ECF 12 - FRACTURE FROM DEFECTS

EXPERIMENTAL AND NUMERICAL INVESTIGATIONS ON CRACK
DEVELOPMENT IN A CONCRETE SLAB SUBJECTED TO THERMAL SHOCK

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In this investigation a material law based on the results of the foregoing experimental and numerical studies was developed to describe the fracture mechanical behaviour of concrete under thermal load. This material law was applied in a numerical analysis of crack formation in a concrete slab subjected to thermal shock induced by a sudden weather change. Herein, the heterogeneity of concrete was simulated by means of a stochastic distribution of the material properties throughout the structure. To check the validity of the FE calculations a series of full-scale experiments was performed. The results of the experiments concerning the temperature distribution as well as the quantity, the periodicity and the development of cracks were found to be in a good agreement with the predictions of the numerical analysis.

INTRODUCTION

In the practice of construction the majority of concrete structures are exposed to more or less severe weather changes. The existing codes generally consider only the linear components of the stresses induced by the temperature or humidity fluctuations and neglect the eigenstresses caused by the non-linearity of the corresponding temperature or moisture gradients. The main reasons for this are evidently the lack of the general analytical solution algorithm to calculate the eigenstresses induced by such gradients and the difficulty to predict the damage evolution (crack development) caused by highly non-linear temperature and moisture fields. Consequently, for the concrete members under thermal load a numerical analysis has to be performed in order to allow the prediction of the eigenstresses and of their consequences. For this, both, a better knowledge of the fracture mechanical behaviour of concrete under such conditions and a suitable algorithm for the numerical calculations are required.

Thermal shock is the most severe case of the loads induced by weather changes. In the foregoing studies the temperature distribution in a concrete slab initially heated by sunshine and subsequently cooled by a sudden rain and hail was investigated by means of the FE method (Mechtcherine (1)). The numerical analysis of the temperature distribution and the corresponding thermal deformations in the slab during the thermal load provided parameters, in particular temperature and strain rates for the subsequent experimental investigation on concrete specimens. From the gained test data the characteristics of the material response, i.e. uniaxial tensile strength $f_t$, modulus of elasticity $E_0$, fracture toughness $K_I$, and fracture energy

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were evaluated (Mechtcherine et al. (2)). In addition, numerical calculations were performed considering specimens with the same geometries as used in the actual tests. In order to take into account the heterogeneity of the concrete, different calculation methods within the frame of the smeared crack concept were investigated (Mechtcherine (3)).

In this paper a material law to describe the fracture mechanical behaviour of concrete under thermal load is developed on the basis of the results obtained from the experimental and numerical investigations. Further, this material law is applied in a numerical analysis of crack development in concrete slabs subjected to thermal shock. Finally, the computer simulations are validated by means of a series of full-scale experiments.

**MATERIAL LAW FOR CONCRETE UNDER THERMAL LOAD**

The constitutive equations to describe the fracture behaviour of concrete under consideration of the effects of temperature and strain rate as observed in the experiments (2) were formulated within the frame of the cohesive crack type models (Hillerborg et al. (4), Bazant and Oh (5)). Hereby, a linear \( \sigma - e \)-relation was used to represent the stress-strain behaviour of the material outside the crack region. It is defined by the tensile strength \( f_t \) and the modulus of elasticity \( E_0 \) (Figure 1, above). The stress-crack opening relation for uniaxial tension was described as a bilinear curve (Figure 1, above). Following the test results, the characteristic "knee" of the \( \sigma - w \)-relation was defined to be at a tensile stress \( \sigma = 0.7 \) MPa for all loading conditions. The area under the complete stress-crack opening curve corresponds to the fracture energy \( G_f \) obtained from the uniaxial tension tests. The area under the first, steeper part of the descending branch extrapolated to the stress \( \sigma = 0 \) is defined as specific crack formation energy \( G_f^t \). This value was derived by means of an inverse numerical analysis of the fracture mechanical experiments (Mechtcherine (3)).

Analysing the experimental and numerical findings equations (1) to (3) were developed to describe the functional dependence of the \( f_t, E_0 \) and \( G_f^t \)-values from temperature and strain rate. Using these equations average values of the fracture mechanical parameters can be calculated for the temperature \( \theta \) and strain rate \( \dot{e} \) from the corresponding material parameters \( f_t^0, E_0^0, G_f^t \), which have been determined experimentally for a reference temperature \( \theta_0 \) at a reference strain rate \( \dot{e}_0 \):

\[
f_t = \left( f_t^0 + a_{f_t} (\theta^0 - \theta) \right) \left( 1 + b_{f_t} \cdot \log \left( \frac{E}{E_0^0} \right) \right)
\]

\[
E_0 = \left( E_0^0 + a_{E_0} (\theta^0 - \theta) \right) \left( 1 + b_{E_0} \cdot \log \left( \frac{E}{E_0^0} \right) \right)
\]

\[
G_f^t = \left( G_f^t_0 + a_{G_f^t} (\theta^0 - \theta) \right) \left( 1 + b_{G_f^t} \cdot \log \left( \frac{E}{E_0^0} \right) \right)
\]

where \( a_{param} \) and \( b_{param} \) are coefficients depending on the concrete composition and determined experimentally for the corresponding material parameters \( f_t, E_0 \) or \( G_f^t \).
Figure 1 (below) shows as an example the stress-strain and stress-crack opening relations for temperatures of 2 °C, 23 °C and 50 °C at a strain rate of $10^{-6}$ s$^{-1}$ determined from equations (1) to (3) for a normal strength concrete (compressive strength $f_c = 53$ MPa).

The first numerical analysis of a concrete slab under thermal shock using the described material law (which was implemented into the crack band model (5), compare Figure 1, above) led to the result, that in case of an uniform thermal load acting over the entire upper surface of the slab strains larger than the failure strain $\varepsilon_f = f_c/E_0$ arise in all elements of the slab surface, irrespective of the fineness of FE discretization. Therefore, in each element one or several „cracks“ develop and all these „cracks“ grow in the same way towards the interior of the slab. Hence, no information could be obtained on the quantity, periodicity and development of cracks. To obtain this information, the heterogeneity of concrete, which is usually neglected in comparable analyses, must be considered.

The heterogeneity was introduced considering the tensile strength $f_t$ to be an independent random variable following a Gaussian distribution. Its standard deviation was assumed to be equal to the standard deviation of the $f_t$-values obtained from the tension tests (2). The modulus of elasticity $E_0$ and the critical strain $\varepsilon_c$, were kept constant for all finite elements. For a better handling, the elements were subdivided into nine groups according to their tensile strength. To each group its own material law was assigned.

**CRACK DEVELOPMENT IN A CONCRETE SLAB SUBJECTED TO THERMAL SHOCK**

The derived material law modified by the stochastic approach as described above was applied in a new FE analysis of the damage evolution in a concrete slab under thermal shock. First, preliminary numerical investigations were performed to study the effect of the thermal load intensity and the slab geometry on the deformation and cracking behaviour of concrete. As a result, the cracks induced by thermal shock in thicker slabs were deeper as in the thinner slabs. This tendency is valid up to a slab thickness of approx. 600 mm. The minimal representative length of the structural member required for a reliable analysis of the periodicity of cracks was estimated by means of the FE analysis as well.

Based on the numerical analysis a concrete slab with the length of 2000 mm, the width of 800 mm and the thickness of 660 mm was chosen for full-scale experiments. The preliminary numerical investigations showed furthermore, that due to thermal shock numerous fine cracks will develop (crack openings at the surface of approximately 0.02 - 0.05 mm), while their location could not be foreseen. This finding showed the difficulty to quantify the opening of such cracks in an experiment. Because of that, a 15 mm deep notch was introduced in the middle of the slab to enable the permanent measurement of the opening of this pre-existing „crack“.

The numerical calculations, assuming an usual sunshine intensity, provided for normal weight concretes maximum temperatures at the upper slab side of approximately 50 °C (1), and showed no dramatic crack development as a result of the following sudden cooling (3). For this reason, for the full-size tests a more severe heating of the concrete slab up to a surface temperature of 80 °C was chosen in order to induce higher thermal gradients during the cooling phase and, hereby, to be on the safe side when assessing the possible damage of the concrete members exposed to sudden weather changes.
In the experiments the concrete slab was heated six hours by means of twelve infrared lamps with an entire radiation intensity of approx. 1350 W/m². Subsequently, the upper surface of the slab was cooled for 50 min by an ice-water mixture with a temperature of 4 °C. To achieve an uniform propagation of the temperature front toward the interior of the slab, its side areas were insulated by thick glasswool platinis. The temperature was measured by thermocouples throughout the vertical cross-section of the slab.

Parallel to the experiments numerical simulations were performed. In the FE analysis the slab was modelled by a mesh with an element length of 10 mm. The calculated and measured temperature distributions in the slab just before cooling as well as 10 and 50 minutes after the beginning of cooling are shown in Figure 2. Due to the sudden temperature change high thermal gradients arise, which lead to considerable eigenstresses. When reaching the tensile strength of the concrete these eigenstresses cause the formation of fine surface cracks. Due to the sustained cooling the cracks propagate—following the propagation of the cold front—toward the interior of the slab. Figure 3 shows the calculated distribution of stresses over the distance from the cooled surface for the vertical cross-section where a crack develops. The position of this cross-section is marked in Figure 4 by an arrow.

The distribution of strains \( e \) exceeding the ultimate strain \( e_u = f_t/E_0 \) over the concrete slab after 50 minutes of cooling gives information about the distribution, depths and widths of cracks caused by the thermal shock (Figure 4, above left). The crack widths at the surface of approx. 0.05 mm, the crack depths up to 50 - 60 mm and the distances between two larger neighbour cracks of approx. 150 - 200 mm are typical results of the numerical simulations. The crack pattern obtained in the full-scale tests corresponds very well to the numerically predicted crack pattern (Figure 4). The crack widths measured in the experiments are also in a good agreement with the calculated crack openings. Furthermore, a characteristic phenomenon was observed in the experiments as well as in the numerical simulation: after the termination of cooling the cracks close again to a great extent.

SUMMARY AND CONCLUSIONS

On the basis of foregoing experimental and numerical investigations a material law for concrete under thermal load was developed within the frame of the cohesive crack models. This material law was applied in a numerical analysis of crack development in a concrete slab subjected to thermal shock. Furthermore, a series of full-scale tests was performed to verify the calculations. The experimental findings were found to be in a good agreement with the results of the numerical simulations. The analysis of the results allows the conclusion, that severe thermal loads induced by weather changes may cause the development of fine cracks only, which will not exert any significant adverse effects on the durability of concrete structures. However, such cracks could have a negative effect on the bearing capacity of concrete members subjected to bending or tension caused by other loading cases (Maliha et al. (6)).

REFERENCES

(1) Mechtcherine, V., "Concrete structures under thermal loads", Diploma thesis (in German), Institute of Concrete Structures and Building Materials, University of Karlsruhe, 1992.

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**Fictitious Crack Model**

- \( f_t \) - Tensile strength
- \( \varepsilon_0 \) - Strain at \( f_t \)
- \( \sigma \) - Stress
- \( w \) - Crack opening
- \( G_c \) - Cohesive energy
- \( G_f \) - Fracture energy

**Crack Band Model**

- \( f_t \) - Tensile strength
- \( \varepsilon_0 \) - Strain at \( f_t \)
- \( \sigma \) - Stress
- \( w \) - Crack opening
- \( G_c \) - Cohesive energy
- \( G_f \) - Fracture energy

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Figure 1, above: cohesive crack models for the description of the behaviour of concrete subjected to tension: the fictitious crack model (left) and the crack band model (right); below: stress-strain and stress-crack opening relations for temperatures of 2°C, 23°C and 50°C at a strain rate of 10^-6 1/s for a normal strength concrete (compr. strength \( f_c = 53 \) MPa)
Figure 2 Calculated and measured temperature distributions over the distance from the slab surface after six hours of heating and following sudden cooling.

Figure 3 Calculated distributions of the main stresses (parallel to the length of the slab) over the distance from the slab surface for different times after the beginning of cooling.

Figure 4 Distribution of the strains exceeding the failure strain $\varepsilon_{f}$ over the concrete slab after 50 minutes of cooling (above left) and the typical crack pattern observed in the full-scale experiments: side view of one half of the concrete slab (below right).