The governing equations for elastic-plastic materials describing the damage accumulation, crack initiation and growth under SSCC conditions are proposed. Damage evolution equation generalized Kachanov's equation in creep and two-criterial approach to analyses of fracture process are discussed. Both the threshold stress and the ductility loss values in uniaxial tests and the threshold stress intensity factor in cracked members are found. The change of failure surface mode during SSCC conditions is described.

INTRODUCTION

Sulfide stress corrosion (SSCC) in metals is not completely understood yet despite intense experimental investigation over the world. SSCC is manifested in various parameters that are used as measures of the mechanical properties of material such as elongation, failure, yield and tensile strength, time to failure, fracture toughness etc. SSCC may change also the mode of fracture from ductile transgranular to brittle intergranular one. SSCC in metals is usually a manifestation of hydrogen embrittlement, which is caused by hydrogen generated during corrosion reaction in H₂S containing environments. It is assumed that hydrogen enters metal continuously and interacts with the lattice and defects. That is the basis of the damaging effect by SSCC. The exact description of that process represents a meaningless task. Instead of trying to reproduce the fine details of that process it appears reasonable to introduce some internal variable reflecting only the main features of the damage accu-

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mulation. That approach in creep of metals has been
done by Kachanov [1] and Rabotnov [2]. The objective of
this paper is to generalize the Kachanov’s model for
SSCC conditions and to analyze from that point of view
some features of failure both in uniaxial case and in
multiaxial one.

GOVERNING EQUATION

Damage evolution equation. Damage accumulation in me-
tals both under creep and under SSCC conditions can be
described by scalar internal variable (damage para-
meter) \( \omega \) [1,2]. One form of the evolution equation for
\( \omega \) proposed for the creep damage accumulation process in
ref. [3] can be rewritten as follows:

\[
\omega = A(\omega_* - \omega)^m, \quad \omega(0) = 0
\]

(1)

where \( A, \ m \) are material parameters, \( \omega_* \) is upper bound
for damage accumulated. The damage accumulation equa-
tion can be derived from (1) in form

\[
\omega(t) = \omega_* \left[ 1 - \left( 1 - (1 - m)\omega_* \right)^{m-1} \right]^{1/(1-m)}
\]

(2)

The value of upper bound \( \omega_* \) depends on stress state,
corrosion environment, temperature etc. The simplest
approximation for \( \omega_* \) vs \( \sigma \) dependence can be written as
the linear function

\[
\omega_* = \alpha \sigma + \beta
\]

(3)

where \( \sigma_0 = \sigma_{kk} \) is the first invariant of stress tensor,
\( \alpha, \ \beta \) are material parameters. The first invariant \( \sigma_0 \)
has been preferred here because of diffusional manner of
damage accumulation process, the parameters \( A, \ m, \ \alpha \) and
\( \beta \) depend possibly on hydrogen ion concentration, partial
pressure of hydrogen sulfide, environment temperature.

Elastic-plastic constitutive equations. Taking into
account the difficulty of the SSCC process let us
consider the elastic-perfectly plastic theory

\[
\varepsilon = \begin{cases} \frac{\sigma_{e}}{E}, & 0 < \sigma_{e} \leq \sigma_* (\omega) \\ \varepsilon_* (\omega), & \sigma = \sigma_* (\omega) \end{cases}
\]

(4)

where \( \sigma_* (\omega) \) is the yield strength, \( \varepsilon_* (\omega) \) is ultimate
tensile strain, \( S_{ij} \) is the deviator of stress tensor,
\( \sigma \) and \( \varepsilon \) are the effective stress and strain. \( E \) is the
elastic modulus.
Fracture criterion. The constitutive equations ought to complete the fracture criterion. For elastic-perfectly plastic theory (4) it can be only the deformation type criterion \( \varepsilon = \varepsilon_0 \omega \). A simple approximations for \( \sigma \) and \( \varepsilon \) vs \( \omega \) dependencies can be written in the following manner:

\[
\sigma_0(\omega) = \sigma_0(1 - k_1 \omega) \tag{5}
\]

\[
\varepsilon_0(\omega) = \varepsilon_0(1 - k_2 \omega) \tag{6}
\]

where \( k_1 \) and \( k_2 \) are experimentally defined constants.

Taking into consideration the dimensionless character of damage parameter \( \omega \) one of constants can be equated to 1, i.e. \( k_1 = 1 \) and \( k_2 = k \).

**ANALYSIS OF SSOC FAILURE**

**Standard SSOC tests.** SSOC is examined usually by uniaxial tension load tests (ULTL), Shell-type 3-point bent beam test (SSBD), double cantilever beam test (DCBT) [4] and slow strain rate tensile test (SSRT) [5] in NACE solution [4].

**SSOC threshold stress.** The ULTL are performed on cylindrical specimens using dead-weight-type constant tester. The maximum initial stress at which no failure occurs within 720 h is referred to as SSOC threshold stress \( \sigma_{th} \) and is used as the criteria of SSOC.

From theoretical point of view described above by equations (1)-(6) the threshold stress \( \sigma_{th} \) is

\[
\sigma_{th} / \sigma_0 = (1 - \beta) / (1 + \alpha \varepsilon_0) \tag{7}
\]

Theoretical dependence of \( \sigma_{th} / \sigma_0 \) vs \( \varepsilon_0 \) illustrated in the Fig.1 corresponds to the experimental data [6]. So, the experimental relation between \( \sigma_{th} \) and \( \sigma_0 \) permits to find the material parameters \( \alpha \) and \( \beta \).

The material parameters \( A \) and \( m \) can be obtained from experimental data for long time SSOC failure as time to fracture vs tensile stress relation. The last material parameter \( k \) can't be found under the stress-controlled tests. For its evaluation it is necessary to make the strain-controlled SSOC tests.

**SSOC ductility loss.** The SSRT are performed on cylindrical specimens using a tensile machine where specimens are loaded with a constant strain rate.
The ductility loss $I = \frac{L_0 - L}{L_0}$ is measured as a parameter for evaluating the degree of degradation caused by SSSC ($L_0$ and $L$ are the elongations in atmospheric tensile tests and in SSRT). The value of ductility loss $I$ can be estimated as follows:

$$1 - I = C \left[ 1 - \left( \frac{\alpha \sigma + \beta}{\alpha \sigma + \beta + 1} \right) \right]^{1-m}$$

(8)

where $C = \frac{\varepsilon_0}{\varepsilon_0 + 1} / A(1 - m)(\alpha \sigma + \beta)^n$.

obtained by integrating (1)–(6) under additional condition $\varepsilon(t) = \varepsilon_0 t$. Theoretical behavior of $I$ vs $\sigma$ illustrated in Fig.2 also corresponds to experimental data [5,6]. Equation (8) can be used for estimation of the last unknown material parameter $k$. So, to describe the SSSC process on the whole it is necessary to know the elastic-plastic material parameters $E$, $\sigma_0$, and $\varepsilon_0$, and SSSC material parameters $\alpha$, $\beta$, $A$, $m$, and $k$. To define them it is necessary to make two kinds of independent tests such as UTIT and SSRT.

Multiaxial failure conditions. Governing equations (1)–(6) allow to describe the SSSC process both in uniaxial and multiaxial stress state. The threshold stress $\sigma_0$ defined for uniaxial loading as maximum nonfailed stress corresponds to the threshold surface, i.e. that surface in the stress space within which no failure occurs during 720 h. As it follows from (3) and (5) the equation of that surface is

$$\sigma_0 + \alpha \sigma_0 = (1 - \beta) \sigma_0$$

(9)

Equation (9) shows that initial cylindrical yield surface $\sigma_0 = \sigma_0$ changes its form during SSSC process to conic one. For plane stress conditions, that is usually realized in thin-walled tubes equation (9) can be rewritten as follows

$$\sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2} + \alpha \sigma_0 (\xi_1 + \xi_2) = 1$$

(10)

where $\xi_1 = \sigma_1 / (1 - \beta) \sigma_0$ and $\xi_2 = \sigma_2 / (1 - \beta) \sigma_0$ is the normalized principal stresses. The form of that surface represented for various values of $\alpha \sigma_0$ in Fig.3 changes from elliptical yield surface for to parabolic,
hyperbolic and straight line ones. From microstructural point of view it means that the character of failure can be changed from ductile transgranular one by shear to brittle intergranular one by tear.

Crack initiation and growth. It should be noted that parameters $a$, $b$, $A$, $m$, $K$, $C$ and $k$ aren't sufficient to describe the crack initiation and growth under SSSC conditions. That is a common situation in strength and fracture mechanics when to define the fracture condition in nonuniform stress field it is necessary to introduce an additional parameter such as $K_{fC}$, $J_c$ or $\sigma_c$.

All of them are equivalent to some linear parameter $d$ referred to as the distance from crack tip to point where local fracture criterion (6) is valid. Hence, for small scale yielding condition that criterion can be written in the following manner:

$$K_I = \frac{E\sigma_y}{\sqrt{2\pi d}}$$

Equation (11) allows to estimate both the threshold stress intensity factor $K_{ISC}$ below which the initial crack doesn't propagate

$$\frac{K_{ISC}}{K_{IC}} = \lambda(1 - k\beta)$$

and equations for crack initiation and growth. Time to crack initiation $t_1$ vs $K_I$ dependence can be written as

$$t = \left( \frac{A}{\lambda K_{IC}} \right)^{1-m} \left( \frac{K-I - K_{ISC}}{\lambda K_{IC}} \right)^{1-m}$$

where $K_{IC} = \frac{E\sigma_y}{\sqrt{2\pi d}}$ is the fracture toughness estimated from (11) as $\lambda = 1/(1 + E\sigma_y)$. For the unknown crack length $l(t)$ Volterra integral equation is obtained.

Unfortunately that equation is very complicated and therefore will be omitted there. Analysis of sub-critical crack growth under SSSC conditions will be done in the forthcoming paper.

**DISCUSSION AND CONCLUSIONS**

As it has been shown above, the proposed governing equations (1)-(6) describe some experimentally observed features of SSSC process both in uniaxial and multi-axial stress states. The main ones are

1) The existence of threshold stress $\sigma_{th}$ and its dependence on the material strength level;
2) The embrittlement of material in sulfide corrosive
environment that is referred to the loss of ductility with increase of strength level;
3) the change of failure model during SCCC process that is reflected in changing the form of the threshold surface in stress space;
4) the delayed character of cracking that means the presence of some time to crack initiation depending on initial stress intensity factor \( k_0 \);
5) the existence of threshold stress intensity factor \( k_{ISSC} \) and its dependence on material strength level.

Some of the results obtained show that the knowledge of \( \sigma_{th} \) experimentally obtained data for tested material isn't sufficient to decide whether the material tested is resisted or not to SCCC process. Only the whole experimental procedure involving both strain-controlled and stress-controlled tests can adequately solve this problem. It is clearly from \( \sigma_{th}/\sigma_* \) vs \( \sigma_* \) and \( k_{ISSC}/k_{IC} \) vs \( \sigma_* \) follows from relations when high level of \( \sigma_{th}/\sigma_* \) ratio doesn't mean automatically that \( k_{ISSC}/k_{IC} \) ratio will be also high enough [7].

REFERENCES

(4) NACE Standard TM-01-77-90, National Association of Corrosion Engineers (NACE), Houston, TX, 1990.
(6) H2S Corrosion in Oil and Gas Production. - A Compilation of Classic Papers (Eds. R.A.Tuttle and R.B.Kane), NACE, Houston, TX, 1981.
Figure 1. Theoretical dependence for $\sigma_{th}$.

Figure 2. Theoretical dependence for $I$.

Figure 3. Threshold surface in the stress space for various values of $\varepsilon_{th}$. 

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