THE CORRELATION BETWEEN CHARPY FRACTURE ENERGY AND FRACTURE TOUGHNESS FROM A THEORETICAL POINT OF VIEW

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Empirical correlation formulas are often used to estimate fracture toughness from the Charpy fracture energy - although they are known to be in general of poor accuracy and valid only with restrictions to certain classes of materials. In the present paper these relations are dealt with from a theoretical point of view, by extending an approximate single specimen method to determine dynamic J-R-curves from impact bending tests to the case of a classical Charpy test. Surprisingly - since there seems to be no direct physical relationship between crack initiation and the ductile fracture energy - a mathematical relation was found to hold between these two parameters. As expected, this relation is not unique, but dependent on some material properties like tensile strength and uniform fracture strain. The relation turned out to be in good agreement with experimental data and the current empirical correlation formulas, giving some theoretical support to the latter and confirming the present model.

INTRODUCTION

It often happens in practical applications of fracture mechanics that the fracture toughness of the considered material is not available, and, due to a lack of testing material or time, it can not be determined experimentally. In these cases, the only possibility is to estimate it from correlations with the Charpy energy, since the latter is known for most structural materials. Several empirical correlation formulas are suggested in the literature for this purpose [1-4] However, lacking a sound theoretical basis, they are known to be not generally valid, but to hold only for certain families of materials. Accordingly, they should be used with appropriate caution.

From a theoretical point of view, the existence of a mathematical relation between fracture toughness (in terms of $K_I$ or $J_I$) and Charpy fracture energy, $K_V$, seems to be unlikely, because they represent physical processes that are rather different. In $K_V$, there are energy contributions from crack tip blunting, ductile crack initiation and, mainly, tearing crack growth and plastic deformation of the ligament, whereas $J_I$ or $K_I$ characterise just initiation of crack growth. However, as shown in the present paper, by using a simplified two-parameter model of ductile crack extension, a theoretical relation

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between the total fracture energy and the initial J-R-curve can be established. It is based on the evaluation procedure to estimate a J-R-curve from an instrumented impact test in three-point bending as described in [5, 6]. By some modifications and simplifications that are discussed in the following closed form formulas could be derived that relate Charpy fracture energy to the fracture toughness. Comparison with experimental data and the current empirical correlation formulas showed good qualitative and quantitative agreement.

**DETERMINATION OF J-R-CURVES FROM A LOAD-DISPLACEMENT DIAGRAM**

![Fig. 1: Mechanical system of a Charpy-type impact test.](image)

Before turning to the classical Charpy test we first consider the case of an instrumented impact bending test performed on an edge-cracked beam in three-point bending. (Fig. 1). In [5] a simple analytical procedure is presented that enables the J-R-curve to be estimated from a single continuous force-displacement record (Fig. 2). Since it plays a key role in the present analysis, the derivation is shortly outlined in the following. It is based on the assumption that ductile crack initiation and growth up to the point of maximum force, \( F = F_{m} \), is J-controlled, following a potential law of the form

\[
J(\Delta a) = C \cdot \Delta a^{p} \quad \text{for } \Delta a < \Delta a_{m} \tag{1}
\]

(range I of the J-R-curve, see Fig. 3), where \( C \) and \( p \) are material-dependent constants, \( \Delta a \) the crack extension and \( \Delta a_{m} \) its value at maximum force \( F = F_{m} \). The second assumption is the tearing crack growth process beyond the J-controlled range (range II in Fig. 3: \( \Delta a > \Delta a_{m} \)) to be governed by a constant crack tip opening angle CTOA, which enables one to find a relation between the tearing crack extension at \( \Delta a > \Delta a_{m} \) and dissipated energy \( E_{d}(\Delta a - \Delta a_{m}) \), which determines the extension of the J-R-curve at its transition from the J-controlled region to the CTOA-controlled region at \( \Delta a = \Delta a_{m} \) by

\[
J(\Delta a) = \frac{\eta \cdot E_{m}}{B \cdot b_{0}} - \frac{\eta \cdot E_{d}(\Delta a - \Delta a_{m})}{B \cdot (b_{0} - \Delta a_{m})} \quad \text{for } \Delta a > \Delta a_{m} \tag{2}
\]

where \( \eta \) is the well known \( \eta \)-factor, being about 2 for deep cracks and, according to [6],

\[
\eta = 13.81 \cdot \frac{a}{W} - 25.12 \left( \frac{a}{W} \right)^{2} \quad \text{for } 0 < a/W < 0.275 \tag{3}
\]

The matching and continuity conditions between (1) and (2) at \( \Delta a = \Delta a_{m} \) represent the equations to determine the unknowns \( C, p \) and \( \Delta a_{m} \). One obtains.
\[ C = \left( \frac{2}{p} \right)^\gamma \cdot \frac{n(a)}{B_p [W - a]^{1 - \gamma}} \cdot E_{mp}^{1 - \gamma} \cdot E_{mp} \]  

\[ p = \frac{3}{4} \left( \frac{E_{mp}}{E_{tot}} \right)^\gamma \]  

\[ \Delta a_n = \frac{E_{mp} \cdot p \cdot b_0}{2E_{tot}} \]  

Thus, the J-R-curve in the form (1) is determined by only two experimental parameters, \( E_{mp} \) and \( E_{tot} \), which both can be obtained very easily as the corresponding areas from the force-displacement diagram that is delivered by the instrumented pendulum hammer (Fig. 2). The agreement of J-R-curves estimated by (1) and (4), (5) with "exact" J-R-curves evaluated by multi-specimen techniques was shown to be relatively good [5, 8].

**Fig. 2:** Force-Deflection diagram of an instrumented Charpy test  
**Fig. 3:** J-R-curve and definition of near-initiation values \( J_{0.2\%} \) and \( J_0 \)

**ESTIMATION OF THE J-R-CURVE FROM THE FRACTURE ENERGY**

In the case of a non-instrumented impact test like the classical Charpy test, the only available experimental parameter is the total fracture energy \( E_{tot} \), whereas \( E_{mp} \) is not available. By using further conditions and assumptions it is possible to eliminate the latter from (4) - (6). At \( \Delta a = \Delta a_n \) the condition for maximum force \( F \) requires

\[ \frac{dF}{d\Delta a} (\Delta a = \Delta a_n) = 0 \]  

According to [10], the general functional relationship between \( J \) and \( F \) is
\[ J = \left( \frac{F}{F_0} \right)^{\gamma/a} \]  

where \( n \) is introduced as the hardening exponent according to the material law

\[ \sigma = A \cdot e^a \]  

(9)

The reference force \( F_0 \) is suitably assumed to be the plastic limit load, which depends on the actual ligament width \( b_0 - \Delta a \) by

\[ F_0 \propto (b_0 - \Delta a)^2 \]  

(10)

With (8), (10) one obtains from (7) in a first order approximation (since \( n < 1 \) and \( p < 1 \)):

\[ \Delta a_n = \frac{n \cdot p \cdot b_0}{2} \]  

(11)

By comparing (6) with (11) one finds

\[ E_{mp} = n \cdot E_{tot} \]  

(12)

Therewith, \( E_{mp} \) can be eliminated from (4) to (6), leaving the total fracture energy \( E_{tot} \) as the only experimental parameter therein. The hardening exponent \( n \) is known to be approximately equal to uniform fracture strain \( \Delta \epsilon_p \), which is the plastic strain at maximum load of a uniaxial tensile test (strictly speaking the true strain, but regarding the approximate nature of the present derivation, we neglect this difference), thus

\[ n = \Delta \epsilon_p. \]  

(13)

With (12) and (13), one obtains from (1) and (4) to (6)

\[ J(\Delta a) = \frac{2}{p} \eta e^{a} B (W - a)^{1/p} \cdot E_{tot} \cdot \Delta \epsilon_p \cdot \Delta a^e \]  

(14)

where

\[ p = \frac{3}{4} \left( 1 + \frac{1}{\Delta \epsilon_p} \right)^{-1} \]  

(15)

According to an experimental investigation [9] the total fracture energy in ductile fracture is not sensitive to the shape of the initial crack or notch. Thus, this procedure to estimate the J-R-curve is approximately applicable to standard Charpy tests as well. In this case, \( E_{tot} = KV \), \( B=W=10 \text{mm}, \Delta \epsilon_p=2 \text{mm} \) and \( \eta=1.76 \). The exponent \( p \) is not very sensitive to \( \Delta \epsilon_p \), thus, in order to get an equation as simple as possible, \( p \) is set to be \( p=2/3 \). Therewith one obtains

\[ J(\Delta a) \equiv 11.44 \cdot KV \cdot A_{\Delta \epsilon} \cdot \Delta a^{2/3} \]  

(16)

(J in N/mm, KV in J)

This equation enables the J-R-curve to be estimated just from the upper shelf Charpy energy \( KV \), and the uniform fracture strain of the uniaxial tensile test, \( \Delta \epsilon_p \).
DETERMINATION OF FRACTURE TOUGHNESS

From the estimated J-R-curve near-initiation J-values can be determined analogously to standards in quasi-static testing. However, as noticed in [5, 11], small specimens tend to deliver too high \(J_{0.2a}\). Three reasonable alternatives \(J_0\), \(J_{0.2}\) and \(J_{0.2a}\) are defined in Fig. 3. The first one is obtained by the intersection of the linear extrapolation of the J-R-curve from the range \(\Delta a>\Delta a_0\) to the intersection with the blunting line, the second the value of J at \(\Delta a=0.2\text{mm}\) and the third its value at a distance 0.2mm from the intersection with the blunting line. According to [12] the blunting line is given by (with KV in J, J in N/mm):

\[
J = s_1 \cdot \Delta a = 3.75 R_m \cdot \Delta a
\]

(17)

(where \(R_m\) denotes the ultimate tensile strength) which seems to be conservative even for Charpy-type specimens [11]. Working out \(J_0\), \(J_{0.2}\) and \(J_{0.2a}\) as defined in Fig. 3 mathematically on the basis of (16) and (17) leads to

\[
J_0 = \frac{7.33 \cdot A_p \cdot KV}{1 - 1.47 \cdot \frac{KV}{R_m}} \]

(18)

\[
J_{0.2} = 3.92 \cdot A_p \cdot \frac{\sqrt{\gamma}}{K} \cdot KV
\]

(19)

\[
J_{0.2a} = 11.44 \cdot A_p \cdot \frac{\sqrt{\gamma}}{K} \cdot \left[ \left( \frac{3.05 \cdot KV}{R_m} \right)^\gamma + A_p + 0.2 \right]^{\frac{\gamma}{2}}
\]

(20)

Which of these three candidates is the most appropriate to characterise fracture toughness is investigated in the next section by comparing them with experimental data and with current empirical correlation laws, which are also based on experimental data.

COMPARISON WITH EXPERIMENTAL RESULTS

In Fig. 4 the comparison of (18) - (20) with the empirical correlation formulas suggested in [2], [3] and [4] is shown for a typical structural steel with \(R_m=650\text{MPa}\) and \(A_p=15\%\). Fig. 5 shows the comparison with data from the literature and own data. Fig. 6 shows a comparison of with the experimental data of [1], which are upper shelf values of relatively high strength steel in the range of 700 MPa < \(R_m\) < 1700 MPa.

Generally, the agreement between the analytical formulas and experimental data is surprisingly good - considering the
fact that no major empirical adjustments was introduced in the theoretical derivation. It seems that (18) performs best for low- to medium strength steel, whereas (19) and (20) give better (i.e. less conservative) results for higher strength steels with R_p>760 MPa.

**Fig. 5** Comparison with experimental data for typical structural steel (R_p<700MPa) **Fig. 6** Comparison of estimated and measured fracture toughness data (from [11]) for higher strength steel (R_p>760MPa)

**CONCLUSIONS**

This investigation shows that there are some unexpectedly strong theoretical relations between Charpy fracture energy and fracture toughness. First comparisons with experimental results exhibits relatively good agreement, which confirms that the used simple model is basically well suited to analyse ductile fracture and gives theoretical support to the existing empirical correlation formulas, therewith strengthening the confidence in the latter. Unlike the purely empirical formulas, the analytical equations additionally quantifies the effect of further material properties, like the uniform fracture strain and the tensile strength, which corresponds qualitatively to experimental results. It is expected that on the basis of the present analytical results and further experimental data, improved semi-empirical correlation formulas can be developed.

**REFERENCES**