A HYBRID METHOD FOR DETERMINING STRESS INTENSITY FACTORS.

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A hybrid method for the mixed-mode stress intensity factors (SIF) determination has been proposed. The application of the method for cracked stiffened or unstiffened plates studies is presented briefly.

INTRODUCTION

The primary requirement for the successful application of fracture mechanics methods is accurate predictions of SIF in cracked structural components. The current paper concerns itself with development of theoretical-experimental method. The method is based on the fundamental solutions for plates of simplified geometry and experimentally measured relative crack surface displacements.

FORMULATION OF THE METHOD

Consider elastic plate of arbitrary configuration with a crack. The coordinates of crack tips are $\alpha$ and $\beta$. Different loadings are applied to the plate. Not all of them are known. The problem is to determine mixed-mode SIF $K_{\alpha,\beta}$ and $K_{\beta,\beta}$.

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Using superposition principle the original problem is substituted by additional one, which fundamental solution is known. An additional problem has a simplified geometry with no loadings in external region and \( p(x) \) loading is acting on crack surface. Thus, additional and original problems have equal SIF and crack surface relative displacements \( g(x) = \Delta U + i \Delta V \). Using the fundamental solution \( g(x) \) and SIF can be expressed in term of \( p(x) \) with help of singular integral operator \( \Gamma \) and functions \( D_0, D_8 \):

\[
\Gamma p(x) = g(x)
\]

(1)

\[
K_{10, b} - i K_{II, b} = \int D_0 b [\rho(x)] dx.
\]

(2)

Let the displacements \( y_1, y_2, \ldots, y_n \) were experimentally measured in the crack points \( x_1, x_2, \ldots, x_r \). Then \( \rho(x) \) can be found from equation (1).

Present \( \rho(x) \) in the form

\[
\rho(x) = \sum_{i=1}^{m} A_i F_i(x) + \sum_{d=0}^{n} B_d T_d(w(x)),
\]

(3)

where \( F_i(x) \) are known functions. Unknown part of loading is approximated by Chebyshev polynomials \( T_d(w(x)) \). The coefficients \( A_i \) and \( B_d \) are obtained in the least-squares sense

\[
\sum_{k=1}^{n} \left| g_{xk} - \sum_{i=1}^{m} A_i F_{ix} - \sum_{j=0}^{n} B_j T_{xj} \right| f_{ix} = 0, \quad i = 1, m,
\]

(4)

\[
\sum_{k=1}^{n} \left| g_{xk} - \sum_{i=1}^{m} A_i F_{ix} - \sum_{j=0}^{n} B_j T_{xj} \right| t_{jk} = 0, \quad j = 0, n,
\]

\[
f_{ix} = \Gamma F_i(x), \quad t_{jk} = \Gamma T_j(w(x)).
\]

The best solution of \( N \) is based on empirical risk criteria. Following this approach SIF are calculated from formula (2).

Two experimental techniques are used for crack surface displacements measurements: first of which is based on the small-scale clip gages. A computer connection makes it possible to perform the measurements in real time.
Glografic interferometry is used in the cases of small cracks.

RESULTS AND CONCLUSIONS

Results some of the solved problems, which shown in figures 1,2,3, are presented here. Structural components were loaded in pure tension 10 kg/mm². The Young's moduli of the plates were 6540 kg/mm². Fundamental solutions for infinite and semi-infinite cracked plates given by Savruk (1) were employed in the SIF determination. Crack profiles have been investigated by speckle-interferometry in the first problem (Fig.1) and by clip gages in other two (Fig.2,3).

Table 1 summarizes SIF obtained by new method and analytic results. Proposed method can provide accurate results for structural components of complicated geometries and loading. The method can be used as in the research and in practice.

**TABLE 1 - Comparison of SIF (kg/mm)** obtained with help of proposed method and from analytic solutions.

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>SIF</th>
<th>Analytic Method</th>
<th>Proposed Method</th>
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<tr>
<td></td>
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<td>Result</td>
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<tr>
<td></td>
<td>$K_{1c}$</td>
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<tr>
<td></td>
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<tr>
<td>3</td>
<td>$K_{1a}$</td>
<td>(3) 61.0</td>
<td>63.0</td>
</tr>
</tbody>
</table>

REFERENCES

(1) Savruk, K. P. "Two-Dimensional Problems of Elasticity for Bodies with Cracks" in Russian, Naukova Dumka, Kiev, USSR, 1981.


Figure 1. Plate with two holes.

Figure 2. Cracked plate stiffened by strips.

Figure 3. Stiffened panel.