THE FATIGUE DAMAGE MECHANICS OF
CARBON FIBRE COMPOSITES

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Fatigue damage in a notched cross-ply carbon fibre-epoxy laminate consists of splits in the 0° ply growing from the notch tips, delaminations between the 90° and 0° plies, and transverse ply (matrix) cracking in the 90° plies over the regions bounded by the splits and the specimen edges. During load cycling, damage grows in a self-similar manner; hence damage can be characterized by a single dimension, e.g., split length. In other words, fatigue damage consists of several interacting, planar cracks whose formation and growth depend primarily on the properties and behaviour of the matrix.

FATIGUE DAMAGE GROWTH IN (90/0)s CFRP

For multiple cracking that constitutes damage propagating from a notch in a composite laminate (Fig. 1), the strain energy release rate $\Delta G$ offers a simple method of analysing the driving forces for individual crack propagation (1). The crack growth-rate equation is of the form:

$$\frac{da}{dN} = A(\Delta G)^{m/2} \quad (1)$$

However, this is insufficient for describing the present example of combined split and delamination growth. As split length $l$ increases, the associated delamination area grows according to $l^2$, which implies an increasing resistance to further crack advance. It would be more appropriate, therefore, to use an equation of this form:

$$\frac{dl}{dN} = A \left[ \frac{\Delta G}{G_c} \right]^{m/2} \quad (2)$$

where $G_c$ is the current critical strain energy release rate or apparent toughness for damage growing under monotonic loading.

Damage growth in static loading is governed by (2):

$$P^2 = \frac{2}{(\delta G/\delta l)} \left[ G_{s1} + G_{g2} \tan \alpha \right] \quad (3)$$

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where the applied stress, $\sigma_{\text{ap}}$, on the specimen is directly proportional to the applied load, $P$. $\delta C/\delta t$ is specimen compliance change with split length, and $G_t, G_d$ are the energies per unit area for splitting and delamination.

The parameters $l_s, l_t$ and $\alpha$ are defined in Fig. 2.

Now the relationship between $\Delta G$ and the change in compliance is well known:

$$\Delta G = \frac{(\Delta P)^2}{2t} \frac{\delta C}{\delta l}$$  \hspace{1cm} (4)

$$\Delta P = (2)(W/2)(\Delta \sigma_{\text{ap}})$$ for one half thickness (two ply) quadrant of the specimen of width $W$ and $\Delta \sigma_{\text{ap}}$ is the applied cyclic stress range. Since the apparent toughness of the composite is (2):

$$G_c = G_s + G_d \left( \frac{l \tan \alpha}{t} \right)$$  \hspace{1cm} (5)

eqn. (2) becomes:

$$\frac{dl}{dN} = A \left[ \frac{1/2(\Delta P)^2 (\delta C/\delta l)}{G_s + G_d \tan \alpha} \right]^{m/2}$$  \hspace{1cm} (6)

Values of $G_s$ and $G_d$ have already been determined by Kortshoof and Beaumont (2) for this material to be 158 and 400 Jm$^{-2}$ respectively. The delamination angle, $\alpha$, has been measured at about $3.5^\circ$, the initial split length,$l_a$, can be determined using the analysis of Kortshoof and Beaumont (2) for quasi-static damage growth. The constants $A$ and $m$ can be obtained by experimental calibration. For carbon fibre-epoxy, $m = 14$. Finally, it is necessary to evaluate $\delta C/\delta l$, for which a finite element model is required (2).

Figure 3 shows split growth data for several $(90^\circ/0^\circ)_k$ specimens subjected to three different peak cyclic stresses ($R = 0.1$). The split lengths are normalised with respect to the notch size, $a$. Also shown is the integrated form of the crack growth equation for these stress levels. A good fit is obtained using $A = 5.0 \times 10^{-6}$ and $m = 14$. Furthermore, choosing the appropriate values for $G_s, G_d$ and $\alpha$ for the carbon fibre-PEEK system (3), eqn. (6) accurately predicts fatigue damage growth in this example also (Fig. 4).

**RESIDUAL STRENGTH**

The model described by Kortshoof and Beaumont can be used to predict the residual strength of fatigue damaged material. It consists of a two-part Weibull model to account for the statistical variability of laminate strength. The Weibull parameters are found by performing strength tests on unnotched unidirectional 0$^\circ$ specimens. Essentially, the Weibull model reduces to (2):

$$\sigma_{\text{ef}} = \sigma_0 \left[ \frac{V_o}{0.0018 (l/a)^3 (a/t^0)} \right]^{1/\beta}$$  \hspace{1cm} (7)
$\sigma_{0f} = \text{failure strength of a volume V of the } 0^o \text{ ply in the damaged zone of the cross-ply composite,}$

$\sigma_0, V_0 = \text{reference stress and volume from experiments on unnotched } 0^o$ specimens.

$\sigma_0 = 1.88 \text{ GPa, } V_0 = 7.4 \text{ mm}^3.$ $\beta = \text{Weibull modulus } = 20 \text{ from unnotched } 0^o \text{ strength tests, } t_0 \text{ is } 0^o \text{ ply thickness.}$

Residual strength, $\sigma_{res}$, can therefore be calculated as a function of split length. The model assumes that laminate fracture occurs when the longitudinal tensile stress in the $0^o$ ply of the cross-ply laminate exceeds its strength:

$\sigma_{res} = \frac{\sigma_0}{K_1}$

where the notch tip stress concentration factor $K_1$ is $f(l/a)$ (Fig. 5) (2):

$K_1 = 8.16 \ (l/a)^{-0.29}$

Figure 5 shows the residual strength data plotted as a function of split length. The predicted residual strength dependence on damage is superimposed. Clearly, Kortschot and Beaumont’s model accurately predicts the residual strength when applied in conjunction with the growth-law described above. The notch tip blunting effect is the dominant factor governing residual strength, with residual strength increasing throughout the duration of the fatigue test (Fig. 6).

FINAL COMMENTS

The model developed in this work is fairly simple in form and compares well with the data available. Most of the parameters which appear in the model have a clear physical meaning and can be determined experimentally.

Further experimental work is needed to investigate the range of conditions (temperature, humidity, strain-rate, state of stress, etc.) over which the model applies. Given the general nature of the approach it may also be possible to extend the work to various lay-ups and geometrical configurations.

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REFERENCES


Fig. 1 A photomicrograph showing splitting, delamination and transverse ply cracking in a $(90/0)_s$ carbon fibre-epoxy laminate (2).
Fig. 2 A model of the various forms of notch tip damage with the critical failure parameters defined (2).

Fig. 3 Predicted damage growth (eqn. 6) with load cycles compared with experimental data for a \((90^\circ/0^\circ)_2\) carbon fibre-epoxy laminate \((W = 24\, mm; a = 4\, mm)\). Each data point refers to a different specimen (1).
Fig. 4   A similar plot to Fig. 3 of experimental data and theoretical prediction for the carbon fibre-PEEK laminates (AS4/ APC2) (3).

Fig. 5   Predicted residual strength as a function of dynamic split length together with experimental observation (W = 24 mm; a = 4 mm; R = 0.1) (1).
Post-Fatigue Residual Strength

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\frac{\varepsilon}{\sigma} = \frac{1}{C_d \tan \phi} \left[ \left( \frac{\Delta K}{A} \right)^{m/2} \left( \frac{b}{a} \right)^{n} \left( C_{d0} + C_{d1} \tan \phi \right) \right]^{2/(m+2)} \]

\[
= \frac{C_d}{\Delta K} \left[ \Delta K^N + C_d \right]^{2/(m+2)}
\]

\[
\sigma_{ef} = C_d \left[ 1 + C_d \left( \frac{\Delta K^N + C_d}{\sigma_{ef}} \right)^{1/2} \right]^{2/(m+2)}
\]

where \( C_d, C_f \) and \( C_b \) are combinations of material constants

Fig. 6  Predicted residual strength as a function of load cycles (N) together with experimental observations (W = 24mm; \( a = 4\text{mm}; R = 0.1 \)) (1).