RELATION BETWEEN THE CRACK TIP DISPLACEMENT VECTOR AND THE J-INTEGRAL IN MIXED MODE DUCTILE FRACTURE

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The relationship between the crack tip displacement vector $\delta$, and the $J$-integral is derived within the HRR-theory and verified with FE-calculations and experimental results. The implications for fracture initiation are discussed on the basis of available data. It is shown that the $\delta_0 = \text{constant}$ assumption predicts the decrease of initiation-$J$ with increasing mode II components of ductile steels. A correlation between $\delta_0$ and the magnitude of maximum equivalent plastic strain ahead of the crack was not found. At high mode II loading a critical combination of strain and triaxiality is attained at the crack tip, which causes a void-sheeting fracture at relatively low $J$-values.

INTRODUCTION

In previous papers it has been shown by the authors (1-3) that the crack tip displacement vector

$$\delta = \sqrt{\delta_1^2 + \delta_2^2}$$  \hspace{1cm} (1)

is more appropriate to characterise ductile mixed mode I/II crack initiation and stable crack growth than the $J$-integral. Experiments carried out on biaxially loaded cruciform specimens with slanted cracks and small compact-tension-shear (CTS) specimens, Figure 1, of the structural steel StE 550 (yield plateau $\sigma_y=580$, ultimate strength $\sigma_u=650 \text{ MPa}$) showed approximately constant technical ($\Delta a = 0.2 \text{mm}$) $\delta$-initiation values over almost the entire investigated mixed mode range. The shear crack resistance curves in terms of $\delta$, were independent of specimen geometry and mixed mode ratio. On the other hand increasing mode II load components led to decreasing $J$-initiation values and lowered the level of the J-R-curves, which fell even under the mode I $J_a$-curve of C(T) specimens in case of near mode II loading. Also other authors found this decrease of the mode II $J$-integral tearing resistance in ductile ferritic steels with low work hardening capability or yield plateau (4-6).

In this paper the $\delta$-$J$-relationship of the HRR-field will be derived under the

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\( \delta_s \) = constant assumption. The implications for fracture initiation are discussed on the basis of available data. To get further insight to the \( \delta_s \)-J-relation plane stress and plane strain finite element calculations were performed and some preliminary results are presented.

**FINITE ELEMENT CALCULATIONS**

Within the framework of mixed-mode fracture characterisation of StE 550 two-dimensional, plane stress and plane strain finite element (FE) analyses of CTS specimens were performed using conventional small strain theory. In Figure 1 the FE model of a CTS specimen loaded at \( \phi=75^\circ \) is represented. Details on the fixture modelling and load introduction are found in (2). A core of square-shaped elements with a length of about 1/32000 the crack length surrounded the crack tip. Suitable scaling of element dimension was obtained by biasing nodes towards the border of the focused region. The angular discretisation consisted of 24 elements per ring. Though this choice allowed to generate regularly shaped elements, it resulted posteriorly to be much too rough for an accurate estimation of stresses and strains in crack propagation direction.

Alike the experiments (2), the FE-models were loaded until a \( \delta_s = 0.19 \) mm at a position 0.5 mm behind the undeformed crack tip was reached. This value corresponded to the physical initiation value obtained from back-extrapolation of the available experimental data and metallographic investigations of sections of the crack tip region (7). The drop in technical \( \delta_s \) of pure mode II loaded specimens, see below, was not considered.

**J-\( \delta_s \)-RELATIONSHIP IN THE HRR-FIELD**

As proposed by Amstutz (8) a relationship of the J-integral and the crack tip displacement vector \( \delta_s \) is obtained from the plane strain mixed-mode HRR-fields (9) by evaluating \( \delta_s \) at the distance

\[
 r = \sqrt{u_x^2 + u_y^2} \quad (2)
\]

behind the crack tip. The polar displacement components \( u_x \) and \( u_y \) are evaluated at \( \theta = 180^\circ \) and \( -180^\circ \). Therefore two different reference points are obtained for the upper and lower flank of the crack under mixed-mode loading. The final result is however conform to the well known expression (10):

\[
 \delta_s = (\sigma_y / \sigma_u)^{\alpha} D_{yx} J / \sigma_u \quad (3)
\]

where \( \sigma_u, \sigma_0, \alpha \) and \( N \) are the Ramberg-Osgood yield stress, yield strain, constant and strain hardening exponent respectively. For typical metallic materials the dependence of \( \delta_s \) on \((\alpha \sigma_0)^{\alpha N}\) is weak. Figure 2 depicts the variation of \( D_{yx} \) with mode mixity for medium to low work-hardening. The \( J-\delta_s \)-relationship obtained from plane strain and plane stress finite element (FE) calculations described in previous section are also displayed in Figure 2 together with experimentally measured values of the 4 mm thick specimens (2). The \((\alpha \sigma_0)^{\alpha N}\)-factor for StE 550 was assumed as 0.81.

Even if different \( \delta_s \), measurement positions were used, the general trend and the actual \( D_{yx} \)-values in Figure 2 are quite similar. The increase of the crack-tip displacement vector with increasing mode II components suggests a decrease of constraint which was also found in other finite element calculations (11, 12).
Figure 3 highlights the decrease of the mixed-mode J-initiation values \( J_i \) of ductile steels (2, 5, 6, 13, 14) and a finite element calculation based on damage mechanics (12) with increasing mode II load components. The mixed-mode fracture data was normalised by the mode I \( J_{1c} \) obtained from mode I loaded mixed-mode specimens (if available). The lines in Figure 3 show the \( J_{1c}/J_{1c} \)-relationship of the plane strain, mixed-mode HRR-fields derived under the \( \delta_i = \) constant assumption for two work hardening exponents \( N \). As may be expected from the uniform slopes of the curves in Figure 2, the normalised curves in Figure 3 display little effects of the strain hardening exponent. The results of the FE-calculations are also shown in Figure 3. At least the trend of the initiation-J-decrease with increasing mode II components is predicted correctly with the \( \delta_i = \) constant assumption and the HRR fields. Even for the special case of StE 550, the FE calculations apparently did not match the experimental results more accurately. With respect to the very low \( J_{1c}/J_{1c} \)-ratios for pure mode II loading of StE 550 and A-503-8, it should be noted here, that also the \( \delta_i \)-initiation values of StE 550 evaluated at \( \Delta a = 0.2 \) mm were reduced by 30% compared to the mode I crack tip openings (2).

**PRELIMINARY RESULTS OF THE RELATION OF \( \delta_i \) WITH STRESS/STRAIN DISTRIBUTION AT THE CRACK TIP**

Systematic experimental investigations and FE-calculations (for example in (15)) have demonstrated that ductile cracks initiate when a critical combination of triaxiality of stress and equivalent plastic strain \( \varepsilon_{eq} \) is reached in a certain region ahead of the crack tip. The stress triaxiality is usually expressed by the ratio of the hydrostatic stress to the v. Mises equivalent stress \( \sigma_{eq}/\sigma_0 \). In this section the implications of the \( \delta_i = \) constant assumption on the damage parameters are presented. Again the values are taken for a load yielding the initiation \( \delta_i \) of 0.19 mm at a distance of \( r = \delta_i = 0.19 \) mm ahead of the crack tip. The HRR-results were calculated with \( N = 20 \).

Figure 4 shows the maximum triaxiality and equivalent strain for plane strain. The values had to be taken at different positions ahead of the crack tip. For increasing mode II the constraint or stress triaxiality decreased and the effective plastic strain increased. In contrast to what was stated by (14) a constant \( \delta_i \) was not equivalent to a constant maximum \( \varepsilon_{eq} \) for any mode mixity. For plane stress (not included in Figure 4) the triaxiality varied slightly with \( M_s (\sigma_{eq}/\sigma_0 = 0.58) \), whereas the equivalent strain showed no consistent trends. Probably the localised regions with intense straining were not matched by the coarse angular mesh.

More than the maximum stresses and strains, the values in crack propagation direction are of interest. The stable crack deflection angles \( \alpha \) (positive for tensile crack growth) in Figure 5 were measured on the broken specimen halves. The stable crack extended in the maximum shear or maximum equivalent strain direction at crack sliding components of \( M_s < 0.68 \). Relatively high crack tip opening displacements caused a crack path deviation in the direction normal to the maximum tensile stress. The constant offset between the HRR and the measured shear deflection angles together with the dimples on the shear crack surfaces detected in (1) suggested that the so called „shear crack“ was in fact a combination of sliding off and void coalesce. The crack deviated from the zero or negative hydrostatic stresses in the shear band, to allow the growth of some microvoids. Final rupture then occurred by a microvoid shearing mechanism (16). Therefore the combinations of triaxiality and strain taken in crack growth direction displayed some amount of positive
triaxiality also in case of shear crack growth. This data is plotted in Figure 6 together with
damage curves measured on notched specimens of similar steels (17, 18). Even if the absolute
values of $\sigma_c/\sigma_e$ and $\varepsilon_{eq}$ are sensitive to the microstructural critical distance at
which the data is extracted, some kind of more or less horizontal\(^1\) cut-off for a maximum
strain is insinuated from the behaviour of the shear crack values in Figure 6. In the ductile
steels the critical crack tip displacement can not reach the high values implied by a mixed-
mode independent J-integral. Under predominant mode II loading a critical strain, which
(together with a small amount of positive triaxiality) causes a void sheeting mechanism, is
attained before the J-integral reaches the high mode I $J_c$-values.

CONCLUDING REMARKS

The relationship between the crack tip displacement vector $\delta$, and the J-integral was de-

erived within the HRR-theory and verified with FE-calculations and experimental results. It

was shown that the decrease of initiation-J with increasing mode II components of own

results and literature data is qualitatively predicted under the $\delta_c = \text{constant}$ assumption.

On the basis of plane strain FE-calculations and the HRR-field estimates of triaxiality

$\sigma_c/\sigma_e$ and equivalent strain $\varepsilon_{eq}$ at incipient crack propagation were made. The results

clearly showed that there is no straightforward relationship between $\delta_c$ and $\varepsilon_{eq}$ as stated

by (14), even though the FE-models had a rather coarse angular meshing and the plane

strain assumption is too strict for thin specimens (19).

As proposed by Teirlink et al. (20) in their fracture maps, the change in fracture

mechanism from tensile fracture to shear fracture adds an additional line in the diagram of

critical $\sigma_c/\sigma_e-\varepsilon_{eq}$ combinations.

AKNOWLEDGEMENTS

Helpful discussions with Dr. H. Amstutz of the University of Darmstadt are gratefully

acknowledged.

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\(^1\) It should be recalled, that the points for pure mode II loading (smallest $\sigma_c/\sigma_e$) are too high

in Figure 6 since a smaller $\delta_c$ was measured in the experiments.


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Figure 1: CTS-specimen, loading device and FE-model.

Figure 2: Variation of $D_{\alpha}$ with mode mixity (HRR-field, FE and expm. results).
Figure 3: The decrease of $\dot{J}$, with increasing mode II is predicted by imposing a constant $\delta$, in the HRR-field.

- cruciform spec.
- CTS spec.
- HRR, $N=10$

Figure 4: Maximum triaxiality and equivalent strain in function of mode mixity.

- $\dot{\sigma}_m$ HRR, pl. strain
- $\dot{\sigma}_m'$ FE, pl. strain
- $\varepsilon_{v,pl}$ HRR, pl. strain
- $\varepsilon_{v,pl}'$ FE, plane strain

Figure 5: Experimental crack deflection angles of Ste 550.

- max. tangential stress
tensile crack growth

- max. equivalent strain
shear crack growth

Figure 6: Combinations of plane strain triaxiality and equivalent strain at initiation.

- HY 130 (17)
- Ste 690 (18)
- Ste 550 (18)

- HRR, plane strain
- FE, plane strain

shear crack growth

tensile crack growth