The probabilistic model for fracture toughness prediction has been elaborated on the basis of the local criteria for cleavage and ductile fracture criterion proposed by authors and the approximated solution for problem about stress-strain state near the crack tip. This probabilistic model allows one to predict the dependence $K_{IC}$ on temperature for any given probability of brittle fracture; to describe the effect of the specimen thickness on $K_{IC}$ to determine the scatter band for the brittle to ductile transition temperature. Comparison of experimental and calculated results has been performed as applied to 2.5Cr-Mo-V nuclear pressure vessel steel.

**INTRODUCTION**

The new local criterion for cleavage fracture has been formulated as (Margolin et al. (1))

$$\sigma_1 + m_1 \sigma_{eff} \geq \sigma_s$$  \hspace{1cm} (1a)

$$\sigma_1 \geq S_e(\omega)$$  \hspace{1cm} (1b)

where $\sigma_1$ - the maximum principal stress, $m_1$ - parameter which may be interpreted as the concentrator coefficient for stress in the dislocation pile-up tip, the effective stress $\sigma_{eff} = \sigma_{eq} - \sigma_Y$, $\sigma_{eq}$ - the equivalent stress, $\sigma_Y$ - the yield stress, $\sigma_s$ - strength of carbides or another particles, on which the cleavage microcracks are nucleated, $S_c$ - the critical brittle fracture stress, $\omega = \int \delta e_p$ - Odqvist’s parameter, $\delta e_p$ - the equivalent plastic strain increment. Condition (1a) in the local cleavage fracture criterion is the nucleation condition for cleavage microcracks, condition (1b) - the propagation condition for cleavage microcracks. Parameter $m_1$ may be represented as $m_1 = m_1 m_n$, where parameter $m_1$ depends on temperature $T$ only and decreases as temperature increases and parameter $m_n$ depends on $\omega$.

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As the local criterion for ductile fracture the plastic collapse criterion has been formulated as (Margolin et al. (2))

$$\frac{dF_{el}}{d\varepsilon} = 0$$

(2)

where $F_{el} = \sigma_{el}(1 - S L)$. $SL$ - the relative area of voids. The value of $SL$ is calculated by equations for void nucleation and growth which have been obtained in (2).

On the basis of the above local criteria for cleavage and ductile fracture the deterministic model has been proposed for prediction of fracture toughness $K_{IC}$ in (Margolin et al. (3)). The proposed model gives a very adequate prediction for the dependence $K_{IC}(T)$ as applied to nuclear pressure vessel steels. Being the deterministic the proposed model allows one to predict only the average values for fracture toughness, but does not allow one to evaluate the scatter for experimental data.

The aim of the present work is to formulate the local criterion for cleavage fracture (1) in probabilistic statement and to elaborate the probabilistic model for fracture toughness prediction.

ANALYSIS FOR STOCHASTIC SENSITIVITY OF CRITICAL PARAMETERS IN CLEAVAGE FRACTURE CRITERION.

It has been shown in (1) that brittle fracture of cylindrical smooth and notched specimens from nuclear pressure vessel steels may be controlled by different conditions. Brittle fracture of smooth specimens is controlled by the cleavage microcrack propagation condition (1b) and the condition (1a) is fulfilled at some stress less than the critical one. On the contrary, brittle fracture of notched specimens is controlled by the condition of cleavage microcrack nucleation (1a) and the condition (1b) is fulfilled at stress less than the critical one. This interpretation allows one to resolve the known contradiction, which arises under comparison of test results for smooth and notched specimens. It is known (1), Beremin (4) that the maximum principal stresses $\sigma_1$ in the ruptured section of specimen for the fracture moment are different for smooth and notched specimens and $\sigma_{1\text{notch}} > \sigma_{1\text{smooth}}$. It is usually taken (4), Knott (5)) that brittle fracture is always controlled by condition $\sigma_{1} = \sigma_{C}$, therefore $S_{C} = \sigma_{C}$. Then the conclusion have to be drawn that $S_{C\text{notch}} > S_{C\text{smooth}}$. This conclusion contradicts the known statement about invariance of $S_{C}$ to stress triaxiality (Davidenko (6)). Attempt has been made to explain such a contradiction by stochastic nature for parameter $S_{C}$ (4); it has been taken that the volume of plastic deformed metal in smooth cylindrical specimen is larger than in notched specimen with the same net-section diameter. This explanation contradicts to experimental results on brittle fracture of corset specimens with 3.0 mm diameter and notched specimens (the notch radius R = 0.5 mm) with the minimum diameter of 4.75 mm. In both specimens plastic deformation occurs on the whole net-section and localizes on a small height adhering to the net-section. Test results show that the average and maximum fracture stresses are larger for notched specimen than for corset specimen (Fig. 1), although the diameter of notched specimen is larger than the corset specimen diameter.
It should be pointed out that when using the criterion (1) the above contradiction does not appear as brittle fracture of smooth (and corset) specimens is controlled by condition (1b), and brittle fracture of notched specimens - by condition (1a).

Brittle fracture is well known to be of a stochastic nature. The question is now which critical parameter may be taken as stochastic parameter. Now we consider the stochastic sensitivity of parameters in the local cleavage fracture criterion (1). To evaluate the stochastic sensitivity of some parameter the effect of the size scale on this parameter may be studied. Results represented in (Margolin et al (7)) show clearly that parameter \( S_c \) is a low sensitive to specimen size, and, hence, we can conclude that this parameter has a small scatter.

The critical parameters \( \sigma_a \) and \( n_1 \) controlling the cleavage microcrack nucleation according to condition (1a) may be determined by testing cylindrical notched specimens (1). These tests show that fracture stresses for notched specimens depend on the size of specimen. The performed in (7) analysis has shown that parameter \( \sigma_a \) appears to be the stochastic parameter.

Thus, from three parameters \( S_c, \sigma_a \) and \( n_1 \) controlling the cleavage fracture only one parameter \( \sigma_a \) has to be taken as the stochastic parameter.

FORMULATION OF THE LOCAL CRITERION FOR CLEAVAGE FRACTURE IN PROBABILISTIC STATEMENT

Main Considerations

1. The polycrystalline material is viewed as an aggregate of unit cells. The size of the unit cell \( \rho_u \) is never less than the average grain size. Stress-strain state in the unit cell is assumed to be homogeneous.

2. As the local criterion for cleavage fracture in probabilistic statement it is taken the criterion (1) in which it is assumed that the parameter \( \sigma_a \) is the stochastic parameter and the rest of parameters are the deterministic ones.

3. The model of the weakest link is used to describe the brittle fracture.

4. The minimum strength of carbides in the unit cell which nucleate the cleavage microcracks is assumed to obey the Weibull distribution

   \[
   p(\sigma_d) = 1 - \exp\left( - \frac{\sigma_d}{\bar{\sigma}_d} \right)^\eta,
   \]

   where \( p(\sigma_d) \) - the probability to find in the unit cell a carbide with the minimum strength less than \( \sigma_d, \bar{\sigma}_d \) and \( \eta \) - the Weibull parameters.

5. It is taken that the brittle fracture may happen only in the unit cells for which the condition \( \sigma_a \geq \sigma_1 \) is satisfied and which locate in the plane of fracture propagation (for example, in the plane of the net-section for notched specimen).

6. The probability of non-fracture (as an occurrence is opposite to fracture) of the unit cell is taken to equal to one if for this unit cell the condition \( \sigma_f < S_c(\infty) \) is satisfied.
Calculation for Probability of Brittle Fracture of Specimen

Now we calculate the brittle fracture probability for specimen in which stress-strain state is heterogeneous. Let this stress-strain state be characterized by three parameters $\sigma^i_1$, $\sigma^i_{det}$ and $\varepsilon_i$ (here $i$ - the number of unit cell) which satisfy to item 5.

The brittle fracture probability $P_i$ of specimen which consists of $k$ unit cells is determined according to item 3 as

$$P_i = 1 - \exp \left( -\frac{1}{(\bar{\sigma}_d)^\eta} \sum_{i=1}^{k} (\sigma^i_{acc})^\eta \right),$$

(4)

where $\sigma^i_{acc} = \sigma^i_1 + m_1 m_c (\bar{\sigma}_d) \sigma^i_{det}$, if for some $i$-th unit cell the condition $\sigma^i_1 < S_c (\bar{\sigma}_d)$ is fulfilled then this unit cell is excluded from consideration and in (4) it is taken $\sigma^i_{acc} = 0$.

The procedure for determination of parameters $\bar{\sigma}_d$ and $\eta$ has been elaborated in (7). This procedure is based on data about fracture loads for two size scales of cylindrical specimens with circular notch for which brittle fracture is controlled by condition (1a). Parameters $\bar{\sigma}_d$ and $\eta$ have been determined as applied to 2.5Cr-Mo-V nuclear pressure vessel steel: $\bar{\sigma}_d = 9700$ MPa and $\eta = 13$.

THE PROBABILISTIC MODEL FOR $K_{IC}$ PREDICTION

Main Considerations for Analytical Description of $K_{IC}(T)$ Curve.

1. The proposed local criteria for cleavage and ductile fracture are used.
2. The probability of brittle fracture is calculated according to (4).
3. The brittle to ductile transition at given temperature happens for the value $K^*_T$ at which

$$P_T(K^*_T)_{T=const} = \max P_T(K_i).$$

(5)

This condition may be explained by the following reason. The probability of brittle fracture is not a monotonous increasing function of $K_i$. The function $P_T(K_i)$ has the maximum value as the probability of brittle fracture is determined by two opposite factors. As the value $K_i$ increases the value $\sigma^i_{acc}$ increases and, consequently, the probability of brittle fracture of $i$-th unit cell increases. At the same time the dependence $\sigma^i_1(K_i)$ becomes a decreasing function over some range $K_i \geq K^*_T$ due to the effect of the crack tip blunting, that results to condition $\sigma^i_1(K_i) < S_c$. For subsequent loading the probability of brittle fracture for such an $i$-th unit cell is equal to zero. So, as the value $K_i$ increases two processes happen: the increase of the probability of brittle fracture of unit cells for which $\sigma^i_1(K_i) \geq S_c$ and the exclusion from the probabilistic process of those unit cells for which $\sigma^i_1(K_i) < S_c$. Some examples for the described processes will be
given below. Thus, the condition (5) may be physically interpreted that for subsequent increase of $K_i$, the brittle fracture does not happen and the ductile fracture is possible only.

5. The stress-strain state near the crack tip is calculated according to the modified variant of the approximated analytical solution represented in (7).

Fracture Toughness Prediction for 2.5Cr-Mo-V Nuclear Pressure Vessel Steel

The dependences $K_{ec}(T)$ calculated for given probability $P_i$ of brittle fracture are represented in Fig. 2 for standard specimens of various thicknesses $B$. As seen from Fig. 2, the brittle to ductile transition is described by the unique curve $K_{ec}^*(T)$ for specimens with various thickness (for example, $B = 50$ and $150$ mm). The curve $K_{ec}^*(T)$, as it has been determined above, is a locus of values $K_c(T)$ with the maximum probability of brittle fracture. The dependences $P_i(K_i)$ are represented in Fig. 3 for $B = 150$ mm and various temperatures. As seen from Fig. 3, the probability of brittle fracture decreases as temperature increases.

To verify the proposed probabilistic model the comparison of the calculated and experimental curves $K_{ec}(T)$ has been performed. Experimental data for 2.5Cr-Mo-V nuclear pressure vessel steel, obtained by testing the compact and bending specimens with thickness $B = 50$, $100$ and $150$ mm are represented in Fig. 4. The curves $K_{ec}(T)$ calculated for various specimen thicknesses and various probabilities of brittle fracture are also given in Fig. 4.

REFERENCES

Figure 1. $\sigma_1$ vs $r$ for the net-sections of specimens 1 and 2 in the fracture moment.

Figure 2. The calculated curves $K_C(T)$ for cracked specimens with various thickness.

Figure 3. $P_r$ vs $K_t$ for cracked specimen with thickness $B=150$mm.

Figure 4. Comparison of test data and the calculated curves $K_C(T)$.