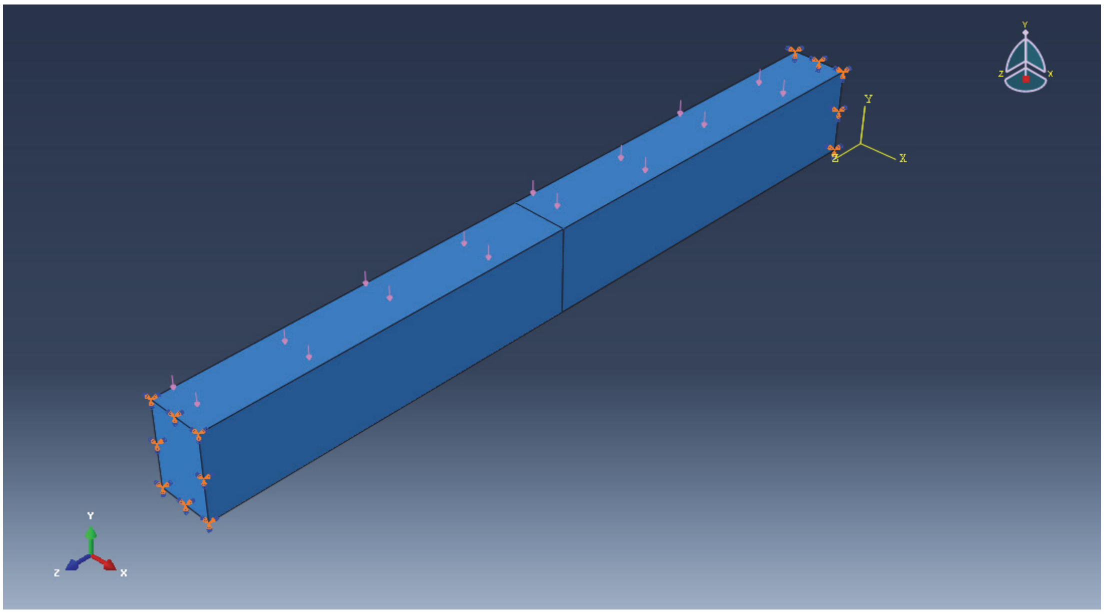
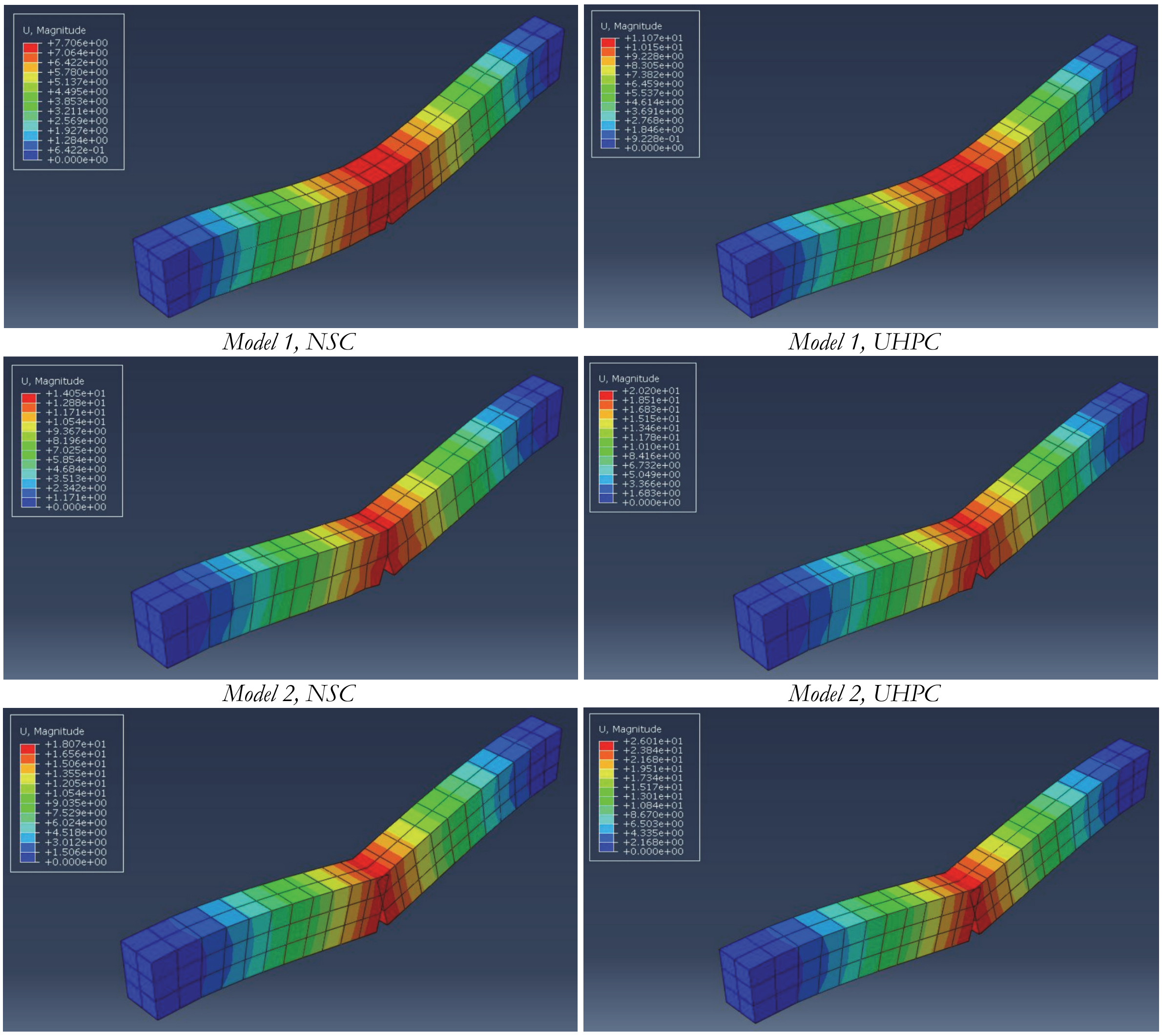


Figure 2: Crack simulated on two sided of fixed beam, and subjected to uniform distributed load.

RESULTS AND DISCUSSION

The study based on stress transfer (or release) across the beam during occurrence of crack development on beam from bottom to top has been performed. The numerical simulation has been made according to fracture strengths of plain concrete beam, and simulation of crack mode I and flexural failure of the beam. The fracture resistance and toughness mechanism plain concrete were simulated. The plain concrete beam, was modeled by normal and high strength concrete. Following this modeling, the beam was loaded under cracking moment. Increasing the length of the crack, and the movement of stress along the beam, fracture resistance and toughness of the beam are reduced. The tensile strength has directly related to the fracture resistance and the stress was distributed according to length of crack. It has been observed that the crack develops with stress transfer and effect on stress, von Mises, load reaction, displacement and length of crack. The material's fracture resistance is important in crack extension. The fracture characteristics of beam influence the stress and deformation behavior of beam. The fracture characteristics depend on strength of material. The crack extension depends on elastic strain energy at fracture. With the increase the toughness of plain concrete the brittleness of concrete is reduced and the fracture resistance increases and caused the change of stress path during the beam is under cracking extension process. The fracture toughness of a material depends on material mechanical property. The maximum concrete interaction was observed in fully collapsed beam, it depends on mechanical property of concrete. Accordingly, a design engineer would be understand that this concept help them to predict failure mechanism of concrete structure, which is failed under flexural failure and considering deformation and fracture resistance of the beam material. The strength of the material effects on stress path, fracture resistance and toughness mechanism of plain concrete beam. The failure occurs when elastic energy-release rate equals the rate change in material resistance and completed flexural crack shape on the beam. In Figs. 3-5 are shown displacement, von Mises and load reaction of different models up to final failure. It has been shown that the maximum displacement occurs in the center of the beam. Due to fixed both side of the beam, there is any displacement observed at the end of the beam. In the NSC and UHPC beams, stress and load reaction path are the same, but displacement, von Mises and load reaction magnitude are different. It is observed that there are direct influences between crack length and displacement, von Mises and load reaction of the beam.

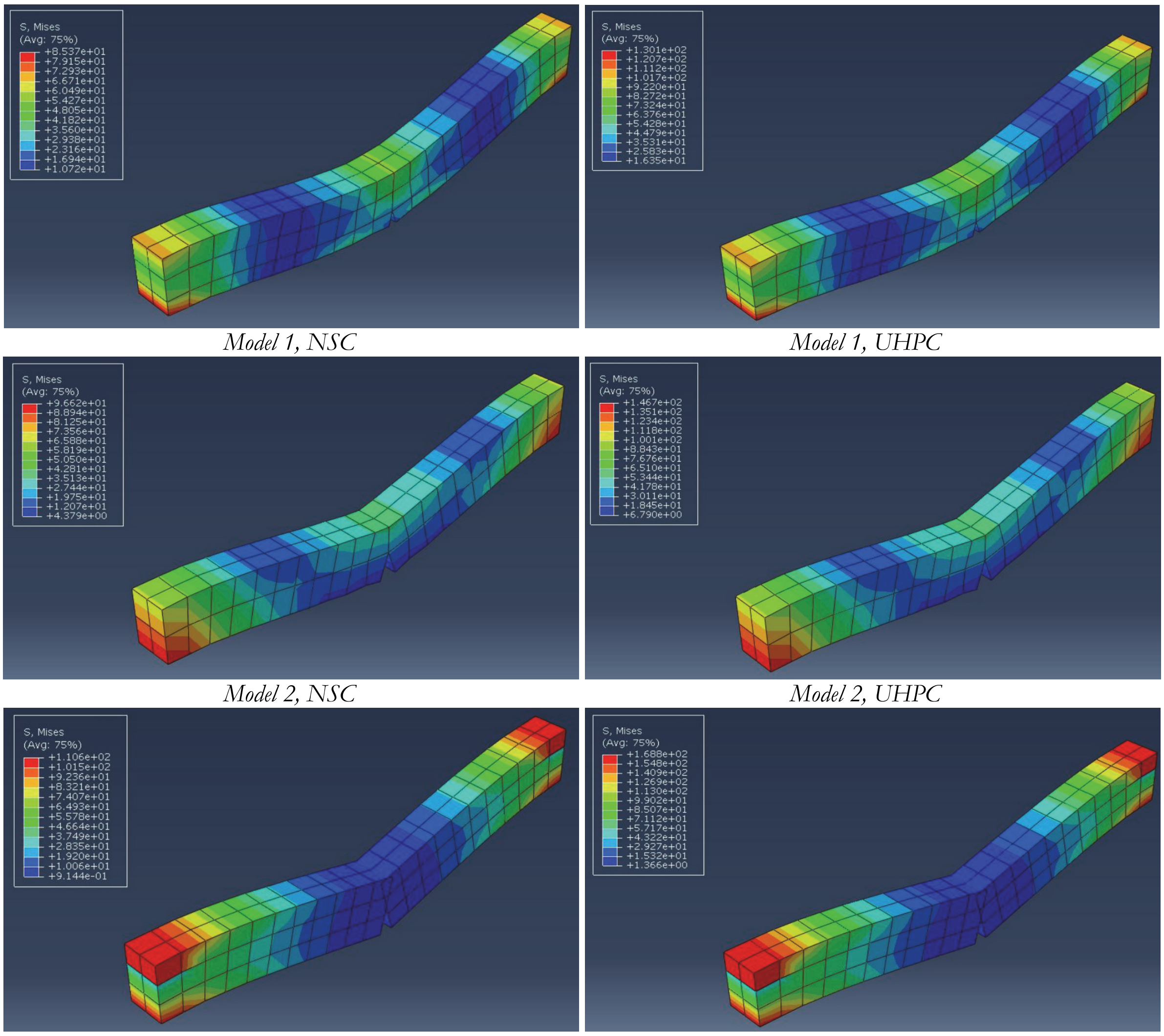


Model 1, NSC *Model 1, UHPC*

Model 2, NSC *Model 2, UHPC*

Model 2, NSC *Model 3, UHPC*

Figure 3: Displacement of different models with final failure Vs crack length.



Model 1, NSC *Model 1, UHPC*

Model 2, NSC *Model 2, UHPC*

Model 2, NSC *Model 3, UHPC*

Figure 4: Von Mises or Distortion-energy criterion of different models with final failure.

According to Tabs. 3-5 and Figs. 6-8 in cracking moment conditions, the UHPC exhibits more displacement before collapse. In both models, the maximum displacement occurs when the length of crack is less than the half of the beam cross section. Furthermore, both models have the same cross sections but, after crack cross of 25% of the beam's height, increasing 30% of von Mises stress, beams are collapsed due to rapid reduction of fracture resistance and toughness. The mechanical properties of concrete are important in mechanism failure of the beam and displacement-crack length, von Mises-crack length and von Mises-displacement of beams.

NSC		*UHPC*	
Crack length (mm)	Displacement (mm)	Crack length (mm)	Displacement (mm)
75	7.71	75	11.07
150	14.05	150	20.20
300	18.07	300	26.01

Table 3: Maximum displacement Vs crack length with applied constant load.

NSC		*UHPC*	
Crack length (mm)	von Mises (MPa)	Crack length (mm)	von Mises (MPa)
75	85.37	75	130.1
150	96.62	150	146.7
300	110.6	300	168.8

Table 4: Maximum von Mises Vs crack length with applied constant load.

NSC		*UHPC*	
von Mises (MPa)	Displacement (mm)	von Mises (MPa)	Displacement (mm)
85.37	7.71	130.1	11.07
96.62	14.05	146.7	20.20
110.6	18.07	168.8	26.01

Table 5: Maximum von Mises Vs displacement.

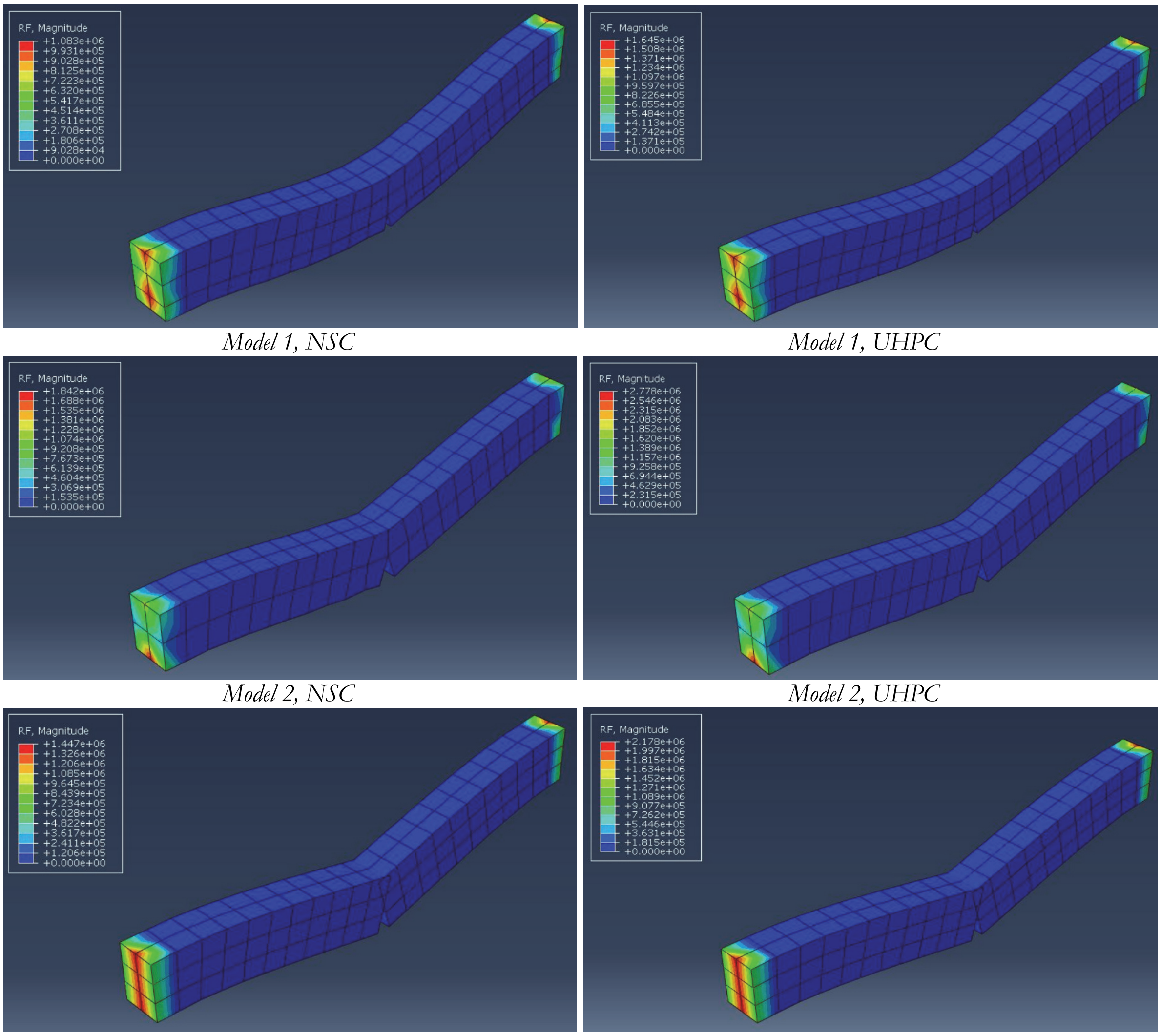


Figure 5: Load reaction of different models with final failure.

The Tab. 6 and Fig. 9, show the pattern of reaction load-displacement in UHPC and NSC are same, with different magnitude. And with collapse of beam, the reaction load decreased, and maximum reaction load occurs when the length of crack is in the half of the beam.

NSC		*UHPC*	
Reaction load (N)	Displacement (mm)	Reaction load (N)	Displacement (mm)
1083000	7.71	1645000	11.07
1842000	14.05	2778000	20.20
1447000	18.07	2178000	26.01

Table 6: Maximum reaction load at fixed support Vs displacement in the center of the beam at different models.

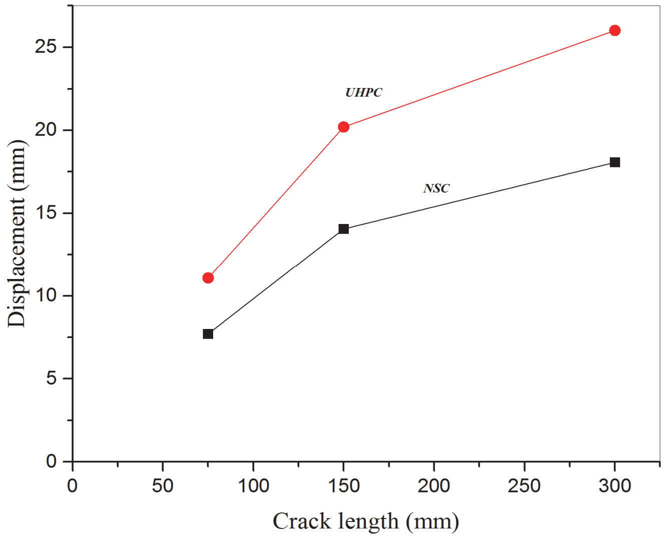


Figure 6: Maximum displacement Vs crack length.

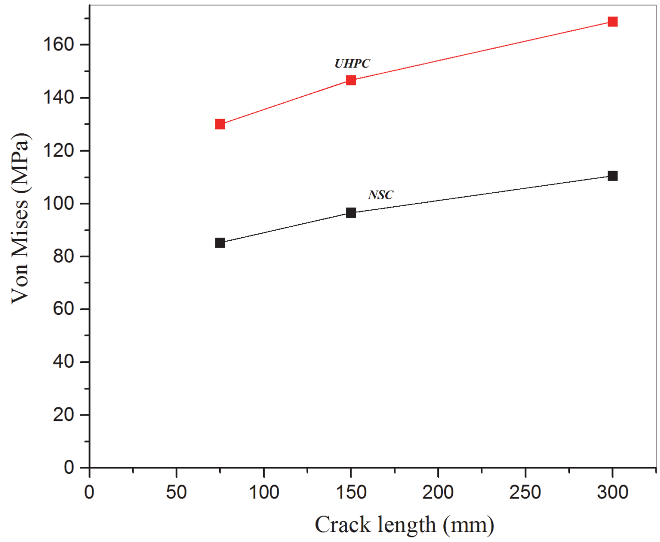


Figure 7: Maximum von Mises Vs crack length.

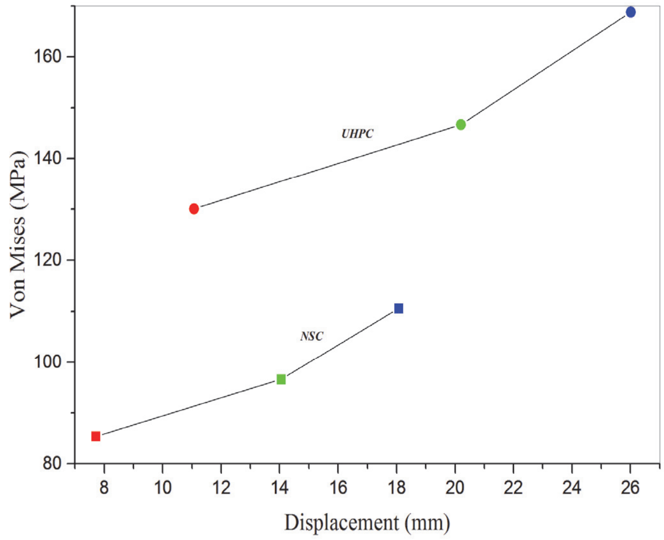


Figure 8: Maximum von Mises Vs displacement.

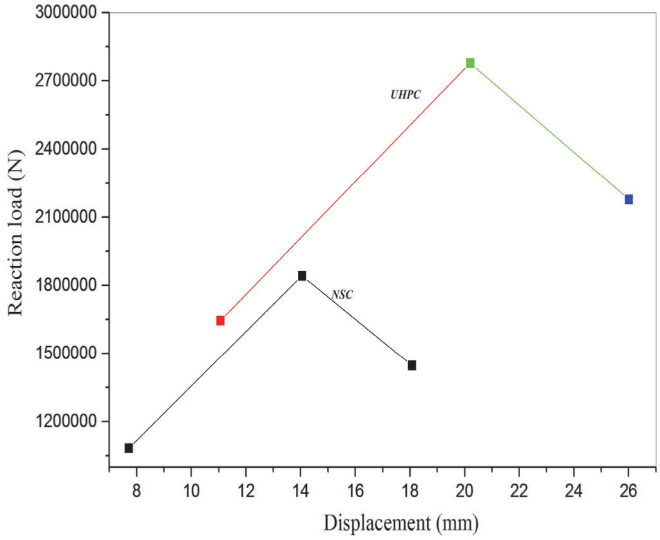


Figure 9: Maximum reaction load at fixed support Vs displacement in the center of the beam at different models.

NONLINEAR ANALYSIS OF BEAMS

The nonlinear forcing frequency with different direction and frequency has been applied on the beam. The force and its frequency are called the exciting frequency or forcing frequency. Beams constructed from NSC and UHPC subjected to forcing frequency. In order to comparative linear collapse mechanism with nonlinear collapse mechanism. The mode 1 of forcing frequency has been selected for nonlinear collapse mechanism of beam. The mode 1 of forcing frequency is in same direction of linear cracking moment. During applied forcing frequency of mode 1, beam displacement-crack exhibit same in both NSC and UHPC models, there is direct relationship between displacement and crack lengths (Fig. 10 and Tab. 7). The displacement increased with increasing crack length. There is not effect of mechanical property in displacement. In this mode, according mechanical propertied of NSC and UHPC, the maximum dynamic toughness has been observed in higher strength concrete. In comparative nonlinear behavior with linear elastic behavior of beams, the von Mises and reaction load exhibited differently (Figs. 10-14 and Tabs. 7-9). In both nonlinear and linear cases, with increase crack displacement increased.

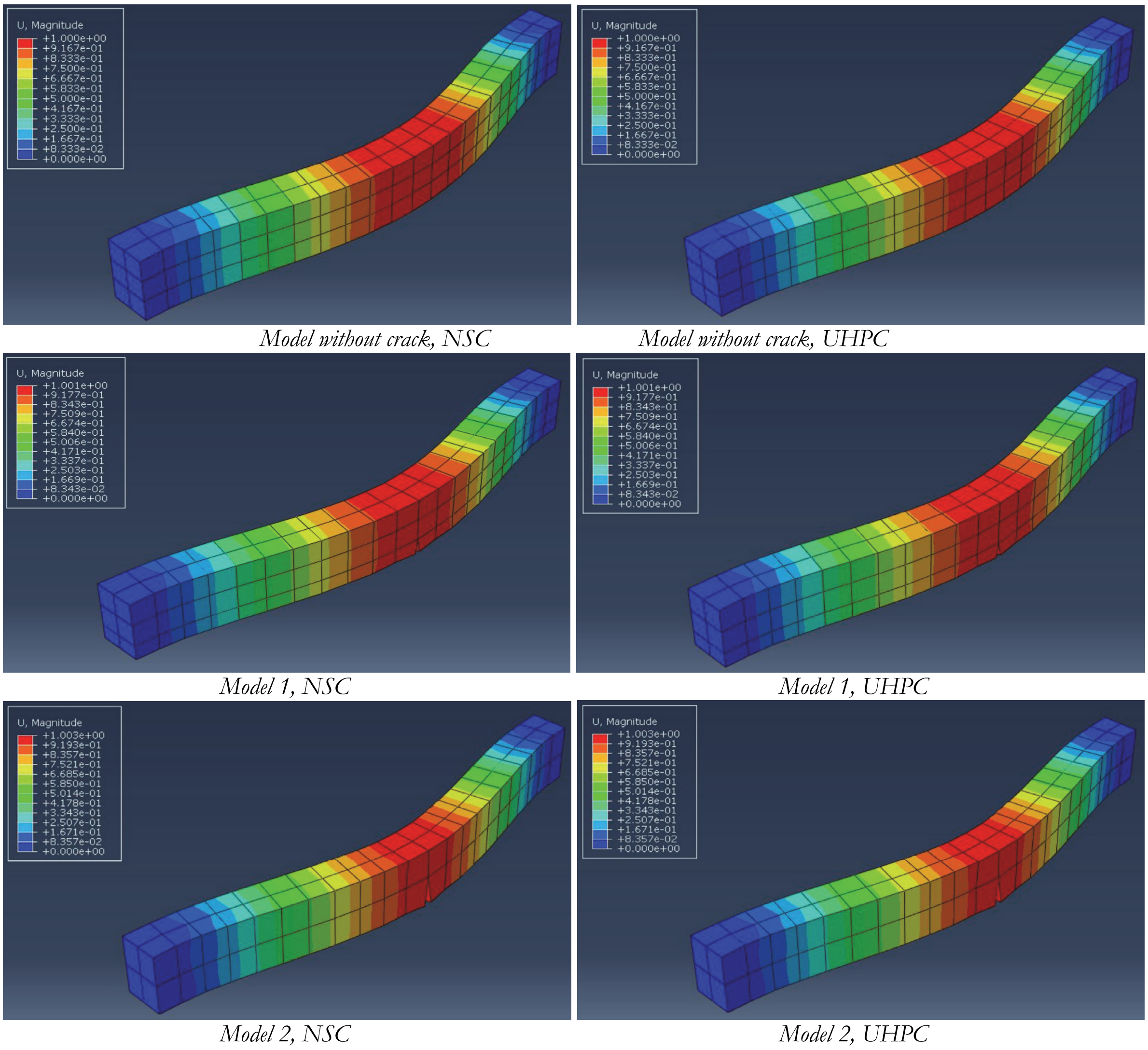


Figure 10: Displacement of different models with applied forcing frequency.

NSC		*UHPC*	
Crack length (mm)	Displacement (mm)	Crack length (mm)	Displacement (mm)
0	1.000	0	1.000
75	1.001	75	1.001
150	1.003	150	1.003

Table 7: Maximum displacement Vs crack length with applied forcing frequency.

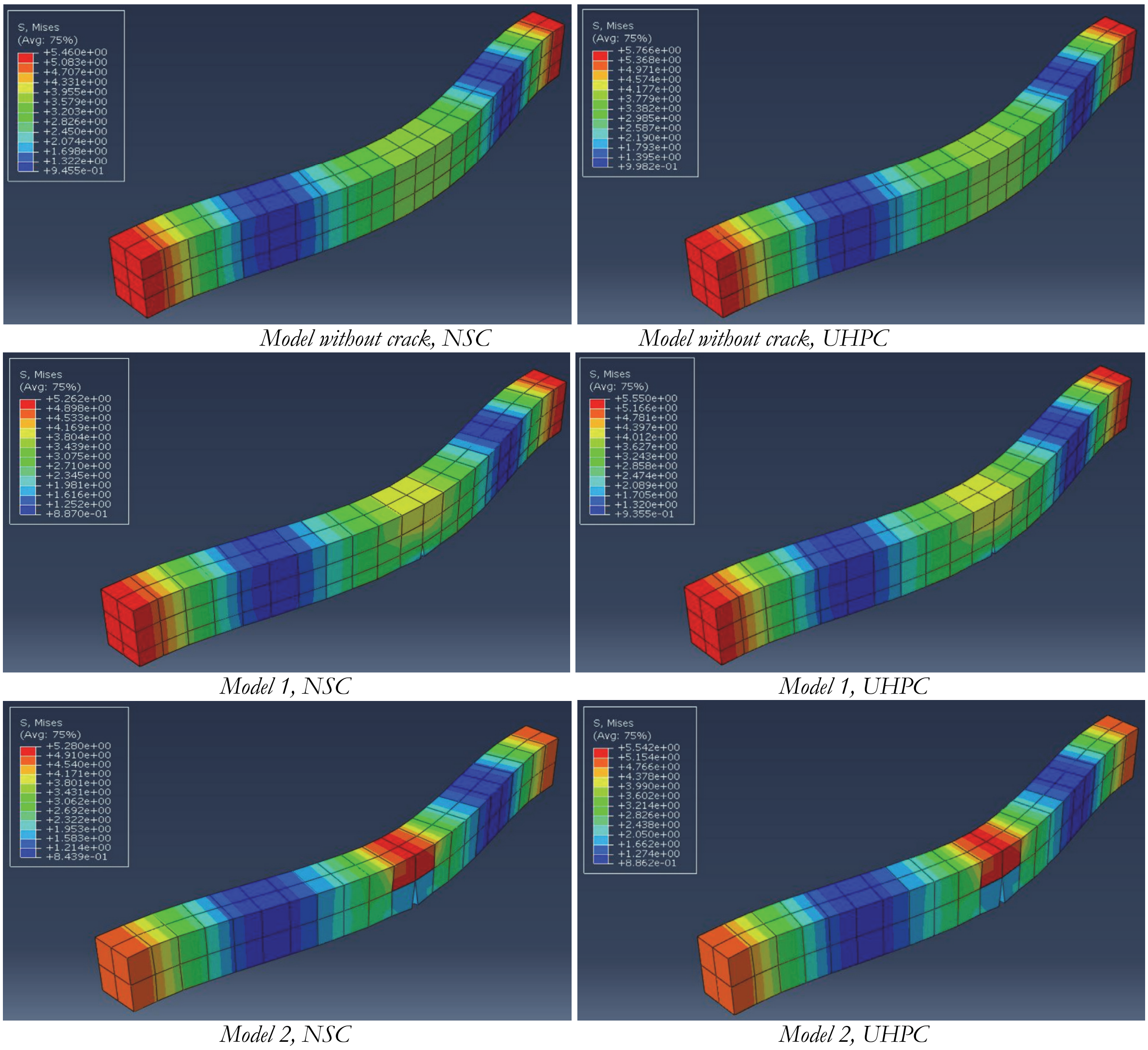
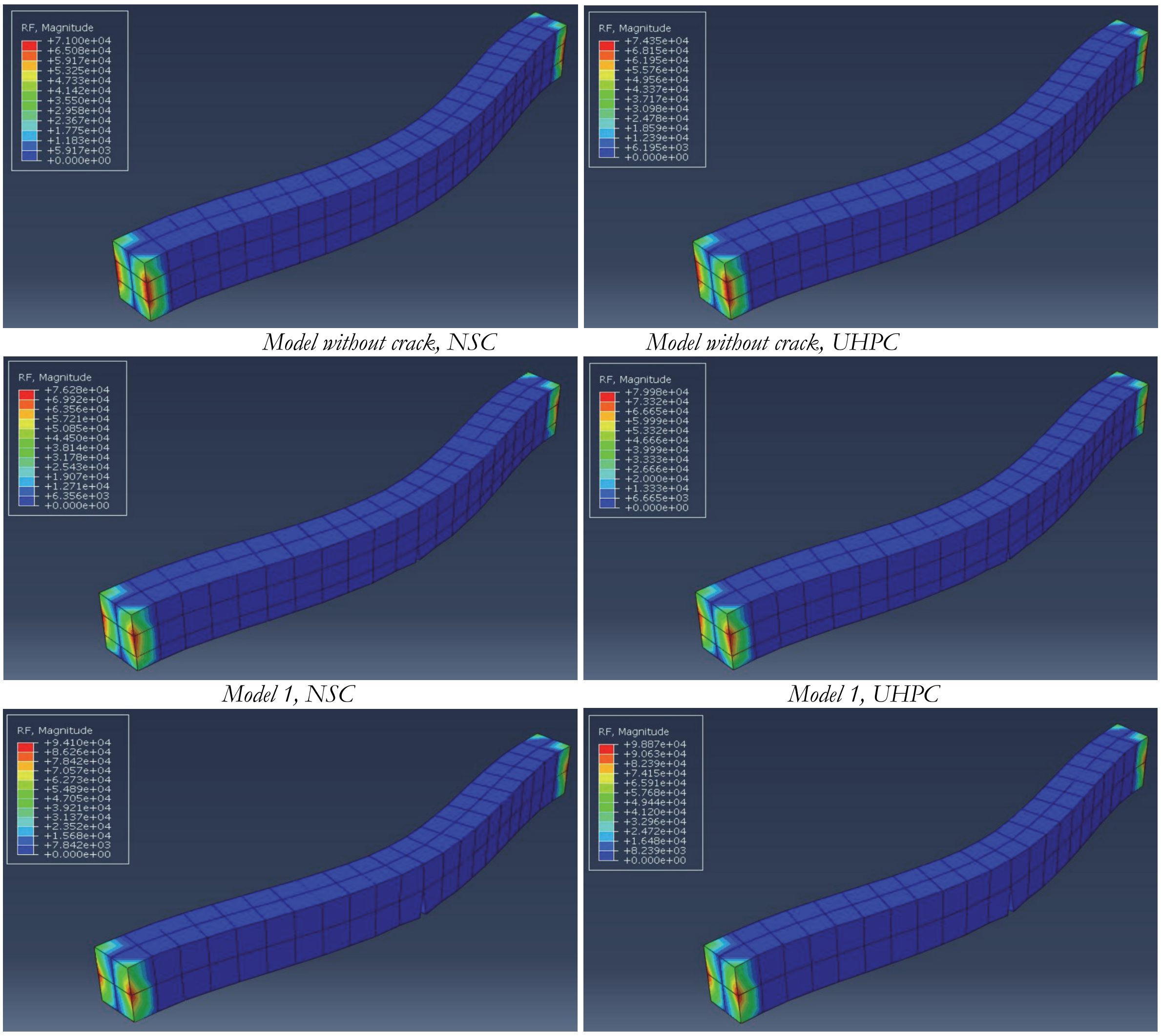


Figure 11: Von Mises of different models with applied forcing frequency.

NSC		*UHPC*	
Crack length (mm)	Von Mises (MPa)	Crack length (mm)	von Mises (MPa)
0	5.460	0	5.766
75	5.262	75	5.550
150	5.280	150	5.542

Table 8: Maximum von Mises Vs crack length with applied forcing frequency.



Model without crack, NSC *Model without crack, UHPC*

Model 1, NSC *Model 1, UHPC*

Model 2, NSC *Model 2, UHPC*

Figure 12: Reaction load of different models with applied forcing frequency.

NSC		*UHPC*	
Reaction load (N)	Displacement (mm)	Reaction load (N)	Displacement (mm)
71000	1.000	74350	1.000
76280	1.001	79980	1.001
94100	1.003	98870	1.003

Table 9: Maximum Reaction load Vs displacement with applied forcing frequency.

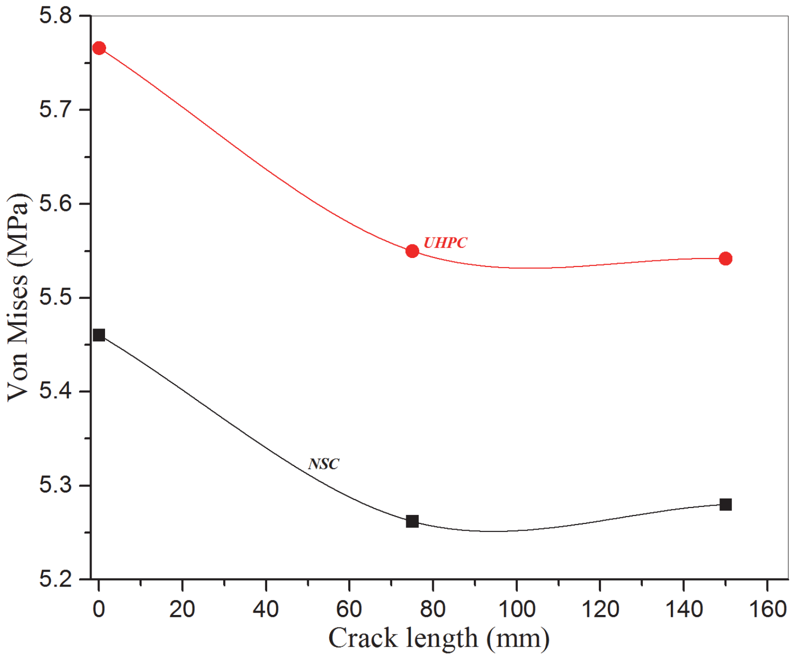


Figure 13: Maximum von Mises Vs crack length with applied forcing frequency.

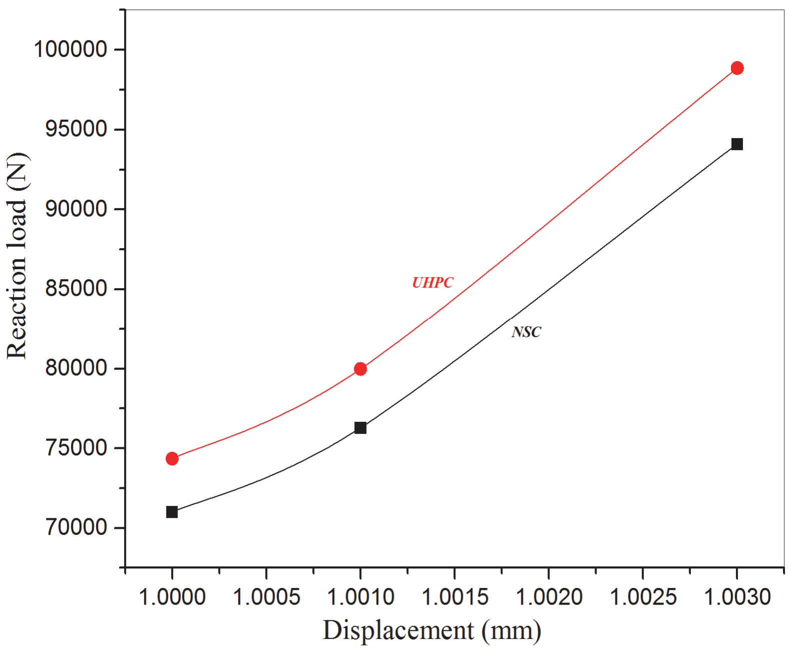


Figure 14: Maximum Reaction load Vs crack length with applied forcing frequency.

Model	Frequency (Hz)	Crack length (mm)	Displacement (mm)
NSC	7.38E-8	0	1.000
	7.28E-8	75	1.001
	7.02E-8	150	1.003
UHPC	7.57E-8	0	1.000
	7.47E-8	75	1.001
	7.21E-8	150	1.003

Table 10: Nonlinear behavior of beams with applied forcing frequency.

Tab. 10 indicates, with improve mechanical property of beam, higher forcing frequency is required to collapse beam. The forcing frequency reduced with developing length of crack. The dissipation of forcing frequency in both models are different, this phenomenon appears more with increase length of crack. Tab. 11 presents frequency of models.

Mode No	Frequency (Hz)					
	Beams NSC			Beams UHPC		
	Model with no crack	Model 1	Model 2	Model with no crack	Model 1	Model 2
1	7.38E-08	7.28E-08	7.02E-08	7.57E-08	7.47E-08	7.21E-08
2	1.15E-07	1.08E-07	8.33E-08	1.18E-07	1.11E-07	8.56E-08
3	1.97E-07	1.97E-07	1.95E-07	2.03E-07	2.03E-07	2.00E-07
4	2.49E-07	2.48E-07	2.36E-07	2.60E-07	2.59E-07	2.46E-07
5	2.95E-07	2.93E-07	2.74E-07	3.03E-07	3.02E-07	2.82E-07
6	3.72E-07	3.65E-07	3.49E-07	3.82E-07	3.75E-07	3.59E-07
7	4.73E-07	4.67E-07	3.66E-07	4.94E-07	4.87E-07	3.80E-07
8	4.75E-07	4.68E-07	3.79E-07	4.95E-07	4.89E-07	3.96E-07
9	4.78E-07	4.71E-07	3.82E-07	4.98E-07	4.92E-07	3.99E-07
10	4.82E-07	4.75E-07	3.87E-07	5.03E-07	4.95E-07	3.99E-07

Table 11: Frequency of all models in different modes.

CONCLUSION

By using ABAQUS, elastic solids plain concrete beam has been modeled. The ABAQUS calculates the stress path during crack development, while the beam is subjected to the allowable cracking moment. The stress transfer has been demonstrated step by step, the process not so easy to understand from the experimental test. This work integrates experimental test and better integrates the explanation failure mechanism of beam and bending crack development. It has been observed that the crack develops with stress transfer, it effects on displacement, von Mises, load reaction and failure mechanism. Furthermore, crack propagation is caused by a transfer of energy from external work and/or strain energy to surface energy. The controlling stress path minimizes crack expansion. The maximum toughness has been observed in higher strength concrete. The forcing frequency reduced with developing length of crack.

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