STRENGTH AND DURABILITY EVALUATION OF STRUCTURAL ELEMENTS USING FRACTURE MECHANICS APPROACHES

A. Ye. Andreykiv*

A general approach to calculations of brittle strength and durability of structural elements in terms of their resistance to crack nucleation and propagation is suggested. It is based on the previous theoretical and experimental results obtained by the author [1, 2, 3, 4].

Application of fracture mechanics methods and approaches to reliability and life assessment of engineering constructions incorporates, as commonly known, the following stages: investigation of in-service conditions of structural elements and shot-pinning of crack-like defects using non-destructive control techniques; the choice of local fracture criterion, corresponding to the fracture conditions of the given material; the choice of fatigue crack nucleation and subcritical growth model; analytical determination of principal parameters of elastic and elastoplastic state of bodies with cracks; determination of crack resistance characteristics of materials; life assessment of structural elements or determination of tolerable dimensions of defects.

The first stage is based on laboratory and in-service observations of the material performance under loading and depends on the qualification of engineering stuff and availability of non-destructive control

*Karpenko Physico-Mechanical Institute of the Ukrainian SSR Academy of Sciences, Iwov, USSR
methods for shot-pinning of defects. In this respect acoustic emission techniques are of special interest.

Local Fracture Criterion. The second stage of calculations is the most important and yet the most complicated one. It practically incorporates all principal concepts of brittle fracture mechanics. Here, universal criterions are of special interest, since they can be successfully applied to describe fracture both of brittle and plastic materials with large or small cracks. The following criterial equation is proposed basing on classical deformational criterion and the theory of cracks of limiting equilibrium

\[ K^2 \alpha^2 - \varepsilon_{\text{nom}}^2 = \varepsilon^2 \]

(1)

This criterion was repeatedly verified using experimental results and calculations obtained due to alternative approaches. Fig. 1 presents calculated (due to criterion (1)) and experimental data of tensile testing of cylindrical specimens with a round crack made of 30KhGSA steel \((G = 1540, \sigma = 1760, \varphi = 0.48, \varepsilon = 0.616, K^2 = 74)\).

Formulation of Nucleation and Subcritical Fatigue Crack Growth Model. The problems arising under durable life assessment calculations of structural elements are inadequately studied and analytically developed. The following calculational models are proposed in the present paper for their solution. A body weakened by a stress concentrator in the form of a smooth crevice, symmetrical to plane \( \xi = 0 \) is considered. The body is subjected to pulse tension normal to plane \( \xi = 0 \). The number of cycles \( N = N^0 \) corresponding to the moment of macrocrack formation at the stress concentrator tip (in the prefracture zone) is to be determined. Let deformation \( \varepsilon \) in the prefracture zone \( (R - R_e) \) be an invariant fatigue fracture characteristic. Considering the crack propagation rate normal to the contour, we obtain the following differential equation relatively to the mobile contour \( \xi^2 = 1 / 4 \xi + 1 / 4 \xi / \xi^2 \)

(2)

under initial condition \( K(\xi, \alpha) = \alpha(\xi) \).

For ideally elastoplastic material, satisfying Tresk-Saint-Venant plasticity condition for \( \varepsilon \) determination, we obtain \( [2] \)

\[ \varepsilon = \varepsilon_0 + \left( \varepsilon_0 \alpha^2 - \varepsilon_0 \alpha - \varepsilon_0 \right) (R - R_e) \]

(3)

The following correlation is proposed for determination of characteristic function of fatigue fracture \( \Phi_{\alpha}(\alpha) \)

\[ \Phi_{\alpha}(\alpha) \]

(4)

Values \( \alpha_0, \alpha_1, \alpha_2 \) are determined experimentally. In Fig. 2 a curve for a plate with two symmetrical concentrators is plotted according to expressions (4), (5)
and (6). Dots correspond to experimental data, which are in good agreement with the calculated curve.

After macrocrack nucleation the task is to determine the number of cycles of its growth up to critical size. Basing on geometrical (Fig. 3) and physical [2] preconditions, determination of the fatigue crack growth kinetics is described by the following set of equations

\[ \frac{dr}{dt} = \beta (a) (K_1^2 / K_0^2), \]

where \( \beta (a) \) is described by relation

\[ \beta (a) \frac{\cos(\alpha) \sin(\alpha)}{K_0 \cos(\alpha) - 3K_0 \sin(\alpha)} = 0. \]

Starting from its initial configuration \( a_0 (q_\infty) \), the crack grows up to its critical size \( a_c (q_\infty) \) determined from expression

\[ a_c (q_\infty) = \frac{1}{\pi} \frac{\cos(\alpha) \sin(\alpha) - 3K_0 \sin(\alpha)}{K_0 \cos(\alpha) - 3K_0 \sin(\alpha)} a_0 (q_\infty). \]

As an example, basing on differential equations (5), growth kinetics of an elliptical fatigue crack under pulse tension of infinite body of 40Kh steel \( (K_0 = 43.4) \) is determined. Crack surfaces are subjected to cyclic loading \( K_0 = 172.6 \). Crack growth curves 1, 2, 3, 4 in Fig. 4 correspond to \( N = 0, N = 128 \cdot 10^3, N = 193 \cdot 10^3, N = 212 \cdot 10^3 \) respectively.

Interpolation method of \( K_S, K_{SS}, K_{SS} \) Determination. Multiparametric Problems. The subsequent stages of strength and life assessment of structural elements in terms of brittle fracture mechanics are connected with determination of stress-strain state parameters of bodies with cracks. The following method is proposed. Let us consider a body containing a system \( m \) of cracks. Suppose, the size of the body and distances between cracks are determined by linear parameters \( c_1, c_2, \ldots, c_m \) and configuration of each \( i \)-th crack \( (i = 1, 2, \ldots, m) \) by \( d^{(i)} \) values \( (j = 1, 2, \ldots, n_j) \) (assuming \( \delta^{(i)} \leq 1 \) \( \delta^{(i)} = 1, 2, \ldots, n_j \); \( \delta^{(i)} \leq 1 \) \( \delta^{(i)} = 2, \ldots, n_j \)). We obtain \[ K^{(0)} (S = 1, 2, \ldots, m) \) determination

\[ K_S = \sum_{i=1}^{m} \sum_{j=1}^{n_j} \left( \frac{\rho_S^{(i,j)}}{\rho_S^{(i)}} \right) \left( \alpha^{(i,j)} \right), \]

where \( \rho_S^{(0)} \) - continuous and continuously differentiated functions \( \rho_S^{(0)} = K^{(0)} \), under \( c_1 \to \infty, \ldots, c_m \to \infty \).

As follows from (6), solution of multiparametric problem consists in solution of corresponding monoparametric problems.
Monoparametric Problems. If in the above discussed problem we assume $m = m^* = n = 1$, it becomes monoparametric. Then, the size of the body and that of the crack are expressed as $C_i$ and $e_i$, respectively, and $e_{ij} = \frac{e_{ij}}{C_i} < 1$. In this case $K_i$ is suggested to be determined by the following interpolational relations

\[ K_i = \frac{g_i}{g_{ij}} \left( \frac{1}{e_i} + \frac{1}{e_{ij}} \right) \]

(9)

\[ \frac{g_i}{g_{ij}} = \left( \frac{g_i}{g_{ij}} \right)_{e_i = 0} \]

(10)

The efficacy and accuracy of the above method were verified based on numerous problems. As an example, we present results of solution for a problem of tension of a rectangular plate with a central crack (Fig. 5) and of torsion of a cylinder with semielliptical surface crack (Fig. 6). Solid lines in Figures are plotted due to equation (8), dotted lines - due to exact solution [2].

In Fig. 6, the curve corresponds to expression (9) and dots are obtained experimentally. Here, $\varepsilon = \varepsilon (\varepsilon'/\varepsilon)$, $\varepsilon'/\varepsilon$, $\varepsilon''/\varepsilon$, $\varepsilon''$ of the function $\varphi_i$ are determined in (4) coordinates.

Experimental Techniques. At present, numerous experimental techniques for crack resistance determination are available. An attempt has been made to develop a universal technique for experimental determination of these parameters under static and cyclic loading based on cylindrical specimen with a round external crack. The use of the above specimen is advantageous in several respects as compared with other methods, namely:

a) due to the absence of the crack contour access to the free surface of specimen fulfilment of the predetermined state (plane deformation) in the prefacture zone along the contour is provided, which corresponds to calculational Griffith-Irwin model; b) due to circumferential position of the crack contour around the specimen cross-section, the plastic yield of material decelerates which renders the testing conditions more strict thus increasing the strength resource; c) manufacturing of cylindrical specimens, formation of round cracks [3] and experimental technique are simple and widely accessible which is very important under static and cyclic crack resistance evaluation of bars.

Using the above technique, $K_i$, $K_e$, $n_i$, $n_j$, $K_{ij}$, of the function $\varphi_i$ are determined in (4) coordinates.

Tolerated defects sizes and critical values of external loading. The performed investigations at all stages of strength and life assessment are interconnected. There are two possible sequences. The first - life assessment of structural elements at a given loading and possible presence of defects; The second - determination of tolerated defects size at the given
life and loading values. Realization of the above sequences of problems incorporates achievements of the mentioned approaches of fracture mechanics.

The proposed approach was used in investigation of various structural elements and verified by results of in-service and laboratory experiments. Results and experimental verification of serviceability of several structural elements are given for illustration. Basing on criterion equation (1) dependency of tolerated pressure upon the size of semi-elliptical surface crack \( R \) was investigated for thin-walled pipe. The same dependency was determined experimentally due to testing of pipes made of 17G15U steel. Calculated values (solid line) and experimental data (dots) are compared in Fig. 7. In Fig. 8, calculated (solid line) and experimental (dots) results were obtained for bending of rail weakened by an oval crack at the head. As evident, the proposed approach to calculation is sufficiently effective.

SYMBOLS USED

- \( K_L, K_R, K_m \) = stress intensity factors for mode I, II and III cracks (MPa\( \sqrt{m} \))
- \( K_{Ic} \) = fracture toughness under static loading (MPa\( \sqrt{m} \))
- \( \varepsilon_c \) = maximum strain under tension at the moment of fracture
- \( \varepsilon_{nom} \) = nominal strain determined due to standard tension testing
- \( \sigma_y \) = yield strength (MPa)
- \( \sigma_u \) = ultimate tensile strength (MPa)
- \( \psi_t \) = relative transverse narrowing
- \( E \) = Young modulus (MPa/m\(^2\))
- \( E^0 \) = linear strengthening modulus (MPa/m\(^2\))
- \( N_{c0} \) = macrocrack nucleation period (cycles)
- \( K_p - R \) = the size of initial plastic zone near the concentrator (m)
- \( \varepsilon_{sc} \) = maximum tensile strain under cyclic loading at the moment of fracture
- \( p_a \) = fracture loading under tension and bending of cylindrical specimens (Fig. 9)
- \( \varepsilon_0 \) = maximum strain in the initial prefracture zone near the concentrator
- \( K_{Isc} \) = fracture toughness under cyclic loading (MPa\( \sqrt{m} \))
- \( \Delta \) = normal to the crack surface
- \( \sigma_{nc} \) = normal stresses in netto-section (MPa)
- \( \Delta \) = subcritical fatigue crack growth period (cycles)
yield strength under shear (MPa)

\( \sigma_{\text{ex} \text{h}, \text{neto}} \) = experimental and calculated fracture of netto-section (MPa)

\( \sigma_{\text{ex} \text{h}, \text{max}} \) = normal and maximum netto-stresses

\( \kappa_{b}, \kappa_{a}, \kappa_{g} \) = experimentally determined coefficients

\( \sigma_{G} \) = crack resistance of the material with the investigated crack (MPa)

\( S_{2} \) = crack area at the head (m²)

\( S_{3} \) = rail head area (m²)

REFERENCES


