# A data fusion based approach for damage detection in linear systems

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**ABSTRACT.** The aim of the present paper is to propose innovative approaches able to improve the capability of classical damage indicators in detecting the damage position in linear systems. In particular, starting from classical indicators based on the change of the flexibility matrix and on the change of the modal strain energy, the proposed approaches consider two data fusion procedures both based on the Dempster-Shafer theory. Numerical applications are reported in the paper in order to assess the reliability of the proposed approaches considering different damage scenarios, different sets of modes of vibration and the presence of errors affecting the accounted modes of vibrations.

**KEYWORDS.** Damage Identification; Modal Strain Energy; Flexibility Matrix; Data-Fusion

## INTRODUCTION

Numerous studies concerning the development of damage indicators mainly based on the modal parameters of systems derived from identification processes, in some cases combined with finite-element model update algorithms, are available in the current literature [1-3]. Some of these indicators, coupled with innovative algorithms, provide information on the position and the severity of damage [4-6]. Among these, the relative damage indicator (rdi) and the modal strain energy change ratio ($MSECR_j$) indicators, respectively based on the change of the flexibility matrix and the modal strain energy of systems before and after the damage, are widely used for damage detection in linear systems [7, 8]. They are defined as:

$$rdi = \frac{\left| diag\left( F_e^d - F_e \right) \right|}{\left| diag\ F_e \right|} \tag{1}$$

$$MSECR_j = \frac{1}{N} \sum_{i=1}^{n} \frac{MSECR_{ij}}{\max_j \left( MSECR_{ij} \right)} \tag{2}$$

where $\boldsymbol{F_e}$ and $\boldsymbol{F_e^d}$ is the elemental flexibility matrix of the system before and after damage respectively, $MSECR_{ij}$ is the modal strain energy change ratio corresponding to the ith mode of vibration and to the jth element of the system and $N$ is the number of elements composing the system.

The studies available in literature show the ability of these indicators to detect the presence, the position, and in some cases, the severity of damage. Nevertheless, the same studies have also underlined some drawbacks generally arising when multiple damages occur or noises/errors affect the identified dynamic properties of the systems. In these cases, a reliable prediction of damage requires a significant number of information particularly in terms of number of modes. Recently, in order to improve the ability of classical damage indicators, data information fusion techniques have been extended to structural damage identification [6, 9-11]. The fusion of information derived from different sources allows, indeed, to improving the ability of damage indicators particularly in detecting the damage position.
In this paper, two approaches for damage detection in linear systems based on the use of the rdi and MSECRj indicators combined with the Dempster-Shafer data fusion theory are presented. In particular, while the first approach, denoted in the following DF as Data Fusion, is based on the fusion of the information derived separately from the $MSECR_j$ and rdi indicators, considered as separate and independent sources, the second approach is an innovative procedure, denoted in the following MDF as Multi-stage Data Fusion, consisting in a data fusion implemented in a multi-stage process where the sources are based on the same damage indicator, either rdi or $MSECR_j$, but evaluated on the basis of different combinations of modes of vibration. Numerical applications are presented in the paper to assess the reliability of the proposed approach considering different damage scenarios, different sets of modes of vibration and presence of noise.

## DEMPSTER-SHAFER THEORY

Dempster-Shafer theory [12] represents the first data fusion theory developed by Dempster and Shafer in 1976 and, still, one of the most valuable. Some key definitions of the theory, those used in the approach proposed in the paper, are summarized in the following.
Considering a finite set $\boldsymbol{\Theta} = \{A, B, C\}$ of mutually exclusive and exhaustive propositions, the corresponding power set $2^{\Theta}$ is defined as the set of all the subsets of $\boldsymbol{\Theta}$ which also includes the null set. The theory of evidence assigns a basic probability assignment function, named BPA or m(X), to any subset $\boldsymbol{X}$ of $2^{\Theta}$, defined as:

$$m : 2^{\boldsymbol{\Theta}} \to [0,1] \tag{3}$$

being:

$$\sum_{X \subset \boldsymbol{\Theta}} m(X) = 1 \text{ and } m(\varnothing) = 0 \tag{4}$$

In the framework of the Dempster-Shafer theory [12] the BPA can be interpreted as a generalization of the probability concept being the probability assigned not only to one hypothesis but to a set of hypotheses without any information on how it is distributed among the elements of the set itself.
The Dempster's rule provides a method for combining the basic probabilities assignment of different information sources $S_i$. In particular, given $S_1$ and $S_2$ two information sources and $m_1$ and $m_2$ the BPAs given by the two sources, the fused BPA is given by:

$$m(\overline{S_1}) = \frac{\sum_{S_1 \cap S_2 = \overline{S_1}} m_1(S_1) \times m_2(S_2)}{1 - Q} \tag{5}$$

where Q represents a measure of the degree of conflict between the two sources defined as:

$$Q = \sum_{S_1 \cap S_2 = \varnothing} m_1(S_1) \times m_2(S_2) \tag{6}$$

## DF AND MDF PROPOSED TECHNIQUES FOR DAMAGE DETECTION

The approaches presented in this paper for damage identification of linear systems are developed by combining the use of classical damage indicators based on the modal strain energy through the Modal Strain Energy Change Ratio index (MSECR) and on the flexibility matrix through the relative damage indicator (rdi) with the Dempster-Shafer data fusion theory. In particular, two different approaches are presented. The DF approach is simply based on the

fusion of the information derived separately from the MSECR and rdi indices, while the MDF approach is an innovative procedure using only of the indexes rdi and MSECR in a multi-stage data fusion (Fig. 1).
In the case of the DF approach, modal strain energy change ratios (MSECR) and rdi are evaluated by accounting all the available modes of vibration and assumed as two separate sources of information ($S_1^{rdi}$ and $S_2^{MSE}$). Then, the obtained MSECR and rdi are converted in local decisions $m_1(S_1)$, $m_2(S_2)$ and involved in DF process which provides the BPAs which can be assumed as indices able to detect the damage location.
Regarding the MDF approach, the procedure is developed by considering the rdi index and the MSECR index separately. The main differences with respect to the DF procedure consists in the fact that the sources are composed by the same index (MSECR or rdi) and each source differ from the others by the number of accounted modes of vibrations. Indeed, considering *n* identified modes of vibration for both the undamaged and damaged state of the system, a group of *n* first-level sources, denoted as $S_i$ with $i = 1,\ldots, n$, is defined by considering different sets of undamaged/damaged couples of mode shapes $\overline{\Phi}_i = \left[\Phi_i, \Phi_i^d\right]$, according to the scheme reported in Fig. 1.b and Fig. 1.c. For each first-level source $S_i$, it is possible to evaluate the rdi and the MSECR of the jth element ($j = 1\ldots N$), named $rdi_j^{[Si]}$ and $MSECR_j^{[Si]}$. The local decision $m_i(Si)$ associated to the $i^{th}$ first-level source $Si$ for both the use of rdi and MSECR are, then, obtained:

$$m_i\left(S_i\right) = \left(\frac{rdi_j^{[S_i]}}{\sum_{k=1}^{N} rdi_k^{[S_i]}}\right)_{j=1\ldots N} \tag{7}$$

$$m_i\left(S_i\right) = \left(\frac{MSECR_j^{[S_i]}}{\sum_{k=1}^{N} MSECR_k^{[S_i]}}\right)_{j=1\ldots N} \tag{8}$$

A second group of n-1 second-level sources, denoted as $\overline{S}_i$, is introduced and, for each $\overline{S}_i$, the corresponding local decisions $m\left(\overline{S}_i\right)$ are obtained from subsequent data-fusion operations according to the scheme shown in Fig. 1.

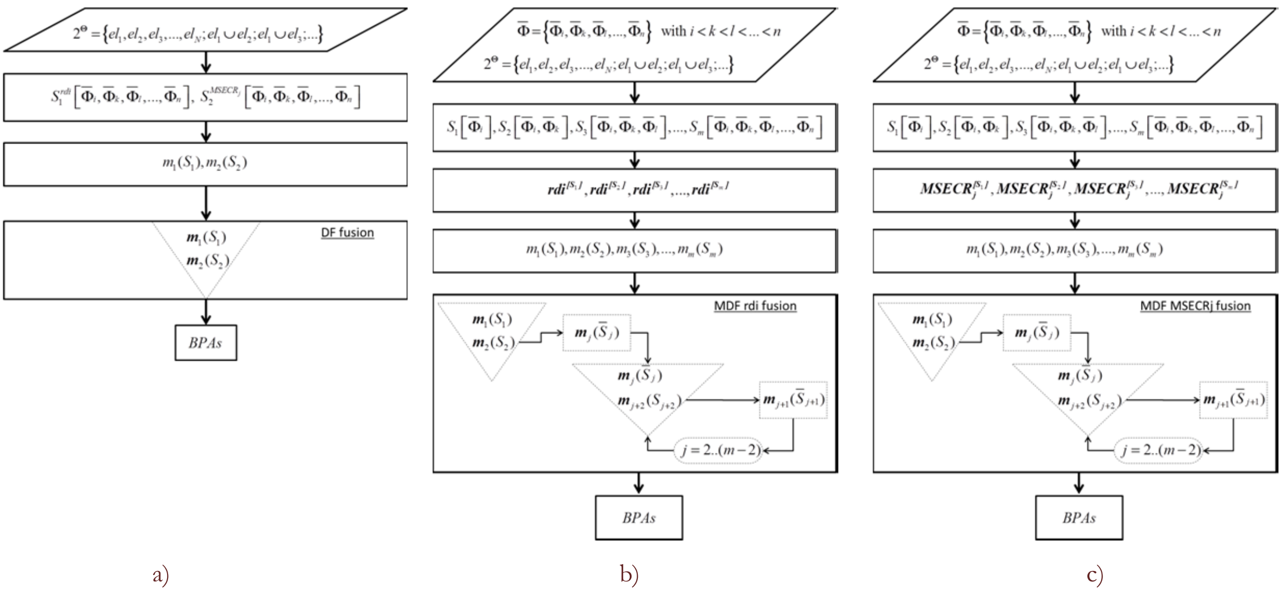



Figure 1: Schematization of the proposed data fusion procedures for the optimization of the damage detection: a) DF fusion technique; b) DF technique based on the rdi indicator; c) MDF technique based on the $MSECR_j$ indicator.

Therefore, the derivation of the second-level sources is based on the fusion of couples of local decisions. In fact, starting, for example, from the first two first-level sources, $S_1$ and $S_2$, a first fusion, developed through the Dempster's rule, provides the vector of local decisions associated to the second-level source $\overline{S}_1$ as follows:

$$\boldsymbol{m}_1(\overline{\boldsymbol{S}_1}) = \frac{\sum_{S_1 \cap S_2 = \overline{S}_1} m_1(S_1) \times m_2(S_2)}{1 - \sum_{S_1 \cap S_2 = \varnothing} m_1(S_1) \times m_2(S_2)} \tag{9}$$

The subsequent fusions for obtaining the local decision vectors of the second-level sources are characterized by the peculiarity that one of the vector of local decisions refers to a first-level source ($\overline{S}_i$) whilst, the other one corresponds to the second-level source derived from the previous fusion ($\overline{S}_{i-2}$), as shown in Fig. 1.
The vector of local decisions corresponding to the last source ($\overline{S}_{n-1}$) is just the vector accounted for deriving information on the damaged members of the system according to the proposed approach:

$$m_{n-1}\left(\overline{S}_{n-1}\right) = \frac{\sum_{\overline{S}_{n-2} \cap S_n = \overline{S}_{n-1}} m_{n-2}\left(\overline{S}_{n-2}\right) \times m_n\left(S_n\right)}{\sum_{\overline{S}_{n-2} \cap S_n = \phi} m_{n-2}\left(\overline{S}_{n-2}\right) \times m_n\left(S_n\right)} \tag{10}$$

The greatest components of this vector indicate the members where the damage is located.

## NUMERICAL APPLICATIONS

The numerical applications reported in the paper are devoted to assess the capability of the Dempster-Shafer theory to improve the ability of classical indices in detecting damages in structures on the basis of the variation of their dynamic properties. In particular, both the DF and MDF techniques proposed by the authors are analyzed with reference to the fixed end beam shown in Fig. 2 [6]. The beam is composed of 12 elements and 13 nodes characterized by only vertical displacements as available DOFs. Each element has a length of 0.6 m, modulus of elasticity of 7.5x10 N/m$^2$, cross sectional area of 0.001 m$^2$, moment of inertia of 7.56x10.7 m$^4$, and mass density of 7800 kg/m$^3$. Two damage patterns are considered and, for both of them, the damage is simulated by a reduction of the stiffness of some elements. The first pattern is a single damage case where only one single element, that is no. 6, is damaged by reducing its stiffness of 15% (denoted in the following as "S6D15"); the second pattern is a multiple damage case where two elements, no. 6 and 11, are both damaged by reducing their stiffness of 10% (denoted in the following as "S6,11D10"). The mode shapes and the frequencies of vibration have been numerically derived through the eigenvalue problem for both undamaged and damaged cases.
The DF and the MDF techniques have been applied to the beam in the different damage patterns considering a limited number of identified mode shapes and not consecutives mode shapes. Moreover, in order to simulate the presence of noises that in real applications generally affect the signals used in the identification process, DF and MDF have been also applied by introducing errors in the numerically evaluated mode shapes.

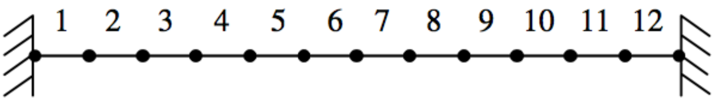



Figure 2: FEM of fixed-end beam

### *Single Damage Case: S6D15*

The results concerning the single damage scenario are shown in Fig. 3 for the case of DF technique and in Fig. 4 for the case of MDF technique, considering in both cases different sets of modes of vibration. In particular, in Fig. 3 are also reported the results deduced by applying the classical damage identification technique based on the use of the rdi and MSECRj indices.
From the figures it is possible to observe that both DF and MDF technique allow to improving the detection of damage with respect to the classical damage identification technique. Indeed, graphically it is evident that the bars corresponding

to the use of DF and MDF more clearly underline the damaged element n.6, particularly in the case of MDF where for all the accounted sets of modes the most significant BPA just corresponds to the damaged element. In the case of the use of the classical techniques, it is possible to observe that for many sets of modes the damaged element is not identified (it is characterized by a BPA less than the ones corresponding to other undamaged elements) and it seems that more than one of the elements composing the beam is affected by damage. For these cases, although the application of the DF technique leads to an increase of the BPA corresponding to the damaged element, the corresponding bar results the highest one, significant values of the BPA also characterize the other elements.

Also in the case of the MDF technique the sets of modes $\Phi=[\phi_3,\phi_5,\phi_7]$, $\Phi=[\phi_3,\phi_4,\phi_6]$ are characterized by a less accurate damage identification although a significant difference characterize the BPA corresponding to the damage element and the BPA of the undamaged ones.

This effect clearly underlines the role of the identified modes of vibration before and after the damage. Indeed, the sets characterized by the better identification of damage are the ones composed of the modes of vibration with significant values of the modal participating mass and by the more sensibility to the damage in terms of variation of the frequency.

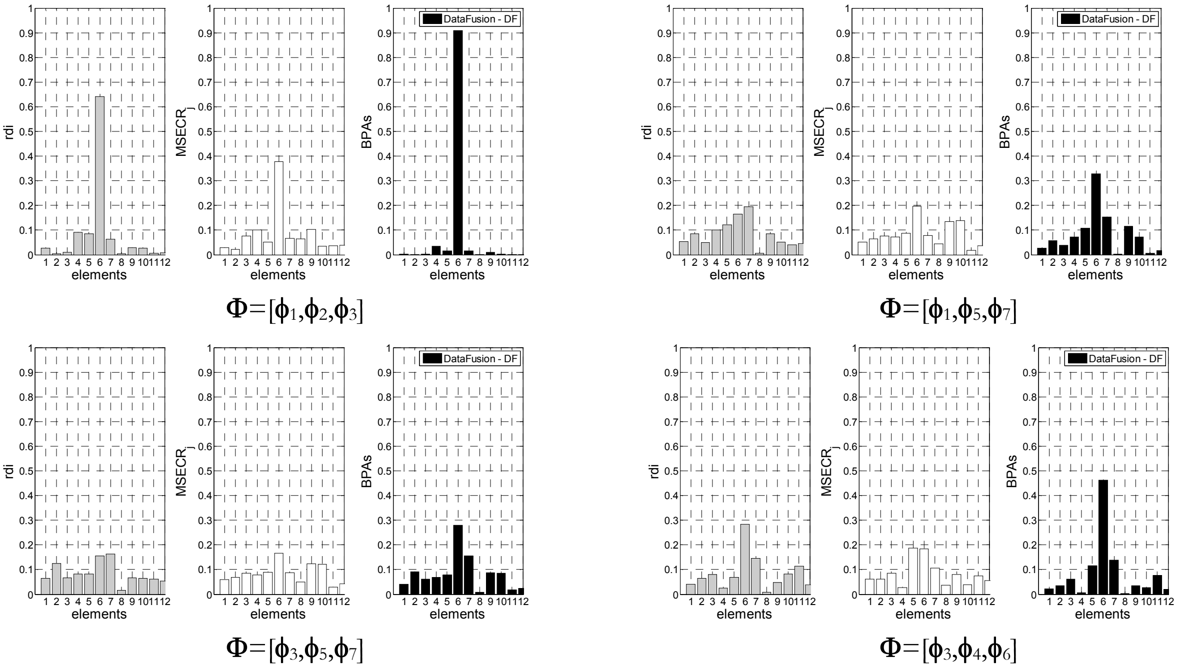



Figure 3: rdi and MSECRj index vs. DF technique – S6D15 damage case

## *Multiple Damage Case: M6,11D10*

In Fig. 5 and 6 are reported the same results considering the multiple damage scenario.

Also for this damage scenario it is possible to observe the capability of the DF and MDF in improving the damage detection with respect to traditional techniques, and, also in this case, it is evident the importance of the selected sets of modes. This last effect becomes more important in the case of multiple damage scenarios.

## *Effect of noise*

The capability of data fusion to improve the damage detection is further assessed by introducing errors in the modes of vibration generally arising when noises affect the signals at the basis of the identification process. In the paper, this effect is simulated by perturbing each mode shape, accounted in the damage identification, as follows:

$$\tilde{\phi}_i = \phi_i \left(1 + r_i \nu\right) \quad \forall i = 1,..,n \tag{11}$$

being $r_i$ a normally distributed random variable with zero mean and $\nu$ the noise level in terms of percentage. In Fig. 7-10 are reported the results derived from the damage identification considering DF and MDF technique, the single damage and the multiple damage scenarios and by accounting 100 samples with a 0.5% noise level.
The obtained results confirm the capability of both DF and MDF to improve the damage detection with respect to traditional approach. At the same, greater dispersion of results characterizes both DF and MDF. This evidence is due to the fact that DF and MDF techniques use several times the same mode of vibration and, consequently, the corresponding errors.

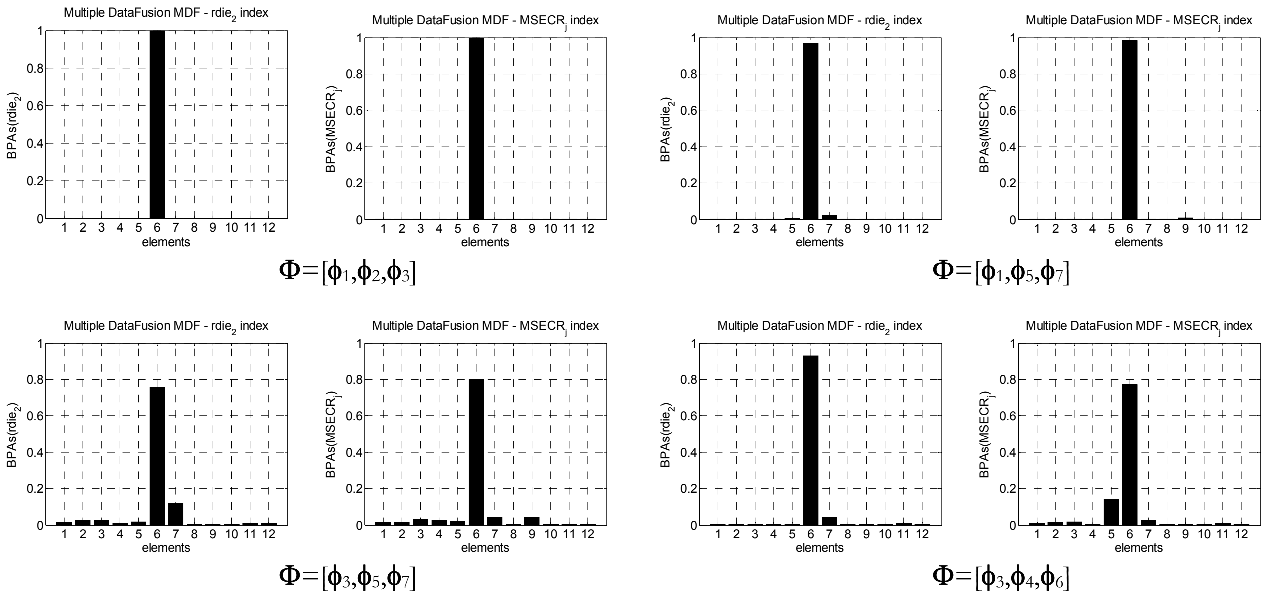



Figure 4: MDF technique – S6D15 damage case

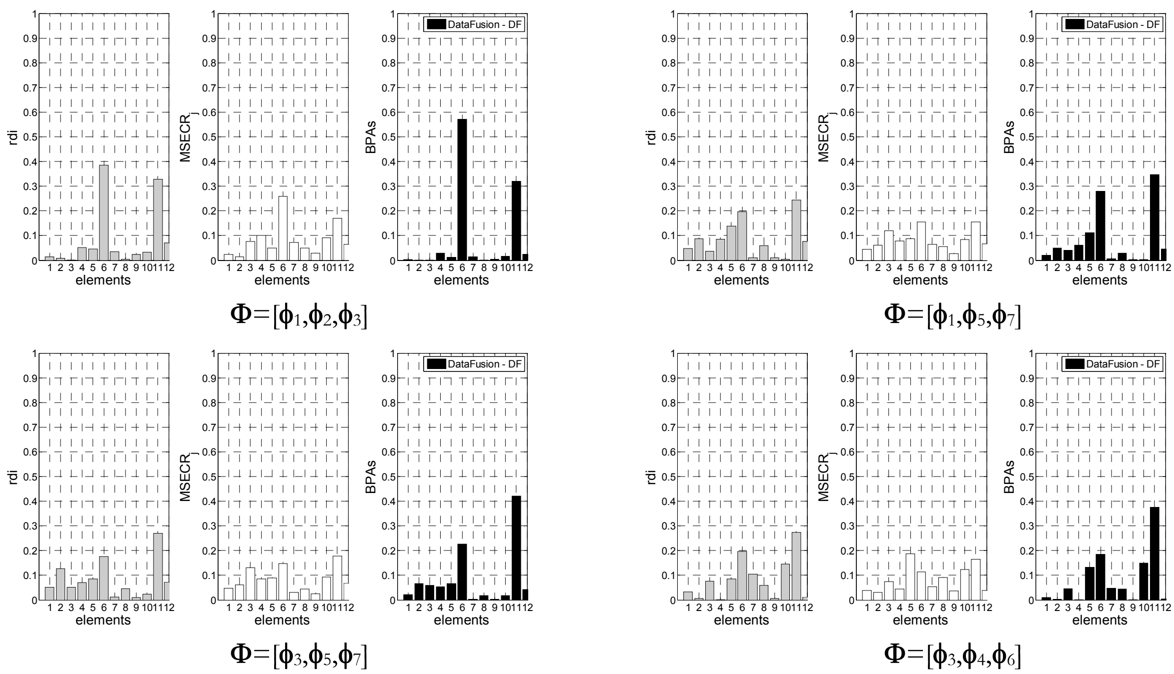



Figure 5: rdi and MSECRj index vs. DF technique – M6,11D10 damage case

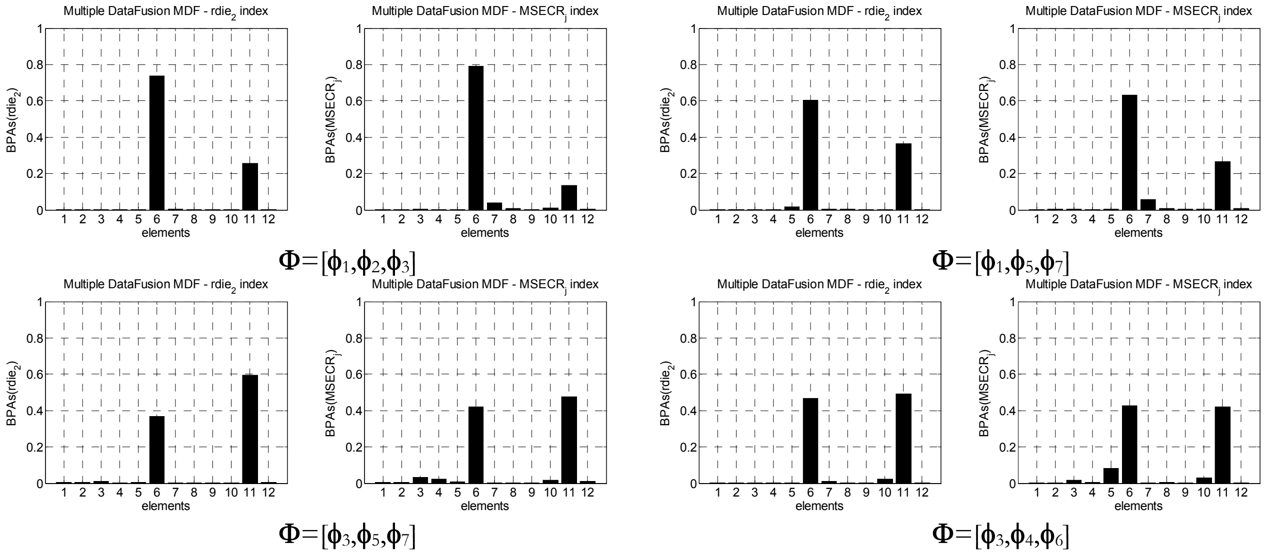



Figure 6: M DF technique – M6,11D10 damage case

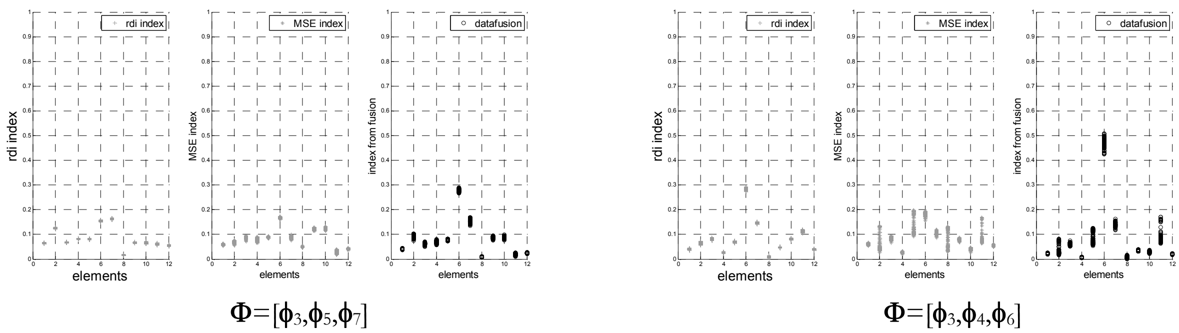



Figure 7: rdi and MSECRj index vs. DF technique – S6D15N0.5 damage case (100 simulations)

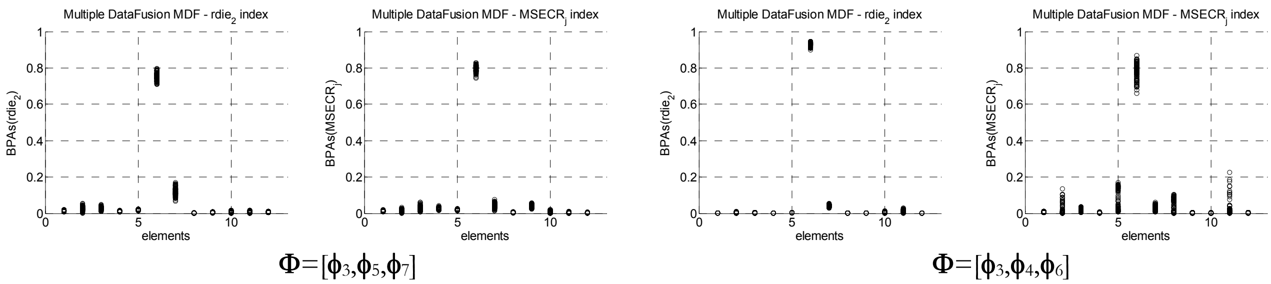
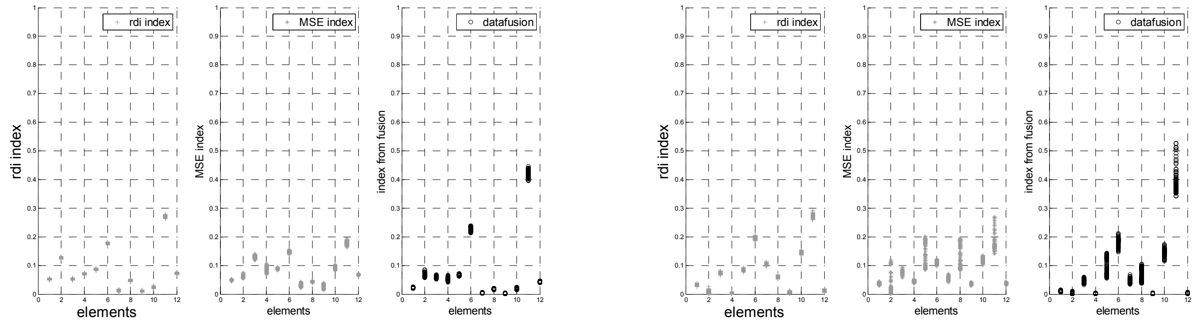
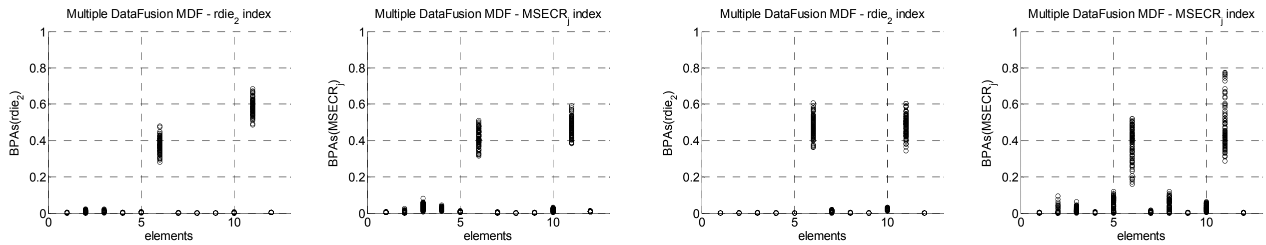



Figure 8: MDF technique – S6D15N0.5 damage case (100 simulations).




$\Phi=[\phi_3,\phi_5,\phi_7]$ $\Phi=[\phi_3,\phi_4,\phi_6]$

Figure 9: rdi and MSECRj index vs. DF technique – M6,11D10N0.5 damage case (100 simulations).




$\Phi=[\phi_3,\phi_5,\phi_7]$ $\Phi=[\phi_3,\phi_4,\phi_6]$

Figure 10: MDF technique – M6,11D10N0.5 damage case (100 simulations)

## CONCLUSIONS

The approaches here proposed combines the use of some damage indicators with the evidence theory based on the Dempster's rule of combination through a procedure devoted to obtain more reliable and evident information concerning the position of damage in linear systems.

The results carried out with reference to the numerical applications reported in the paper have clearly underlined the ability of the proposed approach in improving the performances of damage indicators both in the case of single damage scenarios and in the case of multiple damage scenarios. In particular, the MDF have underlined a better capability to detect the location of damage with respect to the DF where a simple fusion is performed. Indeed, it has been observed that the MDF particularly emphasizes the values of damage indicators corresponding to the damaged members and, at the same time, leads to a significant reduction of the value of the indicators of the undamaged members. This peculiarity is indeed mainly due to the subsequently data-fusion multiple steps procedure which allows to refine the vectors of local decisions. The ability of the proposed approach has been also confirmed in the presence of scenarios characterized by damages with lower damage severity (stiffness reduction less than 5%). Nevertheless, although this aspect has not been analyzed in the present paper, in this case it has been observed a greater sensibility of the data fusion to the set of the selected modes of vibration.

Although the proposed approaches have shown a sensibility to errors affecting the identified modes, as occurs for the classical approach based on the rdi and the MSECR, it has been observed that the most significant variations concern the only indicators corresponding to the damaged members; on the contrary, in the case of the classical approach, the indicators of all the members are significantly affected by the noise.

Finally, as underlined in [6], the ability of the proposed approach in providing an efficient detection of the damage position in structures represents an important basis for the further step of the identification process that is the determination of the damage severity. Indeed, the precision of most of the methods available in the current literature [13] in detecting the damage amount is strongly dependent on the correct identification of the damage position. Moreover, the proposed data fusion approach could be generalized in order to improve not only the identification of the damage position but also the damage extent.

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