SURVEY OF PARAMETERS ESTIMATION METHODS

Classical estimate method

The classical procedure of estimating the parameters has the following form:
1. The ranges of SIF $\Delta K_1$ for the chosen crack length $l_i$ are calculated.
2. The fatigue crack growth rates are determined from the relationship between the crack length $l$ and the number of cycles $N$ by using one of the numerical derivation method.
3. The material parameters $C$ and $m$ involved in the fatigue crack growth law (1) are gained with the help of the linear regression method relating $\log(dl/dN)$ versus $\log K$, i.e. the condition of minimal value of the expression

$$\sum_1 (\log(dl/dN)_i - m \log \Delta K_1 - \log C)^2 \quad \text{(2)}$$

is respected.

If more than one specimen being tested, the results from various specimens are summarized and the sum (2) goes through all the experimental results.

Estimate according to Dittevsen and Olsen

The Dittevsen-Olesen model assumes that the fatigue crack growth can be described by the differential equation

$$\frac{dl}{dN} = X(N.C \Delta K^m) \quad \text{(3)}$$

where $X(N)$ is a stationary stochastic process of white noise type with the mean value one and intensity $\delta$ which becomes the third parameter of the model. The parameter $\delta$ is a measure of the crack growth rate fluctuations about the theoretical curve according to the Paris law (1).

From these assumptions, the authors derive the probability density function for cycle number increments between two subsequent crack lengths. The product of probability densities over all these intervals is the likelihood function. It depends on parameters $C$, $m$ and $\delta$. The maximum likelihood estimations of these parameters are derived from the corresponding extreme of the likelihood function.
The estimators are calculated for each specimen separately and by using them, the relevant mean values, variances and covariances are obtained.

**Estimate method based on the nonlinear regression**

Besides two methods presented above a new third one is suggested in which fatigue crack growth law parameters can be gained. The nonlinear regression analysis is used by assuming the minimization of the expression

\[ \sum_i (N_{ie} - N_{it}(C, m))^2 \] \hspace{1cm} (4)

\( N_{ie} \) being experimental and \( N_{it}(C, m) \) theoretical numbers of cycles for crack lengths \( l_i \).

Having the fatigue crack growth law of Paris's form \( (1) \), the theoretical number of cycles is given by

\[ N_{it}(C, m) = \int_{l_0}^{l_i} \frac{dl}{C.AK(1)^m} \] \hspace{1cm} (5)

where \( l_0 \) is the initial crack length.

We recommend to calculate the parameters for each specimen separately and for a sufficient number of specimens to estimate the variances and covariances of parameters besides the mean values, too.

A more detailed information about this method of estimation is given in reference \( (4) \).

**DISCUSSION**

Any growth law can describe an actual process of fatigue crack growth only approximately. This is also obvious from Fig. 1 containing the application of the three above-mentioned methods on Vinkler's experimental data (more details of this experiment can be found in reference \( (1) \)). The mean experimental curve is constructed from the average values of fatigue crack growth rate in dependence on the JIF range. The dispersion of experimental rates about this curve is represented by maximum and minimum rates. Moreover, Fig. 1 demonstrates three straight lines corresponding with the three above-mentioned methods of parameters \( C \) and \( m \) evaluation. For the second and the third method, the straight lines are related with average values of parameters calculated from 68 specimens.
Fig. 1 Dependence of the fatigue crack growth rate on the range of SIF
Fig. 2 Dependence of the crack length on the number of cycles

Fig. 3 Absolute differences of number of cycles

Fig. 4 Relative differences of number of cycles