IDENTIFICATION OF MATERIAL CONSTITUTIVE PARAMETERS

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Non-linear finite element computations make use of more and more sophisticated constitutive equations. The corresponding equations are very often written as a set of differential equations. The first difficulty encountered by potential users is the gap existing between raw material characterisation on uniaxial specimens and the knowledge of the required parameters. There is very few adapted software to perform this job and mechanical users cannot be always specialists of optimisation techniques. IDENT 1D is an adapted software developed under Matlab® language. The user has only to select the material constitutive equation set and the files which contains the uniaxial experimental characterisation data.

INTRODUCTION

In mechanical area, non-linear finite element computations make use of more and more sophisticated constitutive equations (Lemaitre, 1988). The corresponding equations are very often written as a set of differential equations. The behaviour of the material at a given temperature will be correctly described by a given set of material parameters for the selected constitutive law. The aim is to use these parameters in finite element calculations to predict the mechanical behaviour of a given structure. In general the first drawback encountered by potential users is the gap existing between raw material characterisation on uniaxial specimens and the knowledge of the required parameters. One must therefore use optimisation tools, but the results can be depending of the initial guess and on the way followed by the user.

IDENT 1D is a software developed under Matlab meta-language (Grace, 1995). As material characterisation is mainly performed with uniaxial specimens, the software is only working on 1D formulation of the constitutive equations. Its originality is that no initial estimation of the material parameters will be requested. Two main examples are described in this article. The first one is the Lemaitre & Chaboche coupled damage viscoplasticity model (Lemaitre, 1988). The second example is a non unified cyclic law with a separation of plastic and viscous strain terms which is called DDI model (Contesti, 1989).

IDENTIFICATION OF LEMAITRE & CHABOCHE CREEP MODEL

To improve the assessment of reactor pressure vessel life under severe accident, we are using a model due to Lemaitre & Chaboche with predictive capability in the field of

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viscoplastic flow, material damage and failure. Starting from the experimental creep curves for the 16MND5 RPV steel performed at constant load, a set of parameters is identified.

The Lemaitre & Chaboche creep law

The Lemaitre & Chaboche model (Lemaitre, 1988) is a multiplicative creep law for the viscoplastic strain rate. The evolution law for damage (1-b) has been proposed by Kachanov. The model is described here in its uniaxial and isothermal formulation:

$$
\dot{\varepsilon}_p = \frac{1}{(1-D)^{\alpha+1}} \left( \frac{\sigma}{K \varepsilon_p^{1/3}} \right)^K \quad \text{and} \quad D = \left( \frac{\sigma}{A(1-D)} \right)^r
$$

(1)

where: \( D \) is the isotropic damage (cavities proportion in the material), \( \varepsilon_p \) is the viscoplastic strain and \( \sigma \) is the Von Mises stress.

This law needs five parameters: \( K, M, N, A \) and \( r \), which are temperature dependent. It describes the three stages of the creep behaviour (primary, secondary and tertiary). Creep tests are commonly performed with constant load. The reduction of the specimen section due to the deformation makes the real stress increase during the test. For strains larger than 10%, the updated stress with strain must be taken into account in the identification process. The volume of the specimen's homogeneous part remains constant during the test, so we take in equation (1): \( \sigma = \sigma_\text{nom} \exp(\varepsilon_p) \)

(2)

where \( \sigma_\text{nom} \) is the nominal stress (F/\text{S}_{\text{nom}}).

Two main hypotheses can be assumed for the primary and secondary creep stages:

- \( D = 0 \) (damage appears only during the tertiary creep stage)
- \( \sigma = \sigma_0 \) (strains remain small during the first two creep stages)

Thus, for the primary and secondary creep stages, equation (1-a) reduces to:

$$
\dot{\varepsilon}_p = \left( \frac{\sigma_0}{K \varepsilon_p^{1/3}} \right)^K
$$

(3)

By integrating this law for a constant stress level, we obtain the Norton-Bailey creep law:

$$
\varepsilon_p = C_1 \sigma_0^{n_1} t^{n_2}
$$

(4)

This integrated law makes the identification faster and more efficient. Of course, the parameters \( C_1, n_1, n_2 \) are directly related to \( K, M, N \).

The creep law parameters identification

The identification of the five parameters is achieved in two steps:

- identification of \( K, M \) and \( N \) on the primary and secondary creep stages,
- identification of \( A \) and \( r \) on the whole creep curves, using the identified \( K, M \) and \( N \).

Each part is built with the same scheme. First, an automatic initial guess is performed by regression methods. After that, an optimisation of these initial guesses is achieved.

The procedure is applied at each temperature independently. It needs several creep curves corresponding to different initial loads \( \sigma_0 \) at each temperature. It is possible to give different weight to each experimental creep curve. Minimisation is performed using Matlab algorithms from (Grace, 1995).
Determination of primary and secondary creep stages limit

The end of secondary creep stage is detected by variation of the linear regression coefficient of the strain-time experimental set expressed in log-log.

Identification of K, M and N

The initial estimation of the three parameters K, M and N, is given by linear regressions on the Norton-Bailey law (4) expressed in log-log. The optimisation of these initial values is performed by minimisation of the following cost function $F(C_0, N_0, N_1)$, which estimates the difference between the experimental and the calculated creep curves at each level of nominal stress $\sigma_0$, over the primary and secondary creep stages:

$$F(C_0, N_0, N_1) = \sum_{\sigma_0} \left( \frac{1}{t_1} \int_0^{t_1} \left( \frac{\dot{\varepsilon}_{\text{calc}}}{\dot{\varepsilon}_{\text{exp}}} \right)^2 \right)^{0.5}$$  \hspace{1cm} (5)

Identification of $A$ and $r$

The initial estimation of $A$ and $r$ values is based on the Robinson's rule and from the Kachanov law (1-b) integrated with the constant stress hypothesis. The minimisation of the difference between the Robinson's integral and 1, for each level of nominal stress $\sigma_0$, provides the initial values of $A$ and $r$. The optimisation of these initial values is then performed by minimisation of the cost function $F(A,r)$.

Results

Short term creep tests were performed at constant load on 16MND5 RPV steel, at high temperatures, ranging from 500°C up to 1300°C with 100°C step. For each temperature, four levels of nominal stress were carried out. The identification procedure is applied at each temperature independently. There is no smoothing of the results on the temperature range. The following figures show the evolution of the parameter $A$ versus temperature and an example of the comparison of experimental and identified creep curves.

The evolution of the parameters versus temperature is monotonous and seems physically based (logarithmic for $A$). The good agreement between experimental and calculated creep curves is a first step of validation.

![Figure 1: $A$ versus temperature](image1)

![Figure 2: Experimental and calculated creep curves at 1000°C](image2)

**IDENTIFICATION OF DDI MODEL**

Many high temperature structures contain welds which are generally considered to be a life limiting feature due to their observed inferior performance under fatigue and creep-fatigue loading conditions. In order to get a better understanding of the phenomena
involved in this behaviour, a comprehensive characterisation program of welded components has been carried out and a cyclic non-unified law, called DDI model (Contesti, 1989), has been identified using IDENT ID so that to exhibit the relative effect of plastic and viscous effect. This model is based on the assumption that total strain can be separated into 3 terms: elastic, plastic and viscous strains.

An important characteristic of this model is the large number of parameters (around 17). Thus, in this case, the difficulty of the identification is not to find an accurate cost function but to separate the relative effects of each parameter in a pre-identification.

**Description of DDI model**

The total strain range is divided into three terms:

$$\varepsilon_t = \varepsilon_e + \varepsilon_p + \varepsilon_v$$  \hspace{1cm} (6)

where \( \varepsilon_t \) is the elastic strain, \( \varepsilon_p \) is the plastic strain and \( \varepsilon_v \) is the viscoplastic strain.

Each inelastic strain term is described with a non-linear isotropic and a transient non-linear kinematic hardening. The plastic flow rule is based on the Hill assumption and the viscoplastic flow rule is simulated by a secondary creep law. The multi-axial equations of the model are summarised below:

<table>
<thead>
<tr>
<th>Plastic strain mechanism</th>
<th>Viscoplastic strain mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scale yielding equations</strong></td>
<td></td>
</tr>
<tr>
<td>( f_p = J_1(\sigma - X_p) - R_p )</td>
<td>( f_v = J_1(\sigma - X_v) - R_v )</td>
</tr>
<tr>
<td><strong>Flow rules</strong></td>
<td></td>
</tr>
<tr>
<td>( \dot{\varepsilon}_p = \frac{3}{2} \frac{\sigma - X_p}{J_2(\sigma - X_p)} )</td>
<td>( \dot{\varepsilon}_v = \frac{3}{2} \frac{\dot{\lambda}_p}{J_2(\sigma - X_p)} )</td>
</tr>
<tr>
<td>( \dot{\lambda}_p = \dot{\lambda} = \frac{2}{3} \lambda \dot{\varepsilon}_p )</td>
<td>( \dot{\varepsilon}_v = \left( \frac{f_v}{K} \right)^+ )</td>
</tr>
<tr>
<td>( p ) is the cumulated plastic strain</td>
<td>( \dot{v} ) is the cumulated viscoplastic strain</td>
</tr>
</tbody>
</table>

| **Isotropic hardening functions** |                               |
| \( R_p = R_{p0} + Q_p \left( 1 - \exp(-h_p) \right) \) | \( R_v = R_{v0} + Q_v \left( 1 - \exp(-h_v) \right) \) |

| **Kinematic hardening functions** |
| \( \overline{X}_p = \frac{2}{3} C_{p0} \overline{\alpha}_{p0} + \frac{2}{3} C_{p0} \overline{\alpha}_{p0} \) |
| \( \overline{X}_v = \frac{2}{3} C_{v0} \overline{\alpha}_{v0} + \frac{2}{3} C_{v0} \overline{\alpha}_{v0} \) |
| \( \overline{\alpha}_{p0} = \overline{\varepsilon}_p - d_p \overline{\alpha}_{p0} \overline{p} \) |
| \( \overline{\varepsilon}_{v0} = \overline{\varepsilon}_v - d_v \overline{\alpha}_{v0} \overline{\dot{v}} \) |

' is the deviatoric part of tensors, \( \left( \right)^+ \) is the positive part and \( \overline{\cdot} \) the second order tensors.

**Automatic DDI parameters identification strategy**

The 17 parameters are described in details elsewhere (Contesti, 1989). First, in order to perform an automatic identification of the DDI model, it is important to have an appropriate set of experiments that enable to differentiate the respective influence of the parameters. Thus, the main idea of this identification problem is to separate the DDI parameters using adequate uniaxial tests. It is well established that viscoplastic strain
increases when the strain rate test decreases. Thus, high strain rate tests are used for the identification of plastic strain behaviour when low strain rate tensile, creep, fatigue and fatigue relaxation tests are used for the identification of the viscoplastic part.

**Plastic parameters identification**

The initial isotropic parameters are first determined by using a linear regression of the cyclic stress strain curves: $R_0$ is initially determined on the first cycle and $R_{p0} + Q_b$ is determined on the stabilised hysteresis loop.

The stabilisation rate $b_n$ is initially approximated with the yield stress of an $n$th intermediate cycle by using an approached formula. The cyclic stabilised hysteresis loop is then divided into two parts: the linear one (controlled by the isotropic parameters) and the non linear one (controlled by the kinematic parameters). The initial kinematic parameters $C_{pi}$ and $d_{pi}$ are pre-identified with a basic optimisation analytic procedure based on the following approximation for the non linear part of the stabilised hysteresis loop:

$$\sigma = \frac{C_{pi}}{d_{pi}} \left( 1 - \exp \left( -\frac{d_{pi}}{\sigma_p} \right) \right)$$  \hspace{1cm} (7)

As in the whole identification procedure of the DDI model, a least square cost function is minimised with the fmins algorithm of the Matlab optimisation Toolbox (Grace, 1995).

Arbitrary, the initial value of $C_{pi}$ is set to be equal to 2 $C_{pi}$ and $d_{pi}$ to 10 $d_{pi}$. With these initial values ($C_{pi}$, $C_{pi}$, $d_{pi}$ and $d_{pi}$) an optimisation is performed using the full set of kinematic equations. Then the kinematic parameters are fixed and the whole stabilised cyclic loop is taken into account to optimise the isotropic parameters. The isotropic values are then fixed and the kinematic parameters are optimised and so on until we reach a good identification of the cyclic stress strain stabilised curve.

A final optimisation is performed with all the parameters free to vary. Experience shows that this stage is time expensive and that the previous stages are good enough (except for the $b_n$ parameter). The identification steps are summarised in the flow chart.

*Figure 2: Flow chart for the identification of cyclic plastic part*
Next figures give results of the plastic parameters identification for Zirconium at 200 °C.

![Figure 4: 2nd hysteresis loop (3.10^3 s^-1)](image)

![Figure 5: stabilised hysteresis loop (3.10^3 s^-1)](image)

**Viscoplastic parameters identification**

The determination of the viscoplastic parameters is based on the same idea than the plastic one except for the viscoplastic flow rule coefficients: K and n are first approximated using creep tests and determining the secondary creep stage with a linear regression on a Norton creep law. These coefficients are taken into account to pre-identify the viscoplastic isotropic and kinematic parameters in exactly the same way of determining the plastic ones. The kinematic and isotropic parameters are then used to integrate the full creep law and we obtain a quite good approximation of K and n. The isotropic parameters are first optimised, then the kinematic parameters and finally the whole viscoplastic model is used to optimise the viscoplastic parameters.

**CONCLUSIONS AND FUTURE DEVELOPMENTS**

IDENT ID is a novel software running under Matlab environment and is really adapted for an easy identification of material parameters. It is an essential tool for all those who are performing non-linear mechanical computations with finite element method. It is simple to use and no pre-requisite knowledge of optimisation techniques is requested. Modifications (new constitutive equations, other identification approaches, etc.) can easily be handle with very few modifications and by using the powerful mathematical routines of Matlab software. Future developments which are under study are:

- to enlarge the number of included constitutive equations,
- to give an estimation of the degree of the adequation of the model,
- to estimate the uncertainties on each identified parameters.

**REFERENCES**