Focussed on: Fracture and Structural Integrity related Issues

Double peeling of elastic pre-tensioned tapes

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ABSTRACT. Peeling is a physical mechanics involved in the detachment of many natural and industrial applications. In this paper, we investigate the double peeling of an endless elastic pre-stressed tape adhering to a flat smooth rigid substrate. Solutions are given in closed form and their stability is discussed. Critical pull-off force needed to detachment is shown to be higher for pre-stressed tapes. However, when a pre-stress is high, tapes behave differently and may spontaneously detach from the rigid substrate.

KEYWORDS. Adhesion; Peeling; Fracture; Adhesion.

INTRODUCTION

Adhesion governs many natural processes and has to be carefully accounted for in a large variety of industrial applications, where its optimization is indeed a key point to be pursued [1]. As an example, in biology and bioengineering, detachment determines the aggregation of cells and, in parallel, their attachment to extracellular matrix (ECM). Furthermore, adhesion mechanics governs also the hairy attachment systems of insects, reptiles and spiders that show extraordinary adhesive abilities even at the human size scale. On the other side, when focusing on components of industrial interest, we observe that Van der Waals forces, especially in vacuum, are often responsible for sticking in Micro-Electro-Mechanical- Systems and, therefore, for their failure [2].

In this paper, we pay attention to the detachment of an adhering tape from a rigid substrate since it represents a basic but representative model of natural phenomena and industrial components. In this respect, important contributions were given by Kendall [3] that proposed a simple model to explain the detachment of a single elastic tape from a rigid surface, and Cheng [4] that extended the Kendall's model to pre-stressed tapes. These models, which have been developed moving from energetic considerations, complemented by the purely geometric approach proposed in [5] and claiming to be able to take into account also frictional force contributions. Fixed the peeling load, this approach predicts, in accordance with some experimental evidences, smaller peeling angles, but, to some extent, results are affected by a certain degree of arbitrariness since they depend on the initial configuration of the tape [5]. Here, by moving from the pioneering Kendall's model and therefore relying on an energetic approach, we focus on the case of the double peeling of an endless pre-stressed thin tape. We observe that the pre-tension P_0 determines an increase of the critical pull-off force at small peeling angles while decreasing it at large peeling angles. Also beyond a critical value of P_0, the pull-off force vanishes at a corresponding critical peeling angle. In this case, the system may spontaneously detaches. Furthermore, this paper contains a detailed analysis on the stability conditions of the process: particular attention is paid to detect the boundary between stable and unstable adhesion.

FORMULATION

Given an adhesive tape going to be peeled from a flat rigid substrate, the non-contact area can be studied as an interfacial crack, which, during the detachment process, propagates determining the advance of the peeling process. As a matter of fact, by recalling the Griffith criterion, we can observe that, in equilibrium conditions, the total free energy U_{tot} has to be stationary:

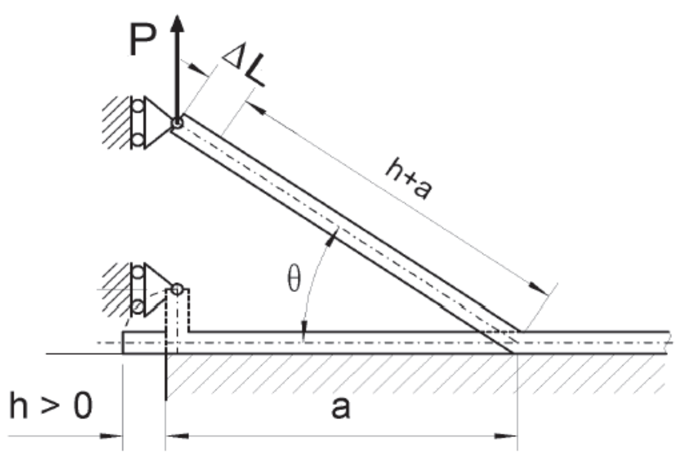
$$G = \Delta\gamma \tag{1}$$

where $\Delta\gamma$ is the work of adhesion and G is the energy release rate at the crack tip and, for fixed load *P*, is equal to:

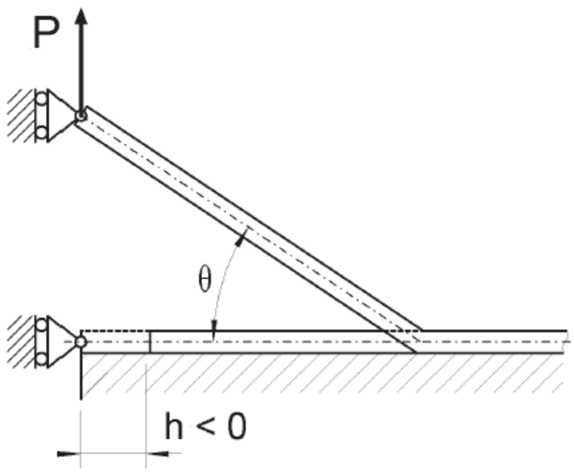
$$G = -\left(\frac{\partial U_{el}}{\partial S} + \frac{\partial U_p}{\partial S}\right)_P \tag{2}$$

with U_{el} and U_p respectively equal to the elastic energy stored in the system and the potential energy associated with the external load P; S is the size of the detached area.

Now, let us focus on the peeling tape. In Ref. [6], it has been shown that, given a symmetric tape loaded with a normal load *2P*, it is possible to study just an half of the tape loaded with a force equal to P. As matter of fact, in the following, we study the reduced system in Fig. 1. More in details, the problem is formulated considering two different initial configurations of the elastic tape having cross section $A_t = bt$ (Fig. 1). In the first configuration (Fig. 1a), a length *h* of the tape is not attached to the substrate and, before applying the external force *P*, it is rotated. In the latter (Fig. 1b), the tape is stretched of the quantity *h* before loading. In both cases, a pre-tension P_0 can be applied to the tape before it is attached to the substrate.



(a)



(b)

Figure 1: Peeling of a load. Initial configuration (a): a length h of the tape is not attached to the substrate and it is rotated before applying the external force P; initial configuration (b): the tape is stretched of a quantity h before loading.

Now, given the system in Fig. 1, elastic and potential contributions will be respectably equal to:

$$U_{el} = \frac{1}{2}\frac{(a+h)}{Ebt}\left(\frac{P^2}{\sin^2\theta} - P_0^2\right) \tag{3}$$

$$U_P = -P(a+h)\sin\theta\left[1 + \frac{1}{Ebt}\left(\frac{P}{\sin\theta} - P_0\right)\right] \tag{4}$$

By recalling Eq. (1) and introducing the following dimensionless quantities

$\hat{\delta} = \frac{\delta}{h}$,

$\hat{a} = \frac{a}{h}$,

$\hat{P} = \frac{P}{Ebt}$,

$\hat{P}_0 = \frac{P_0}{Ebt}$,

$\Delta\hat{\gamma} = \Delta\gamma / Et$,

$\hat{G} = \frac{G}{Et}$,

we can estimate the dimensionless energy release rate:

$$\hat{G} = \frac{\hat{P}}{\sin\theta}\left(1-\cos\theta\right) - \frac{\hat{P}\hat{P}_0}{\sin\theta} + \frac{\hat{P}_0^2}{2} + \frac{\hat{P}^2}{\sin\theta} \tag{5}$$

where we employ the condition $a = \left(a + h + \Delta L\right)\cos\theta$ (see Fig. 1) , leading to:

$$\hat{a} = \frac{\cos\theta\left[1+\left(\frac{\hat{P}}{\sin\theta} - \hat{P}_0\right)\right]}{1-\cos\theta\left[1+\left(\frac{\hat{P}}{\sin\theta} - \hat{P}_0\right)\right]} \tag{6}$$

Notice that this equation is perfectly coherent with relation found by Chen in [3] for single pre-stresses tape. Finally, we can determine the vertical displacement:

$$\hat{\delta} + 1 = \frac{\hat{P} + \sin\theta - \hat{P}_0\sin\theta}{1-\cos\theta - \left(\hat{P}\cot\theta - \hat{P}_0\cos\theta\right)} \tag{7}$$

RESULTS AND DISCUSSION

Our analysis starts discussing the stability of the peeling. In detail, let us determine stable or unstable equilibrium conditions and what happens when it moves from unstable equilibrium.

In this respect, in Fig. 2a, the dimensionless peeling force $\hat{P}$ is plotted as a function of the peeling angle θ at equilibrium. Furthermore, in Fig. 2b, for a fixed load $\hat{P} = \overline{P}$, we show the relative dimensionless energy $\hat{U}$ as a function of the peeling angle. Notice that in this last case the peeling angle is referred also to conditions out of equilibrium. Now, it is possible to observe that, for any load smaller the critical peeling force, that is the maximum load the tape can sustain, two different equilibrium conditions exist. In the region h/a > 0 (corresponding to the tape configuration shown in Fig. 1a), the total energy $\hat{U}$ takes a local minimum at the peeling angles solving Eq. (4) and, therefore, in this case equilibrium (solid line in Fig. 2a) is stable. On the contrary, in the region h/a < 0 (corresponding to the configuration of Fig. 1b), we find a maximum for the total energy $\hat{U}$ and, therefore, unstable equilibrium conditions are present.

It is noteworthy to understand now what happens when we move from non-equilibrium conditions. Given the pull-off load $\hat{P} = \overline{P}$, we focus our analysis on the starting configurations A, B, C, and D shown in Fig. 2a. Let us start from point A. We observe that the minimization of the total energy requires that the peeling angles decreases monotonically towards smaller and smaller values. The end of the process should correspond to vanish the peeling angle but, in this case, the tape, being completely attached to the substrate, would be forced to sustain the vertical load with an infinite stress. This is physically not possible and, therefore, failure will occur before the tape adheres to the substrate. On the other side, when the system starts from point B, the tape peeling angle increases until we have the complete detachment, which corresponds to the red line in Fig. 2. As matter of fact, all the solutions corresponding to the dashed curve in Fig. 2a are physically admissible assuming that the initial configuration is that one reported in Fig. 1b, but they are unstable. Indeed, a small perturbation can lead the tape to failure (point A of Fig. 2a) or to complete detachment from the substrate (point B of Fig. 2a). On the contrary, when the system initially moves from a non-equilibrium configuration in the right-side region (points C and D), a stable equilibrium with a finite detached area will be reached: this will correspond to the local minimum of the total energy.

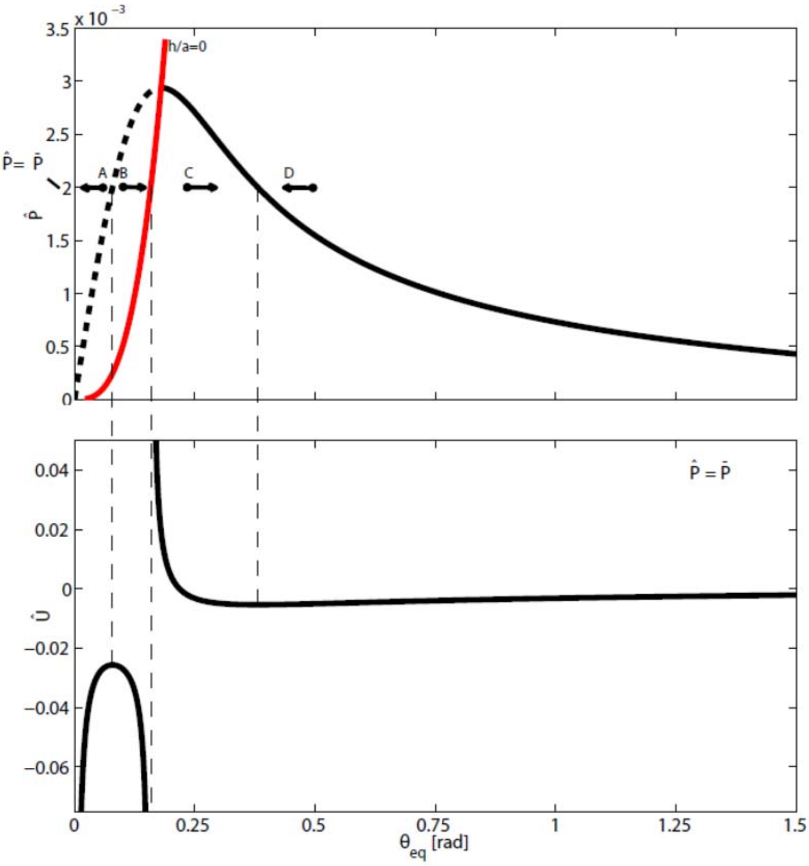


Figure 2: Peeling force $\hat{P}$ as a function of the peeling angle in equilibrium conditions θ_{eq} (a); the total energy as a function of the peeling angle θ_{eq} (even out from equilibrium) (b). The work of adhesion is $\Delta\hat{\gamma} = 4 \cdot 10^{-4}$.

With regards to the curve separating the stable region from the unstable one, this boundary does not depend on the value of the work of adhesion $\Delta\gamma$, but decreasing $\Delta\gamma$ indeed means lowering the limit pull off value $\hat{P}_{lim}$, of the system. On the other hand, the boundary *h/a=0* results affected by the preload $\hat{P}_0$ (see Fig. 3). However, besides a quantitative influence, conclusions previously drawn without any pre-stress keeps on being valid on a qualitative base when a preload $\hat{P}_0$ is present. This keeps on being valid for different values of the work of adhesion $\Delta\gamma$. Interestingly, we observe that, by increasing the pre-tension $\hat{P}_0$, we increase the maximum pull-off force $\hat{P}_{lim}$ the tape is able to sustain. However, it is possible to identify a critical value for the pre-stress $\hat{P}_{0_{cr}} = \sqrt{2\Delta\gamma}$. When this critical threshold is passed, the tape can spontaneously detach without applying any external load and, on the contrary, it is noteworthy to observe that a pull-off load is necessary to make the system to adhere to the surface. For $\hat{P}_0 > \hat{P}_{0cr}$, the peeling angle does not exceeds a critical angle θ_{cr} .

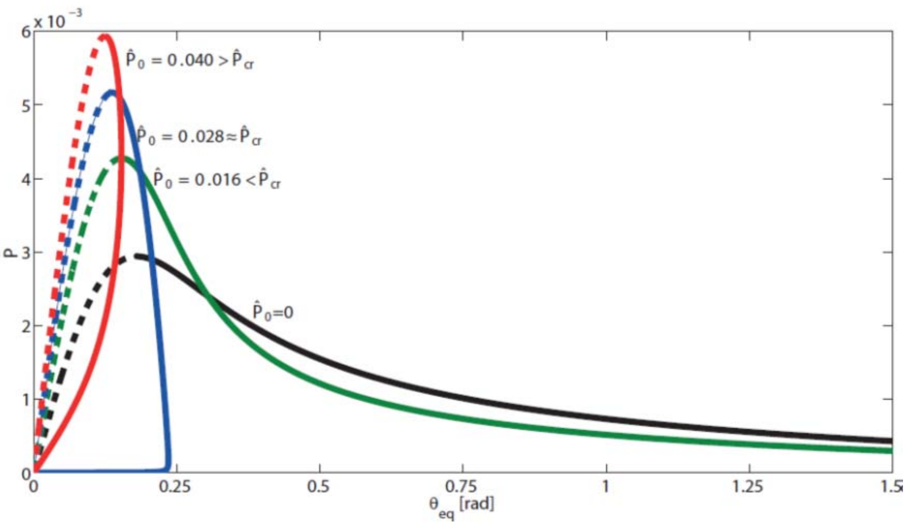


Figure 3: The dimensionless peeling force $\hat{P}$ as a function of the peeling angle at equilibrium θ_{eq} for different values of the dimensionless pre-load $\hat{P}_0$. The work of adhesion is $\Delta\hat{\gamma} = 4 \cdot 10^{-4}$.

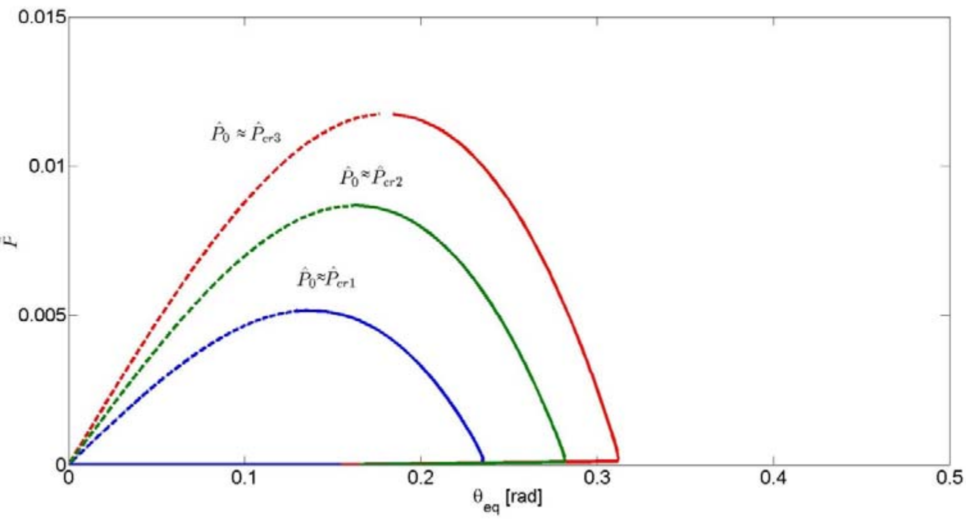


Figure 4: The dimensionless peeling force $\hat{P}$ as a function of the peeling angle at equilibrium θ_{eq} when the load $\hat{P}_0$ is equal to the critical pre-load $\hat{P}_{0_{cr}} = \sqrt{2\Delta\hat{\gamma}}$. Results are provided for different values of the work of adhesion: $\Delta\hat{\gamma}_1 = 4 \cdot 10^{-4}$; $\Delta\hat{\gamma}_2 = 8 \cdot 10^{-4}$, $\Delta\hat{\gamma}_3 = 1.2 \cdot 10^{-3}$.

Now, for both the stable region and the unstable one, we plot the vertical displacement $\hat{\delta}$ as a function of the peeling angle at equilibrium (see Fig. 5). We observe that, when we are close to the limit peeling angle θ_{lim}, the displacement diverges: as shown in Fig. 6, with a finite load equal to $\hat{P}_{lim}$, we are able to completely detach the tape. Notice that, given the configurations in Fig. 1, for pre-load $\hat{P}_0$ smaller than the critical pre load $\hat{P}_{0cr}$, the smallest load, the tape can sustain in the stable region, is that one for which the peeling angle in equilibrium is $\pi / 2$.

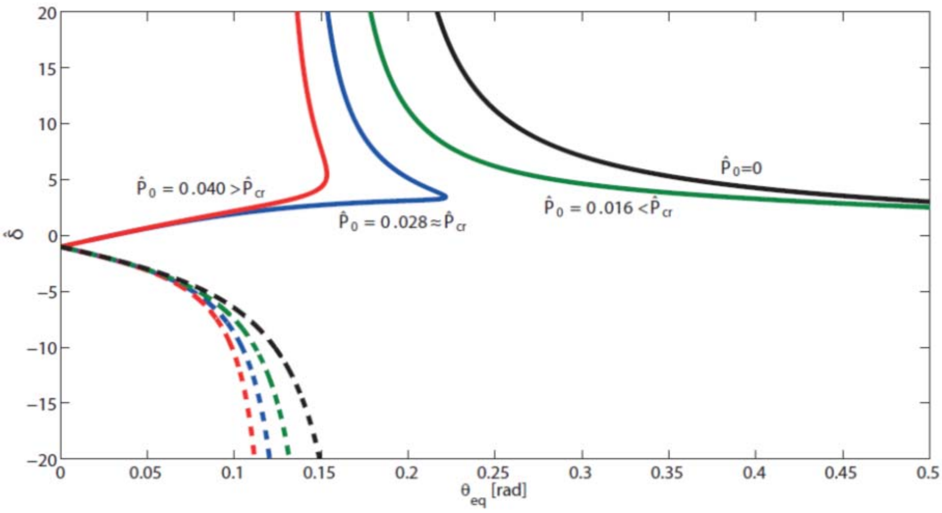


Figure 5: The dimensionless peeling force $\hat{\delta}$ as a function of the peeling angle at equilibrium θ_{eq} for different values of the dimensionless pre-load $\hat{P}_0$.

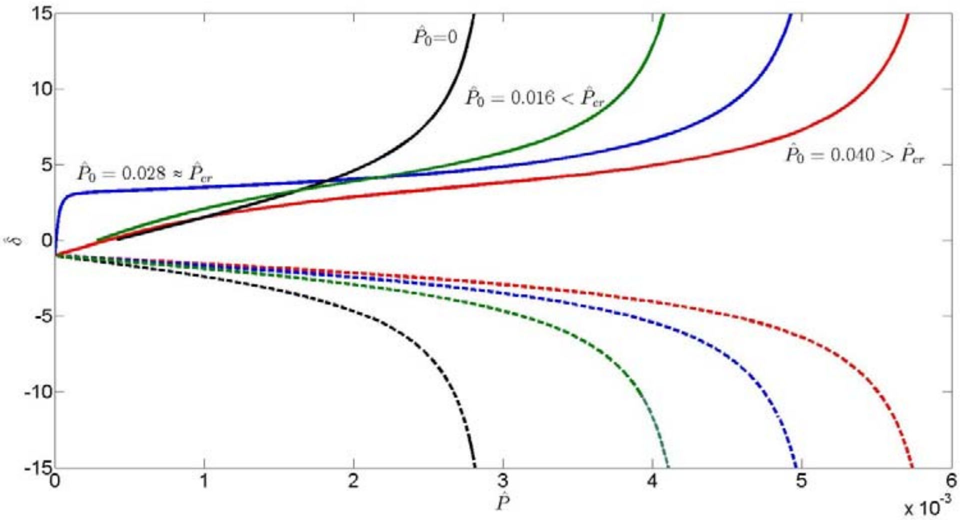


Figure 6: The dimensionless peeling force $\hat{\delta}$ as a function of the peeling angle at equilibrium θ_{eq} for different values of the dimensionless pre-load $\hat{P}_0$.

CONCLUSIONS

In this paper, we develop a model to study the adhesion and the detachment of tapes adhering to a rigid surface. The scheme proposed in Fig. 1 is quite interesting since, as shown in Ref. [6], a symmetric double tape can be reduced to this model.

Conclusions, we carry out by moving from such a scheme, show the presence of two equilibrium regions: one corresponds to values *h/a<0* and is stable; the other one, referred to *h/a>0*, is on the contrary unstable. In this last case, a small perturbation can lead the tape to failure (point A) or to complete detachment (point B). This can be useful to understand experimental results [7, 8] and sudden failure in industrial systems.

Finally, we study the importance of the pre-load $\hat{P}_0$: this does not affect the conclusions on stability previously carried out, but has a quantitative influence on the boundary between stable and unstable regions. Furthermore, we define a critical pre-load $\hat{P}_{0_{cr}}$: once this is overcome, spontaneous detachment can occur and, interestingly, an external load has to be applied to get the tape adhered to the substrate.

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