DETERMINATION OF THE FLOAT GLASS FRACTURE TOUGHNESS
WITH NOTCHED SPECIMEN, MICROSTATISTICAL APPROACH

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The concept of notched stress intensity factor is naturally applied on the determination of the of brittle materials toughness; critical notch stress intensity factor has been used in this study as a measure of the static and dynamic fracture toughness on three-point-bending glassy specimens with various notch radius.

A statistical approach to the fracture of glass is described. A dynamic model is proposed which enables determination of the number of microcracks involved in the fracture process.

INTRODUCTION

For very brittle material like glass, the use of pre-cracked specimens to determine the fracture toughness is practically impossible since pre-cracking by fatigue or static methods often loads to unexpected fracture. For this reason, tests are usually carried out on notched specimens and the results are treated by considering inaccurately the notch as a crack, or by using a correction of the linear fracture mechanics introduced by Creager (1), the validity of which is limited. The use of notched specimens needs to take into account the fact that the stress distribution at notch tip is different from this at crack tip.

It has been seen that the mean value fracture toughness is few sensitive to the loading rate, but the scatter of the obtained value increases with this parameter. The fact that fracture of glass test specimens is caused by microcracks explains the observed scattering and proves that the strength is linked to the volume or surface of the test specimen into consideration and to the probability of finding a defect, attributed to a greater multiactivation of defects.

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In order to explain the loading rate sensitivity, we propose a statistical approach which enables calculation of the number of microcracks that are present in the volume within which fracture occurs, from phenomenological results.

**NOTCH STRESS INTENSITY FACTOR**

Fracture strength called fracture toughness $K_{IC}$ is measured according to a particular procedure defined by standard (2). This standard recommends the use of pre-cracked specimens with a controlled crack length. This is relatively difficult to obtain and notched specimens are preferably used. Test results are obtained considering the notch as a crack which is not perfect or using the correction of the linear fracture mechanics relationship proposed by Creager (1) which has a limited applicability.

Fracture conditions for a notched specimen is characterised by using the real stress gradient at the notch root (3). This stress gradient at notch tip can be characterised by a relationship different from the crack tip stress gradient.

for a regular notch, the notch stress intensity factor is defined by the relationship (1)

$$K_{Ld} = \sigma_{\text{max}} \sqrt{\frac{2\pi X_c}{\sigma_{\text{max}}}}$$

$$\sigma_{\text{max}} = K_{Ic} \sigma_N$$

(1)

Where $\sigma_{\text{max}}$ is the maximum stress at the notch tip, $K_{Ic}$ is stress concentration factor, $\sigma_N$ is critical net stress and $X_c$ is effective distance.

A comparison of the obtained values with those calculated by the classical or Creager formulas is made.

**EXPERIMENTAL STUDY.** The studied material is a Float glass. Single edge notched bending (SENB) specimens are used, the notch depth of which is 8 mm. Some essential physical and mechanical properties are presented in Table 1.

**TABLE 1 - Physical and mechanical characteristics of glass**

<table>
<thead>
<tr>
<th>Young’s modulus (MPa)</th>
<th>Volumetric mass (Kg/m³)</th>
<th>Poisson’s ratio $\nu$</th>
</tr>
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<tbody>
<tr>
<td>70000</td>
<td>2508</td>
<td>0.23</td>
</tr>
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The notch stress intensity factor versus the square root of the notched radius has been determined by the experimental results and those reported in Figure 1, according to Creager's formula, which predicts a linear relationship between the fracture toughness and square root of the notched radius (The reported values are the arithmetic means of 12 measurements per radius in dynamic loading and 6 measurements in static ones).

The results obtained under static and dynamic loading show a notch stress intensity factor increasing with the radius at the notch tip and loading rate (Figure 2).

The fact that the fracture of glass test specimens is caused by microcracks, implies that their fracture strength is essentially random by nature. This explains the observed scattering of the results and implies that the strength must in some way be linked to the volume or surface of the test specimen under study and the probability of finding a critical defect.

This phenomenon of scatter for high loading rate is attributed to the possibility of multi-activation of defects which modifies the probability density for finding a critical defect.

Our approach is based on dynamic three-point-bending tests on glassy test-pieces with various notch radii. The model we propose represents a statistical approach which enables calculation of the number of microcracks which are present in the volume within which fracture occurs, from phenomenological results.

**STATISTICAL MODEL**

Consider a brittle material with an arbitrarily oriented crack. Let \( \Omega \) be the solid angle containing the normal to the planes of cracks (figure 3), together with the normal component \( \sigma_n \) of the applied stress field. The method used for analyzing has been widely described elsewhere (4).

The proposed model is deduced from the theories of Weibull (5) and Batdorf (6) and is based on the following assumptions:
- the material is homogeneous and isotropic and the cracks are randomly oriented,
- the stress normal to the cracks is greater than the critical stress \( \sigma_c \) (characteristic for each crack), causing extension of the crack and hence total fracture (figure 3)
- the macroscopic state of the stresses is not affected by the presence of cracks, i.e. the theory of elasticity of continuous media is applicable.

The critical stress \( \sigma_c \), which is uniaxial, uniform and normal to the plane of crack, causes the cracking.
The number of cracks in the volume $\Delta V$ with a critical stress between $\sigma_c$ and $\sigma_c + d\sigma_c$ is written:

$$dN = \Delta V \left| \frac{dN(\sigma_c)}{d\sigma_c} \right| d\sigma_c$$

(3)

and the probability that the cracks will have a critical stress within the elementary interval $(\sigma_c, \sigma_c + d\sigma_c)$ is given by:

$$P_r(\Delta V, d\sigma_c) = \frac{\Omega \Delta V}{4 \pi} \left| \frac{dN(\sigma_c)}{d\sigma_c} \right| d\sigma_c$$

(4)

The probability of fracture of the total volume $V$ when $\sigma \to \sigma_r$ is:

$$P_r = 1 - \exp \left[ - VN_o \int_{\sigma_o}^{\sigma_r} \left( 1 - \sqrt[3]{\frac{\sigma_c}{\sigma_r}} \right) \left( \frac{1}{\sigma_r - \sigma_o} \right) d\sigma_c \right]$$

(5)

By integrating between the threshold stress $\sigma_o$ and the fracture stress $\sigma_r$, the probability of fracture becomes:

$$P_r = 1 - \exp \left[ - \frac{N_o V}{\sigma_o} \left( \frac{1}{3} \sigma_r - \sigma_o \left( 1 - \frac{2}{3} \sqrt[3]{\sigma_o \sigma_r} \right) \right) \right]$$

(6)

where $N_o V$ is the total number of cracks in the production volume, $\sigma_o$ is the threshold stress and $\sigma_r$ is the theoretical stress.

When the fracture probability of $e$ as given by the ordering statistics and the theoretical fracture stress of the material are known, the threshold stress $\sigma_o$ and the number of cracks involved in the rupture of the specimen can be calculated.
DISCUSSION

The fracture probability density $P_f$ is given by orderly statistics, and the stress $\sigma^*$ is taken as the fracture strength of a defect-free test specimen, and is of the order of 1000 MPa according to Zarzycki (7).

The fracture probability density given by equation (6) gives a better approximation to the minimal critical threshold stress, and the total number of cracks in the production volume involved in fracture decreases with notch radius, but increases with critical fracture bending stress (figure 4).

CONCLUSION

The method to determine the fracture toughness of very brittle material is based on the actual stress gradient at the notch tip. The results obtained on a float glass show an increase of the fracture toughness with the notch radius and loading rate.

The fact that fracture of glass test specimens is caused by microcracks explains the observed scattering and proves that the strength is linked to the volume or surface of the test specimen into consideration and to the probability of finding a defect.

The proposed model represents a statistical approach enabling calculation of the number of microcracks present in the volume of production of the fracture from phenomenological results. The number of cracks involved in rupture decreases with notch radius, but increases with critical bending stress.

REFERENCES

2. ASTM E 399.
FIGURE 1: The influence of the square root of the notched radius on the notch stress intensity factor.

FIGURE 2: Evolution of notch stress intensity factor versus the notch radius.

FIGURE 3: Solid angle $\Omega$ containing the normal to the cracking plane.

FIGURE 4: Mean critical bending stress and number of cracks in the volume of production as a function of notch radius.