FRACTURE MECHANICS METHODS FOR RESIDUAL STRESS EVALUATION IN PLANE PROBLEMS

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The progressive cutting technique for residual stress evaluation based on a fracture mechanics approach is reviewed and extended to a more general plane problem.

A crack-like cut is supposed to be produced in a plane body affected by residual stresses in order to measure relaxed strain. The crack is assumed to be straight but not necessarily perpendicular to the surface or lying in a plane of symmetry. In such a general condition, residual stress released on the crack-faces has both normal and shear components. The relationship between released residual stresses and measured relaxed strains is discussed for a general position of the strain-gage. The corresponding influence functions are obtained by taking into account Mode I and Mode II Loadings and their interaction by the application of the Weight Function Method.

INTRODUCTION

In the recent years, Cheng and Finnie (1,4) have developed the "crack compliance method" for residual stress measurement, based on Linear Elastic Fracture Mechanics (LEFM) concepts. In this method, preexisting residual stresses are realised by the introduction of a narrow cut of increasing length and, consequently, a stress redistribution in the whole body takes place thus generating a strain relaxation that can be measured by strain gages. Due to its narrowness, the cut may be reasonably assumed to be a crack and moreover LEFM methods are supposed applicable to relate the released residual stress along the cut and the measured strain. The method has been successfully applied either for through the thickness or for near the surface residual stress measurement also in the presence of relevant stress gradients. Similar approaches have been presented by Kang and Seol (5), as the "progressive cracking" method, and by Fett (6).

The theoretical foundations and the above mentioned applications of the method were limited to cases where only Mode I Loading was active. In this paper the basis for the extension of the method to a more general plane problem is proposed by assuming a Mixed Mode I-II Loading.

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A CRACK IN A RESIDUAL STRESS FIELD

Let us consider a homogeneous isotropic plane body free from external loading and affected by residual stresses, for instance due to manufacturing processes. The following hypotheses are also assumed: the introduction of the crack-like cut produces no proper residual stress, relaxed strain is elastic at any point and LEFM is applicable.

Pure Mode I loading

Let us consider first a particular case where both the body and the residual stress field are symmetric with respect to the same axis and a straight crack-like cut is introduced along this axis starting from the surface. Due to symmetry, residual stresses on the crack line have only a normal component and only Mode I Loading is produced by the cut. On the basis of the Weight Function Method, the SIF can be obtained by the following relationship:

$$K_I(a) = \int_0^a h(s,a) \cdot \sigma_{res}(s) \cdot ds \tag{1}$$

where $h(s,a)$ is the Weight Function for the geometry in exam and $\sigma_{res}(s)$ is the original residual stress acting along the crack-line. The term "original" indicates the residual stress affecting the body before the introduction of the cut. In the following the symmetry hypothesis will be dropped and an arbitrarily oriented crack not necessarily lying in the centre of the body will be considered (Fig. 1).

Mixed Mode I-II and their reciprocal influence

As a general plane crack problem was assumed, still limited to straight cracks, a Mixed Mode I-II Loading is expected due to two different reasons: (1) the original residual stresses on the crack line can have both normal and shear components and (2) the crack itself may not lie in the symmetry plane (see Fig.2). Indeed only for a crack in a symmetry axis under a general loading, normal stresses produce Mode I Loading, shear stresses produce Mode II and the effects of normal and shear components on the SIFs can be uncoupled. On the contrary, if the crack is not on the symmetry axis (because it is angled or off axis as in Fig.3) the problem is more complex and a general Mixed Mode I-II is produced by either the normal or the shear components. This requires a careful application of the Weight Function technique as recently shown by Beghini, Bertini and Fontanari (7).

In this general case, the SIFs can be obtained by the following expressions in which four different Weight Functions are involved:

$$K_I(a) = \int_0^a h_{I,\sigma}(s,a) \cdot \sigma_{res}(s) \cdot ds + \int_0^a h_{I,\tau}(s,a) \cdot \tau_{res}(s) \cdot ds \tag{2a}$$

$$K_{II}(a) = \int_0^a h_{II,\tau}(s,a) \cdot \tau_{res}(s) \cdot ds + \int_0^a h_{II,\sigma}(s,a) \cdot \sigma_{res}(s) \cdot ds \tag{2b}$$

Although it may sound strange, shear components can produce Mode I Loading, that is displacements normal to the crack line and, vice versa, tractions on the crack faces cause Mode II Loading with shear displacements. A physical explanation of this coupling crack line divides the body.

Unfortunately in many cases, the application of equations (2a) and (2b) is still a purely theoretical approach, due to the luck of available complete Weight Functions including the "mixed" components, $h_{I,\tau}(s,a)$ and $h_{II,\sigma}(s,a)$.

RESIDUAL STRESS MEASUREMENTS

On the basis of the previous general analysis, if the complete set of Weight Functions was available, residual stresses could be obtained having the SIFs for several crack lengths and solving the integral equations (2a, b). However the SIFs cannot be measured directly with the necessary accuracy and it is preferred to make an indirect evaluation on the basis of a sensor having a higher sensitivity. To this purpose strain gages are usually employed and a relationship between the SIFs and the strain at the gage location is required.

Let us consider a strain gage applied on the body contour with an active grid (having length t) between the points P and Q (fig. 3) and an acquisition system which records the strain measured as a function of the crack length. The solution of the problem requires a function relating the measured strain to the crack length and the original residual stress. In the following, the body will be assumed constraint free or, which is equivalent, constrained with not redundancy. Castigliano's theorem may be applied by imposing a couple of opposite forces parallel to the contour each having intensity F, the first one applied in P and the other one in Q. The energy change δU due to a virtual crack extension δa is related to the change in distance between P and Q by the following:

$$\delta\left(u_Q - u_P\right) = \left[\frac{\partial \delta U}{\partial F}\right]_{F=0} \tag{3}$$

where u_Q and u_P indicate the displacement components, in the direction of the forces F, of Q and P respectively. As the strain-gage measures the mean strain on the grid length, the reading due to a crack extension is given by:

$$\delta\varepsilon = \frac{\delta\left(u_Q - u_P\right)}{t} = \left[\frac{1}{t}\left(\frac{\partial \delta U}{\partial F}\right)\right]_{F=0} \tag{4}$$

The quantity δU can be obtained also by the LEFM approach:

$$\delta U = \frac{B}{E'}\left(K_I^{\,2} + K_{II}^{\,2}\right)\delta a = \frac{B}{E'}\left[\left(K_{I,res} + K_{I,F}\right)^2 + \left(K_{II,res} + K_{II,F}\right)^2\right]\delta a \tag{5}$$

where $E'=E$ for plane stress and $E' = E/\left(1-\nu^2\right)$ for plane strain and B is the body thickness. It is worth noting that the SIFs are produced both by residual stresses and by forces F (see Fig. 3). Combining eqn.s (5) and (4), being $F=0$ and $\partial K_F / \partial F = K_F / F$, the following relationship can be obtained:

$$\delta\varepsilon = \frac{2B}{E'F}\frac{1}{t}\left(K_{I,res}K_{I,F} + K_{II,res}K_{II,F}\right)\delta a \tag{6}$$

which gives the effect of residual stresses on the strain gage measurement through the SIF which gives the effect of residual stresses on the strain gage measurement through the SIF values. In order to determine the original residual stress distribution $\sigma_{res}(s)$, $\tau_{res}(s)$ acting on the cracked path, equations (2a,b) have to be considered too:

$$\frac{\delta\varepsilon}{\delta a}\frac{E'}{2B} = \int_0^a \left[h_{I,\sigma}(s,a)\sigma_{res}(s) + h_{I,\tau}(s,a)\tau_{res}(s)\right]ds \cdot g_I(a) + \\ + \int_0^a \left[h_{II,\tau}(s,a)\tau_{res}(s) + h_{II,\sigma}(s,a)\sigma_{res}(s)\right]ds \cdot g_{II}(a) \tag{7}$$

where

$$g_I(a) = \frac{1}{t\,F}\int_0^a \left[h_{I,\sigma}(s,a)\,\sigma_F(t,s) + h_{I,\tau}(s,a)\,\tau_F(t,s)\right]ds \\ g_{II}(a) = \frac{1}{t\,F}\int_0^a \left[h_{II,\tau}(s,a)\,\tau_F(t,s) + h_{II,\sigma}(s,a)\,\sigma_F(t,s)\right]ds \tag{8}$$

In equation (7): $\delta\varepsilon$ is a measurable quantity, δa is imposed during cutting; $h_{I,\sigma}(s,a)$, $h_{I,\tau}(s,a)$, $h_{II,\tau}(s,a)$, $h_{II,\sigma}(s,a)$ are supposed known Weight Functions, $\sigma_F(t,s)$, $\tau_F(t,s)$ are the stress components produced by couple of forces F on the crack path for the uncracked body. These stress functions depend on the dimension and on the position of the strain-gage and have to be determined, for example by Finite Element calculations. In some cases the analytical expressions for $\sigma_F(t,s)$ and $\tau_F(t,s)$ are known, (e.a. for a surface crack in a semiplane or in a ring).

In equation (7) the different contributions on the strain measurement become evident: the effect of residual stress is weighted by a function depending on position and dimension of the strain gage (through $g_I(a)$ and $g_{II}(a)$) and on the crack geometry (through the Weight Functions).

If the functions $\sigma_F(t,s)$ and $\tau_F(t,s)$ are known, $g_I(a)$ and $g_{II}(a)$ can be determined analytically by a simple integration. On the contrary, an experimental approach can be applied as well. A residual stress free specimen having the same geometry and strain-gage arrangement is necessary. The specimen should be loaded in order to produce a known stress. In the case that an original uniform tension σ is produced on the crack path, eqn. (7) becomes:

$$\frac{\delta\varepsilon}{\delta a}\frac{E'}{2B} = \int_0^a h_{I,\sigma}(s,a)\sigma\,ds \cdot g_I(a) + \int_0^a h_{II,\sigma}(s,a)\sigma\,ds \cdot g_{II}(a) \tag{9}$$

where integrals can be easily calculated and a linear equation in the two unknowns $g_I(a)$ and $g_{II}(a)$ is obtained for every crack length. In order to solve this problem two loading conditions are required producing stress distributions not proportional each other.

As the $g_I(a)$ and $g_{II}(a)$ functions have been determined, residual stress components can be obtained by solving the integral equation (7). To this purpose, series expansions of the following form:

can be assumed for the unknown functions, being $\Psi_i(s)$ a suitable basis of known functions and σ_i and τ_i unknown coefficients. By combining eqns. (10) and (7), the following equation holds:

$$\frac{\delta\varepsilon}{\delta a}\frac{E'}{2B} = \sum_{i=0}^{n}\sigma_i\left[g_I(a)\int_0^a h_{I,\sigma}(s,a)\Psi_i(s)ds + g_{II}(a)\int_0^a h_{II,\sigma}(s,a)\Psi_i(s)ds\right]$$
$$+\sum_{i=0}^{n}\tau_i\left[g_I(a)\int_0^a h_{I,\tau}(s,a)\Psi_i(s)ds + g_{II}(a)\int_0^a h_{II,\tau}(s,a)\Psi_i(s)ds\right] \tag{11}$$

that can be formally simplified in the following relationship:

$$\frac{\delta\varepsilon}{\delta a}\frac{E'}{2B} = \sum_{i=0}^{n}\left[\sigma_i\, c_{\sigma_i}(a) + \tau_i\, c_{\tau_i}(a)\right] \tag{12}$$

For any crack length and strain gage, a linear equation containing $2(n+1)$ unknowns σ_i and τ_i can be written. The necessary equations may be obtained by increasing the number p of strain gages and the number m of partial crack lengths. In general, it must be $p \cdot m \geq 2(n+1)$, if the number of equations is larger than the unknowns, the least square solution of the linear system can be looked for. A rational choice for the number of strain-gages and their location can be assisted by an analysis of stability of the system (12).

CONCLUSIONS

The "crack compliance method" for residual stress measurement was proposed for a more general plane problem in order to extend the applicability of the technique to more complex cases. The knowledge of the complete set of Weight Functions for the considered crack geometry is required and both normal and shear residual stress components can be obtained. The procedure leads to a final linear system whose mathematical properties can be used to define optimal experimental arrangements thus improving the sensitivity of measurements.

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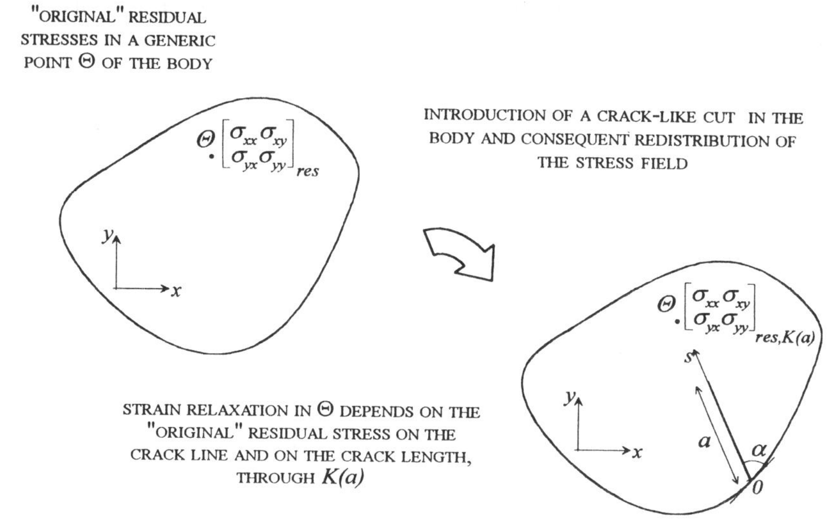
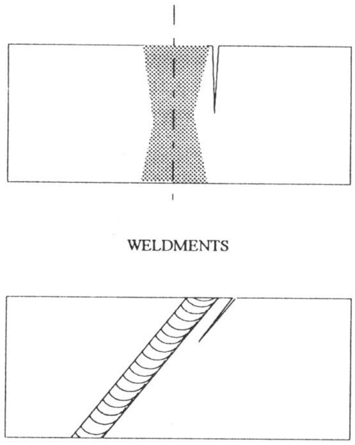


Figure 1. Plane body with residual stress: before and after the introduction of a crack.



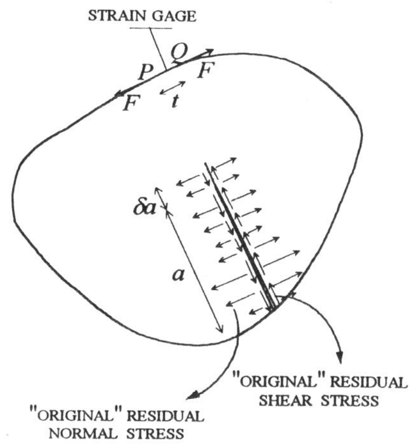


Figure 2. Examples of geometries in Mixed Mode I-II.

Figure 3. Application of Castigliano's theorem; strain-gage is applied along PQ.