DELAMINATION GROWTH IN LAYERED COMPOSITES: NUMERICAL MODELING AND PARAMETER IDENTIFICATION

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1. Introduction

Numerical simulation of interlaminar debonding and, more generally, of fracture phenomena in layered composites is here dealt with in a finite element framework by means of interfaces ([1], [2]). A visco-plastic interface model proposed in [3] is considered, which depends on nine parameters. The rate-dependency has been introduced on the bases of experimental evidences presented e.g. in [4].

Abstract

The present paper deals with parameter identification of interface models conceived for the simulation of decohesion phenomena in layered composites. Specifically, a visco-plastic interface model is considered, which depends on nine material parameters. The extended Kalman filter for discrete time problems is adopted to solve the inverse problem. In order to reduce the difficulties related to the computation of the sensitivity to the model parameters, a simplified procedure is used which is based on a priori interpolation of the response functions in the model parameter space.

Sommmario

Nel presente lavoro si affronta il problema dell’identificazione parametrica di modelli di interfaccia concepiti per la simulazione di fenomeni di decoesione in compositi stratificati. Si considera, ai fini della sua identificazione, un modello di interfaccia visco-plastico dipendente da nove parametri. La tecnica di identificazione adottata è basata sul filtro di Kalman esteso, per problemi non-lineari, nella sua versione a tempo discreto. Al fine di ridurre le difficoltà numeriche connesse con il calcolo della sensitività del sistema ai parametri del modello, si adotta una procedura semplificata, basata su di una interpolazione a priori della funzione di risposta.
Parameter identification is a major problem for interface models, difficulties are increased by the fact that direct experimental results cannot be obtained on interfaces. Solution of the identification problem is here sought via the Kalman filtering technique ([5], [6]), which furnishes the estimation of model parameters along with its error covariance matrix, that is a measure of the uncertainty relevant to the estimated values.

By using the Kalman filtering technique a central role is played by the sensitivity analysis ([7], [8]). In order to overcome the difficulties related to the analytical computation of the sensitivity of the structural behaviour to the model parameters, the sensitivity matrix is here computed after a Lagrangian interpolation of the structural response in the model parameters space, as proposed in [7]. This method allows to partially solve difficulties related to the noise embedded in the numerically computed system response, which arises from the finite accuracy with which they can be estimated in finite element simulations; the same procedure has been recently applied by the Authors in [9] and [10].

2. Visco-plastic interface model

An interface law is here conceived as a relation between interface tractions \( t \) and displacement discontinuities \( [u] = (u_+ - u_-) \), subscripts + and − denoting quantities relevant to the two decohesing surfaces. This law is used to simulate decohesion processes, e.g. due to delamination in layered composites ([1-4]), which occur along a surface \( \Gamma \) with normal \( n \), dividing the body \( \Omega \) in the two parts \( \Omega^+ \) and \( \Omega^- \) (see Fig. 1).

\[ [u] = [u]^t + [u]^p \]  

\[ t = K[u]^t ; K = \text{diag}(K_i) i = 1, 2, 3 \]  

\[ [\dot{u}]^p = \gamma < f(t, \lambda) > N \frac{\partial f(t, \lambda)}{\partial t} \]  

\[ f(t, \lambda) = \sqrt{\left(\frac{t_1}{a_1}\right)^2 + \left(\frac{t_2}{a_2}\right)^2 + \left(\frac{t_3}{a_3}\right)^2} - 1 + h\lambda \]  

\[ \lambda = \int_{0}^{\tau} \sqrt{[\dot{u}]^p} \sqrt{[\dot{u}]^p} dt \]  

![Figure 1. Reference frame for the interface](image)
In Eqn. (1) the additivity of elastic $[u]^e$ and visco-plastic $[u]^vp$ (i.e. irreversible) displacement discontinuities is assumed. The elastic behaviour of the interface is governed by relations (2), where interface elastic stiffnesses $K_i$, with the dimension of a force over a length cube, are introduced. Relation (3) governs the evolution of visco-plastic displacement discontinuities through a law of the Perzyna kind. $f(t, \lambda)$ is the visco-plastic potential defined in (4), which is a function of the interface tractions $t_i$ and of a visco-plastic multiplier $\lambda$, defined in equation (5) where $\tau$ denotes time. In the above Eqns. the symbol $<\bullet>_+$ denotes the McAuley brackets, defined as: $<\bullet>_+ = \bullet$ if $\bullet > 0$; $<\bullet>_+ = 0$ if $\bullet \leq 0$. The positive part of the traction $t_3$ normal to the interface is introduced in Eqn. (4) in order to take into account an unilateral effect, thus avoiding the development of visco-plastic displacement discontinuities in the normal direction when the interface is subjected to a compressive state of stress. The introduced visco-plastic interface model depends on nine parameters; parameter $N$ is non-dimensional, while the other parameters have the following dimensions:

$$K_i \ [F][L]^{-3}; \ a_i \ [F][L]^{-2}; \ h \ [L]^{-1}; \ \gamma \ [F][L]^{-1}[T]^{-1}; \ N$$

where F, L, T respectively denote Force, Length and Time.

When the parameter $h$ is positive, a softening effect is introduced in the model and progressive interface degradation can be described. This circumstance is exploited here in order to simulate delamination and debonding in composites.

3. The Extended Kalman Filter

In this section, the Extended Kalman Filter (EKF) [5], [6] is briefly presented for nonlinear systems which have been discretized in space and time. The output system variables are represented by measurable quantities in the volume $\Omega$ of the structure or on its surface $\partial\Omega$. Measurement is meant as the value of an output variable at a certain instant along the loading process. Dealing with discrete-time systems, the time interval of interest $T \equiv [\tau_0, \tau_{end}]$ is partitioned according to $[\tau_0, \tau_{end}] = \bigcup_{t=1}^{N} [\tau_{t-1}, \tau_t]$. During a typical finite time-step $[\tau_{t-1}, \tau_t]$ one is given the estimate of the model parameters gathered in the state vector $\hat{z}_{t-1}$ (where the hat stands for estimated value of $z_{t-1}$) at $\tau_{t-1}$ and the measurements $y_t$ at $\tau_t$. The EKF processes the information gained from measurements at $\tau_t$ to update the estimate of the state vector.

Let the state vector $z$ be assumed as a steady-state feature of the system, that is:

$$z_t = z_{t-1} \quad (6)$$

Equation (6) implies that no model errors are considered in this paper, namely that the only uncertainties in the numerical description of the system are related to measurements. The behaviour of the non-linear system can be described at time $\tau_t$ by the following measurement equation:
where the vector valued response function \( h_t(z_t) \) contains all the non-linearities of the model. The random vector \( v_t \) describes the measurement noise, which is assumed as a discrete white, uncorrelated random process with zero expected value and characterized by means of the measurement covariance matrix \( R_t \). By assuming that the system response \( h_t(z_t) \) is a continuous and continuously differentiable function of the model parameters, the response function can be expanded in Taylor series up to the first order about the estimate \( \hat{z}_{t-1} \) of the state vector at time \( \tau_{t-1} \). At time \( \tau_t \) the EKF furnishes the following improvement of the previous estimate \( \hat{z}_{t-1} \) by accounting for measurements \( y_t \):

\[
\hat{z}_t = \hat{z}_{t-1} + K_t \left( y_t - h_t(\hat{z}_{t-1}) \right)
\]

(8a)

\[
Q_t = Q_{t-1} - K_t \left( \frac{\partial h_t(z_t)}{\partial z_t} \right)_{z_t = \hat{z}_{t-1}} = Q_{t-1} - K_t H_t(\hat{z}_{t-1})Q_{t-1}
\]

(8b)

\[
K_t = Q_{t-1} H_t^T (z_{t-1}) \left[ H_t(\hat{z}_{t-1})Q_{t-1} H_t^T (z_{t-1}) + R_t \right]^{-1}
\]

(8c)

where the superscript \( T \) stands for transpose. In the above Eqns. \( K_t \) is the Kalman gain matrix; \( y_t - h_t(\hat{z}_{t-1}) \) represents the difference between the actual response \( y_t \) of the system at time \( \tau_t \) and its estimate based on the filtered state vector \( \hat{z}_{t-1} \) at time \( \tau_{t-1} \); \( Q_t \) is the current parameter error covariance matrix, defined as:

\[
Q_t = E \left[ (\hat{z}_t - z_t)(\hat{z}_t - z_t)^T \right]
\]

(9)

\( E[\bullet] \) being the expected value of \( \bullet \).

To initialize the iterative procedure reported by Eqns. (8a-c) the initial guess \( \hat{z}_0 \) of the state vector can be taken as any value of \( z \) belonging to an a-priori assumed state variable domain \( \Gamma_z \), given by:

\[
\Gamma_z = \{ (z) : z^M \leq z \leq z^m \}
\]

(10)

This initialization is finally checked by means of a global iteration technique, that is by adopting the final estimate \( \hat{z}_{\text{end}} \) at time \( \tau_{\text{end}} \) as a new initialization value for \( z \). This procedure is repeated until the attainment of convergence of the final estimate according to

\[
\frac{|(\hat{z}^{(\xi+1)}_{\text{end}}) - (\hat{z}^{\xi}_{\text{end}})|}{(\hat{z}^{(\xi+1)}_{\text{end}}) + (\hat{z}^{\xi}_{\text{end}})} \leq \text{Tol}
\]

being a prescribed tolerance and the left-hand superscripts \( (\xi + 1) \) and \( (\xi) \) denoting two subsequent global iterations.
The computation of the sensitivity matrix $H_t$ proves to be a very difficult task for FE analyses of elastoplastic structures (see, e.g., [8]). A simple approach to overcome the major difficulties has been proposed by Aoki et al. [7], already applied by the Authors in [9] and [10] and followed here throughout. This approach consists of computing functions $h_t$ through FE analyses for a discrete number of a-priori assumed state vectors (i.e. of material model parameters) defining a domain $\Gamma_z$. Within the domain $\Gamma_z$ an $n$-dimensional quadratic Lagrangian interpolation is employed, where $n$ is the number of model parameters to be identified gathered in the state vector $z$. The approximate sensitivity matrix $H_t$ can then be computed by taking the derivatives of functions $h_t$, as given by the Lagrangian interpolation, with respect to the state variable vector $z$.

The multi-dimensional quadratic interpolation requires $3^n$ FE computations. This procedure can be very costly if the number $n$ of model parameters to be identified increases over a critical threshold that depends on the computational burden of each FE analysis.


An example of identification of the visco-plastic interface model of Section 2 is here presented. In order to obtain pseudo-experimental data, which consist of specimen (system) response as a function of interface material parameters, use is made of a Double Cantilever Beam test. Numerical results have been obtained by means of plane strain finite element simulations.

The specimen is characterized by the following dimensions (see Fig. 2).

$$L = 170. \text{ (mm)}; \quad H = 3.5 \text{ (mm)}; \quad B = 20. \text{ (mm)}; \quad a_0 = 59.5 \text{ (mm)}$$

$B$ denoting the specimen width and $a_0$ the initial crack length.

The two arms of the specimen are assumed to behave according to transversely isotropic elasticity governed by the following parameters in the reference frame of Fig. 2.

$$E_{11} = 84766. \text{ (MPa)}; \quad E_{33} = 0.1E_{11}; \quad G_{13} = 1000. \text{ (MPa)}; \quad v_{13} = 0.035; \quad v_{32} = 0.35$$

As observed in Section 2, the visco-plastic interface model under discussion depends on nine parameters. In the example here presented only the behaviour in pure mode I of fracture is considered. Hence, the parameters to be considered are the following: $K_3, a_3, h, \gamma N$

To further simplify the analysis, the interface stiffness $K_3$ is assumed to be a priori known, that is: $K_3 = 100.000 \text{ (N/mm)}^3$.

The parameters to be identified are therefore only $a_3, h, \gamma$ and $N$. It is now important to notice that at the limit, for the imposed displacement discontinuity velocity $[\dot{u}] \to 0$ (or for the viscosity parameter $\gamma \to \infty$) the behaviour of the visco-plastic interface model of Section 2 coincides with that of the relevant time-independent one and is independent on the value of parameters $\gamma$ and $N$. This important feature of the interface model is here exploited in order to decouple the identification of parameters $\gamma$ and $N$ from that of parameters $a_3$ and $h$.

Let us consider first the identification of parameters $a_3$ and $h$. 

**Figure 2**

DCB specimen

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$</td>
<td>84766 (MPa)</td>
</tr>
<tr>
<td>$E_{33}$</td>
<td>0.1$E_{11}$</td>
</tr>
<tr>
<td>$G_{13}$</td>
<td>1000 (MPa)</td>
</tr>
<tr>
<td>$v_{13}$</td>
<td>0.035</td>
</tr>
<tr>
<td>$v_{32}$</td>
<td>0.35</td>
</tr>
</tbody>
</table>

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$L$ = 170 (mm); $H$ = 3.5 (mm); $B$ = 20 (mm); $a_0$ = 59.5 (mm)
The EKF procedure described in Section 3 is applied by constructing the Lagrangian interpolation of the response function on the bases of the following nodal values of parameters $a_3$ and $h$:

$$a_3^{(1)} = 10, \quad a_3^{(2)} = 80, \quad a_3^{(3)} = 150, \quad (MPa)$$

$$h^{(1)} = 100, \quad h^{(2)} = 150, \quad h^{(3)} = 200, \quad (mm^{-1})$$

Hence e.g. node 12 in the Lagrangian interpolation means the response corresponding to parameters $a_3 = a_3^{(1)} = 10; h = h^{(2)} = 150$. The responses of the DCB specimen, corresponding to the nine nodal couples of parameters, are shown in terms of load-time plots in Fig. 3.

Figure 3
DCB. Load-time plots at varying parameters $a_3$ and $h$.

Figure 4
DCB. Pseudo-experimental and identified load-time plots

All responses in Fig. 3 were obtained by imposing the vertical displacement at the loaded point with a constant opening velocity $v = 5 (mm/min)$.

For each plot, 20 measurement stations have been considered for the input of the EKF procedure. The identification was carried out at varying initialisation values of $\hat{z}$ inside the parameter domain: 100 initial values were chosen at a regular spacing in both directions of the 2D parameter domain.

In Fig. 4 a comparison is done between the load-time data used as pseudo-experimental input for the EKF procedure and the identified load-time plot, i.e. the numerical response of the DCB specimen obtained with the identified couple of parameters $a_3$ and $h$ given in Table 1. The data in Table 1 and the comparison in Fig. 4 show that the applied EKF was able to find a correct answer.

Table 1
Exact and identified values of parameters $a_3$ and $h$

<table>
<thead>
<tr>
<th></th>
<th>$a_3$ [N/mm²]</th>
<th>$h$ [1/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>exact</td>
<td>30</td>
<td>170</td>
</tr>
</tbody>
</table>
Let us now consider the identification of parameters $\gamma$ and $N$, having fixed the values of parameters $a_3$ and $h$ as follows: $a_3 = 80. \ (N/mm^2)$, $h = 150. \ (mm^{-1})$.

The following nodal values of parameters $\gamma$ and $N$ have been chosen to construct the Lagrangian interpolation of the response function:

$$\begin{align*}
\gamma^{(1)} &= 10. \\
\gamma^{(2)} &= 55. \\
\gamma^{(3)} &= 100. \ \ (N/mm \text{sec}) \\
N^{(1)} &= 1. \\
N^{(2)} &= 2. \\
N^{(3)} &= 3.
\end{align*}$$

The responses of the DCB specimen in terms of load-time plots corresponding to the nine nodal couples of parameters obtained by combinations of the above values are shown in Fig. 5. Responses in Fig. 5 were obtained by applying the vertical displacement in the loaded point with a constant opening velocity $v = 500 \ (mm/\text{min})$ for a time interval of 3.6 (sec) and then by maintaining the displacement fixed for other 3.5 (sec). In such a way a relaxation behaviour was obtained in the second part of the response. Also in the present case, the EKF procedure was applied by making use of load-time pseudo experimental data obtained through a numerical response corrupted with a measurement error $\delta = 1 \ (N)$.

The couples of parameters $\gamma$ and $N$ chosen to generate the pseudo-experimental response (the exact ones) and the average identified values obtained via the EKF procedure are shown in Table 2. As in the previous case, the identified values are averages over all values obtained at varying initial conditions inside the parameter domain: 100 initial values were chosen at a regular spacing in both directions of the 2D parameter domain.

| identified | 31.9 | 166.2 |
Table 2

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$ [N/mm sec]</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>exact</td>
<td>22.</td>
<td>2.5</td>
</tr>
<tr>
<td>identified</td>
<td>30.7</td>
<td>2.6</td>
</tr>
</tbody>
</table>

In Fig. 6 a comparison is done between the load-time data used as pseudo-experimental input and the load-time plot which corresponds to the numerical response of the DCB specimen obtained with the identified couple of parameters $\gamma$ and $N$ given in Table 2. From Fig. 6 it can be observed that the identified load-time plot is almost coincident with the pseudo-experimental one, in spite of the fact that the identified parameter $\gamma$ is different from the exact one.

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References