INTERACTION OF BRITTLE FRACTURE AND BUCKLING OF
COMPRESSED AND TORSIONED BARS WITH A BISYMMETRIC OPEN
CROSS SECTION

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INTRODUCTION

This paper is an attempt to construct interaction
curves. This can be done by introducing the limit bearing
capacity of the cross-section measured by a parametric
limiting bimoment $B_k$ as a parametric function
of the brittle fracture strength $R_f$ and the axial
force $S$

$$B_k = (R_f + S/A) J_{\omega}/\omega$$

where: $A$ - area of the cross-section, $J_{\omega}$ - sector
inertia-moment, $\omega$ - sector ordinate cross-section.

Bimoment $B < B_k$ as load of cross-section will
be determined from differential equation of the form
(2), shown by Kowal and Kubica [1].

$$E J_{\omega} \phi'' + (5 \epsilon - G J_{\omega}) \phi'' = m_0 + M_0 \delta s_{\omega}$$

where: $\phi$ - torsion-angle of cross-section, $E J_{\omega}$ -
sector stiffness of cross-section, $G J_{\omega}$ - stiffness
of pure torsion, $m_0$ - continuous torsional moment,
$M_0$ - torsional moment, $\delta$ - Dirac's symbol, $i =$
$E (J_x + J_y)/A$.

The bar load by bimoment is calculated from the
differential equation (2), assuming a zero axial force
$S$. Flexural torsional critical bearing capacity $S_{cr}$
of the bar will also be determined from the differential
equation (2) assuming $m_0 = 0$ and $M_0 = 0$, then we
have

$$S_{cr} = \left(\sigma^2 + \sigma' \sigma''\right) E J_{\omega}/l^2 \frac{l^2}{\omega}$$

where: $A = G J_{\omega}/E J_{\omega}$, $l$ - bar span, $n$ - number of
half-waves.

Algorithm of the construction of interaction
curves is shown by the example of a cantilever com-
pressed bar, loaded by a concentrated torsional moment
on the free end of the bar.

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EXAMPLE OF CONSTRUCTING INTERACTION CURVES

Let us take into consideration a bar shown in Figure 1. From the solution of the differential equation (2) for

\[ k^2 = (3G_0 - 6J_S)/EJ_\omega \] 

we have a maximal \( B \)

\[ B = -\frac{M_b (th \ al)}{\alpha} < B_k \]  

We obtain load of support cross-section by bi-

moment \( B \) from the solution of the differential equa-

tion (2) for \( S = 0 \)

\[ B = -\frac{M_b (th \ al)}{\alpha} \]  

Taking into consideration the relationships (3,4,5), we obtain the equation of a family of interaction curves

\[ B = \frac{tq k \ al}{B_k k \ al} \]  

Argument \( k^2 \) occurring in equation (6) will be transformed to the form (7) taking into consideration the relationship (3) and \( n = 0.5 \) for the cantilever bar

\[ (k^2)^{1/2} = \frac{(n T^2 - a^2 t)}{EJ_\omega} = \frac{(0.25 T^2 + a^2 t)}{EJ_\omega} \]  

The final form of the interaction curve in the area

\[ a^2 t^2 (n T^2 + a^2 t) < S/S_{cr} \leq 1, \text{ for } k^2 > 0 \] had the form

\[ \frac{B}{B_k} = \frac{th \ al}{al} \left( \frac{\sqrt{(0.25 T^2 + a^2 t)} S/S_{cr} - a^2 t}{\sqrt{(0.25 T^2 + a^2 t)} S/S_{cr} - a^2 t} \right) \]  

Lower bond of the family of interaction curves only for \( a^2 = 0 \). Then \( B = -M_b \), equation of the inter-

action curve assumes the form (9)

\[ B = \frac{tq (0.25 T^2 S/S_{cr})}{B_k 0.5 \sqrt{S/S_{cr}}} \]  

In the interval \( 0 < S/S_{cr} < a^2 t^2 -(n T^2 + a^2 t^2) \), bimoment \( B \) determined from differential equation (2) is

\[ B = -\frac{M_b (th \ al) b \ l}{b l < B_k} \]  

where:

\[ (b l)^2 = (6J_S - 5a^4)/(EJ_\omega) = a^4 - t^2 -(0.25 T^2 + a^2 t^2) S/S_{cr} \]  

Interaction curve assumes the form:

\[ \frac{B}{B_k} = \frac{th \ al}{al} \left( \frac{\sqrt{(0.25 T^2 + a^2 t)} S/S_{cr} - a^2 t}{\sqrt{(0.25 T^2 + a^2 t)} S/S_{cr} - a^2 t} \right) \]  

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Figure 1 shows limiting curves determined from interaction equation derived for selected examples. Curve 1 for $a_l = 0$ refers to: a cantilever bar loaded by a moment concentrated at the end of the bar, a bar with a forked fix at the ends and a bar rigidly fixed, loaded by a concentrated torsional moment in midspan. Curves 2 refer to the same bars for $a_l = 1$. Curves 3 and 4 refer respectively to the cantilever bar loaded uniformly along the bar's length, for $a_l = 0$ and $a_l = 1$.

REMARKS AND CONCLUSIONS

The introduction of the concept of limit bearing capacity of the cross-section of a non-free torsioned bar, measured by bimoment $B_k$ as a parametric function of axial force $S$ and brittle fracture strength $R$, provide the possibility to construct interaction curves in dimensionless coordinates. The interaction curves depend on the coefficient $a_l$. Lower bond of the interaction curves is obtained for $a_l = 0$.

The characteristic feature of the interaction curves in dimensionless coordinates is their similarity for many cases of bars in spite of their different limit bearing capacity.

REFERENCES

(1) Kowal Z., Kubica E., Second Order Torsioning of the Thin-Walled Bars with a Bisymmetric Open Cross-Section, TU Wroclaw, PNIB 4, Metalowe Dzwigary Specjalne 1, Wroclaw 1971.

Figure 1  Examples of interaction curves