A model of ductile crack propagation based on geometric fracture mechanics concepts and a scheme- 
matisation of the deformation field is introduced. Two parameters (CTOA and “Neck”) defining the onset of unstable ductile crack propagation are defined. Techniques for measuring CTOA and “Neck” in the laboratory are derived. The model indicates that for laboratory specimens the energy of fracture depends on the ligament size. A link with the “Two Parameter Approach” to defining ductile fracture energy is proposed. This link allows the definition of an alternative method for defining CTOA.

INTRODUCTION

A typical fracture pattern for large diameter gas pipelines, even though rare, is the propagation of a ductile crack at high velocity (≥100 m/sec) and for a significant distance from the initiation zone. Such fracture is controlled by the balance between the crack driving force and resistance offered by the pipe; this latter depending either on the toughness of the steel or on the constraints of the external environment. The issues of safety and correct design of gas pipelines entailed in this problem have led to the introduction of post yield fracture mechanics in the evaluation of resistance of materials (1) (2). The first problem that arises in these methods is precisely that of the correct definition of these parameters and their measurement in the laboratory. Since this type of fracture is associated with an extensive field of plastic deformation, the classical parameters of Post Yield Fracture Mechanics, in general valid in the initiation phase, may not be immediately applicable in the following phase of stable propagation. In recent years, two new concepts, based on energetic and geometric models, have been developed to evaluate ductile fracture propagation. The energetic model is a “two parameter approach”

*C.S.M.-ILVA, Rome **SNAM PROGETTI, Milan
(3) based on the energy dissipated during crack propagation, while the geometric model is based on the concepts of crack tip opening angle CTOA and "Neck". CTOA is defined as the angle at which the two new surfaces emerge from the crack tip while "Neck" is defined as the maximum width of the plastic zone adjacent to the crack tip (ref. fig. 2). Instability conditions are developed from the geometric concepts, with analyses demonstrating that the energetic and geometric models are in fact equivalent. This permits not only a refined physical interpretation of the parameters in the energetic approach, but also the definition of a technique for deriving the geometric parameters from the specific energy of fracture.

The Energetic Concept

Work carried out at British Steel (3) indicates that the energy of fracture may be modelled by a two parameter equation:

\[ U/A = R_c + S_c (w-a_0) \]  

where \( U \) is the total energy absorbed by the test specimen, \( A \) is the fracture area and \( (w-a_0) \) is the test ligament. The parameters \( R_c \) and \( S_c \) are associated with respectively, the energy of formation of new fracture surfaces and the energy expended in plastic deformation adjacent to the fracture surface. Expression /1/ has been verified experimentally for ligament lengths up to 150 mm. The values of the parameters \( R_c \) and \( S_c \) have been derived from comparison of the different fracture energies with specimens of different ligament lengths.

The Geometric Concept

The conditions for unstable fracture, as defined by the R and G curves, can be expressed in terms of the strain at the crack tip:

\[ \text{COD}_{app} > \text{COD}_{mat} \]

\[ \frac{d \text{COD}_{app}}{da} > \frac{d \text{COD}_{mat}}{da} \]  

where mat refers to the critical response of the material and app, that value applied by the system.

In fractures where extensive plasticity occurs it is hypothesized that "COD" may be substituted by the parameter "Neck". Further a kinematic approach (illustrated in Fig. 1) supports the following assumption:

\[ \text{CTOA} = 2 \gamma_p = 2 \text{Arctan} \left( \frac{1}{2} \lim_{\Delta a \to 0} \frac{\Delta \delta}{\Delta a} \right) \]  

From this relation it is evident that the derivative of COD may be substituted by CTOA, so that the conditions for unstable propagation become
\[(\text{neck})_{\text{app}} > (\text{neck})_{\text{mat}}\] 
\[(\text{CTOA})_{\text{app}} > (\text{CTOA})_{\text{mat}}\] 

It has been demonstrated experimentally that both these conditions are necessary for unstable fracture (4,5).

**RELATION BETWEEN GEOMETRIC AND ENERGETIC CONCEPTS**

Modelling of the process of plastic deformation in simple laboratory specimens (SENB) allows the development of a link between the geometric and the energetic concepts.

In Zone A (Fig. 2 adapted from Knott (6)), slip occurs on planes perpendicular to the surface of the specimen and extends over a zone with dimensions compatible to that of the ligament. Such a process results in a distortion of the specimen, with insignificant reduction in specimen thickness. In Zone B, adjacent to the crack tip, slip occurs on planes at 45° to the specimen surface, leading to plastic strain and the formation of the necking region. The zone has dimensions equivalent to the specimen thickness.

A division of the energy of fracture can be made with reference to these two distinct deformation patterns, thus the total energy consists of one component \( (E_N) \) arising from the energy expending on the necking process and a second \( (E_D) \) arising from the distortion of the specimen.

So that, given complete plasticity of the ligament, and for a crack advance of \( \Delta a \),

\[
\frac{\text{d}E_N}{\text{d}a} = \text{Neck} \frac{\text{d}F_L}{\text{d}a} \\
\frac{\text{d}E_D}{\text{d}a} = 2 \frac{M_L}{\text{d}a} \frac{\text{d}\theta}{\text{d}a} \\
\]

(necking energy) (distortion energy)

Where \( F_L \) and \( M_L \), are the limit force and limit moment of the specimen and \( 2\theta \) is the rotation of one half of the specimen with respect to the other.

Further

\[ M_L = A \alpha_B (w-a)^2; F_L = M_L/S \]

Where \( S \) - half span of the specimen and \( A \) is a dimensionless constant that depends on the specimen geometry. Experimentally it has been demonstrated that, given an initial ligament of sufficient length, CTOA remains virtually constant throughout crack propagation (4). Using geometric considerations (refer to Fig. 1) it can be shown that

\[
\frac{\text{d}\theta}{\text{d}a} = \frac{\sin \gamma_0 \cos \theta}{\cos (\gamma_0 \theta)r^*(w-a)} \\
\]

If this expression is used in conjunction with expression /3/ a direct dependence between CTOA and the derivative of the rota-
tation angle $\theta$ with respect to crack depth "a" can be established:

$$\text{CTOA(a) mat} = \frac{2 \arctan \left( \frac{r^*(w-a)}{d^0/a} \right)}{\theta}$$

where $r^*(w-a)$ is the distance from the crack tip to the axis of rotation and $r^*$ itself is constant for a given type of specimen (e.g. for SENB specimen $r^*=0.45$).

Using this expression, while assuming that both "Neck" and CTOA do not depend strongly on crack length, equations relating the components of fracture energy to the geometric parameters can be derived.

$$E = A_0 \text{ Neck} + \frac{(A/r^* a_0 \text{ CTOA}/2)}{(w-a_0)}$$

It is thus notable that both this equation (theoretically derived from the geometric approach) and the experimentally derived energetic model of Priest and Holmes (3) have a linear dependence on ligament size. Equating the ligament dependent and independent terms of the two approaches:

$$R_c = A_0 \text{ Neck} \quad S_c = A_0 \text{ CTOA}/2r^*$$

$R_c$ appears associated with the necking energy and $S_c$ with the energy of distortion and thus CTOA. Further, from values of $R_c$ and $S_c$, determined from the fracture energy of two similar specimens with different ligament lengths, the "CTOA" and "Neck" parameters may be calculated using /8/.

**EXPERIMENTAL VERIFICATION**

To verify the equivalence of the energetic and geometric approach, CTOA values were derived from purely geometric considerations /6/ and were compared with those determined indirectly from $S_c$ values /8/.

For such an analysis, measurements were taken from static fracture tests on SENB specimens, of thickness 18 mm, with three different ligament sizes (38, 66, 95 mm). Test specimens were fabricated from plates of a gas pipeline steel (X70) which at a given temperature demonstrated different Charpy V energies.

The geometric determination of CTOA makes use of the experimental $\theta(a)$ curve and the relation between CTOA and $S_c$ previously presented /8/. In table 1 the details of the comparisons are presented. It is evident that the two methodologies lead to very similar values of median CTOA. Further the derived values of "Neck" are of the expected magnitude. The correspondence of the CTOA values obtained by the different techniques indicates that the extensive data base of $R_c$ and $S_c$ values derived by Priest and Holmes may be used to define "CTOA" and "Neck" values and thus to predict plastic instability.
<table>
<thead>
<tr>
<th>Charpy V (J/cm²)</th>
<th>Neck (mm)</th>
<th>CTOA from Sc</th>
<th>CTOA from geometric relation $\theta - \theta(a)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>3.1</td>
<td>5.9</td>
<td>6.5</td>
</tr>
<tr>
<td>155</td>
<td>4.3</td>
<td>7.7</td>
<td>7.5</td>
</tr>
</tbody>
</table>

**TABLE 1: Comparison of CTOA values.**

**CONCLUSIONS**

Using a geometric approach, the parameters (Neck and CTOA) which define the onset of unstable crack propagation are defined. Further analyses, using a model of the field of plastic deformation, permit the derivation of the dependence of energy of fracture in terms of both these parameters and specimen ligament length. The same dependence of energy on ligament length is found experimentally by Priest and Holmes. This not only provides a significant validation of the geometric approach but also provides a useful method for determining "CTOA" and "Neck".

**REFERENCES**


Fig. 1 Geometrical definition of Crack Tip Opening Angle ($\gamma_p$)

Fig. 2 Deformation Mechanisms during Crack Propagation