PROGRESS OF COMPUTATIONAL FRACTURE MECHANICS IN FINLAND

H. Talja*

Computer codes for assessing values of elastic-plastic fracture parameters in two- and three-dimensional cases have been developed. The validity of the codes has been assessed by evaluating fracture mechanics tests and participating in international round robin programs. Methods of calculating J-integral values have been compared and the effect of different parameters on the numerical results has been extracted.

INTRODUCTION

In Finland the nuclear power is of great economic importance: in 1985 35 % of the electricity consumed in Finland was produced by nuclear power while the average operating factor of our four plants was about 90 %.

To assure the safe operation without unnecessary shutdowns as well in future it is important to be able to assess the effect of possible defects in nuclear power plant components. For this purpose capabilities for experimental and computational fracture analyses have been developed at VTT.

In this paper the work concerning computational fracture mechanics is summarized.

* Nuclear Engineering Laboratory
Technical Research Centre of Finland (VTT)
DEVELOPMENT OF COMPUTER CODES

The development of computing codes for fracture mechanics was started in Finland in the late 70'ies in accompaniment to nuclear safety analyses. The computer codes for fracture assessments are a part of VTT's finite element system consisting of pre-processors, FE-programs and post-processors (Fig.1).

At the very beginning 2-dimensional codes were developed whereas increasing attention is nowadays paid on the use of 3-dimensional models. The first two computer codes 'CRACK' and 'JINT' were independent finite element codes. At present the general purpose program ADINA is being utilized also in the field of fracture mechanics. The postprocessing program 'VTTVIRT' has been developed to calculate J-integral (or energy release rate) values on the basis of the stress and displacement results calculated using the ADINA code.

CRACK and JINT -codes

Both codes adopt 8-noded isoparametric 2-dimensional elements applicable for plane stress, plane strain and axisymmetric cases. Mechanical loads can be given as concentrated or distributed forces or as imposed displacements. Thermal loads can be taken into account, as well.

The CRACK-code was developed for calculating stress-intensity factors in 2-dimensional and axisymmetric cases. It calculates the stress-intensity factor from nodal displacements near the crack tip or from the energy release rate due to a virtual crack extension.

The JINT-code is a two dimensional, nonlinear finite element code for calculating the fracture parameter J-integral. The code is based on the theory of small elastic-plastic deformations according to the von Mises material model. The material stress-strain curve for materially nonlinear analysis can be described with two or more linear parts or with an exponential curve. When purely mechanical loads are concerned, the line integral definition of J is used. In case of thermal loads, a modified definition of J is applied.
In the linear region the program calculates also the linear stress-intensity factor $K_I$. Pre- and post-processors are available to generate the mesh, to visualize the results using x-y plots or equipotential curve plots and to calculate CTOD-values.

**VTTVIRT-code**

The postprocessing program 'VTTVIRT' has been developed to calculate J-integral (or energy release rate) values on the basis of the stress and displacement results calculated using the ADINA code. In 2-dimensional cases the original formulation by Parks (6) according to Eq.1. and the improved method introduced by delorenzi (3) according to Eq.2. can be used. When 3-dimensional models are considered only the VCE-method by delorenzi has been programmed.

The equations for J-calculations are:

\[ J = -\int \frac{\partial \mathbf{u}}{\partial \mathbf{a}} \cdot \mathbf{w} + [J][\mathbf{c}]^T \frac{\partial [E]}{\partial [u]} \, dV \]  \hspace{1cm} (1)

and

\[ J = \frac{1}{\Delta A_c} \int \left[ \text{trace} \left[ \{\delta \mathbf{u}\} \right] - \{w\} [\mathbf{w}] \frac{\partial \mathbf{x}}{\partial \mathbf{x}} \right] \, dV \]  \hspace{1cm} (2)

To make the program user-friendly automatic generation of integration paths has been programmed. This is favourable especially in three-dimensional cases. All data needed in calculations are stored in a working vector in order to reduce the necessary amount of central memory.

At the present state the VTTVIRT-code can handle only mechanical loads. Modifications needed to take thermal loads as well as pressure loads into consideration are being made presently.

**Use of weight function method (VTTSIF-code)**

The stress-intensity factor (SIF) can always be determined by using the finite element method but the calculation may be too expensive for practical purposes. This is truth especially when considering crack growth due to
fatigue, in which one must calculate SIF e.g. along the crack front with various sizes and aspect ratios of an crack. By using the weight function method (WFM) computer costs can be effectively reduced.

The weight function is equal to the stress intensity factor caused by a pair of forces acting in an arbitrary point on the opposite crack surfaces. When the stress distribution in unflawed structure is known the stress intensity factor is obtained simply by integrating the product of the weight function and stress over the crack area.

The only difficulty in utilizing WFM is to determine the weight function values. An exact weight function has been formed only for few cases. In the cases, when the stress-intensity factor along the crack front for some reference case is unknown the weight function must be determined numerically by using the finite element method or the boundary integral equation method.

If the SIF for some stress state in the structure concerned is known then SIF for arbitrary loading can be calculated by the 'VTTSIP' code. At the moment the VTTSIP code is able to calculate SIF for two-dimensional edge cracks, axisymmetric edge cracks both in inner and outer walls and semi-elliptical surface cracks in three-dimensional cases.

**NUMERICAL RESULTS**

To assess the validity of the codes a wide research program has been performed where fracture mechanics tests have been evaluated using the 2-dimensional JINT-code (Refs. 7 and 8). The effects of different material parameter values, different specimen geometries and the side grooving were investigated. The experimental results always fell between plane stress and plane strain solutions. This indicated that 3-dimensional calculations are necessary to get accurate compatibility.

VTU is participating in international round robin programs. The EGF numerical round robin programs have been reported by Larsson (5). For the second EGF round
robin, where an experiment using a CT-specimen was simulated. VTT made only 2D-calculations. Later when the VTTVIRT-code was extended to 3-dimensional cases the calculation was repeated and results comparable with the 3D-solutions in the round robin were achieved. The FE-mesh used in the calculations is shown on Fig. 2. The values of different parameters (e.g. integration order) in 3D-analyses were investigated. When using reduced integration the calculated loading was 2.5 % lower at the final applied displacement level. However, the average J-value along the crack front was a few percent higher. This is expected to be caused by the oscillatory behaviour of stresses typical in FE-calculations and partly by the virtual crack extension technique proposed by deLorenzi (3). Different techniques to model the virtual crack extension are presently being extracted.

For the third EGF round robin we have made several 2D-analyses using varying values of parameters. The length of crack tip elements was about 17 % of ligament size. The J-values were calculated using four element rings whose numbering was started from the innermost one. The values of different parameters used in the ADINA/VTTVIRT-calculations are listed in Table 1. The feasibility of the VCE-method by deLorenzi compared to the original method by Parks was calculationally demonstrated (Tables 2 and 3 and Fig.3).

Using both methods nearly identical average J-values were obtained. When the method by deLorenzi is used, J-values can be accurately calculated from the crack tip elements in the whole applied load range. When full integration is used, the accuracy is remarkably improved near the linear region, but this effect diminishes rapidly when the size of the plastic zone is increasing. Near the linear region best results are obtained when both 1/r and 1/\sqrt{r} terms in strains are included, while it seems to be appropriate to use only the 1/r-singularity when the size of the plastic zone is remarkable. As expected poorest accuracy is obtained when singular terms are omitted. Similar results have previously been reported by Bakker (2).

Using the method by Parks large deviations in J-values calculated from the crack tip elements are
obtained. The deviations depend strongly on the chosen amount of the virtual crack extension. For this reason comparisons of crack tip J-values calculated using different values of the parameters are not very useful. In the present case the virtual crack extension \( \delta = \frac{f_{el}}{320} \) was used.

For the NRC FEM round robin a CT-specimen with extensive crack growth was analyzed. Because our present codes cannot simulate crack growth separate analyses with different crack lengths were performed. The experimental dependence between crack extension and load line displacement was then used to obtain the final results. As regards the J-integral our results were close to the experimental values obtained by the standard ASTM E 813–81 (1) whereas in the solutions where the crack extension was simulated during the calculation the J-values were close to \( J_W \) proposed by Ernst (4). Our load versus load line displacement results deviated quite little from the experimental ones though only the material nonlinearity was taken into consideration (Fig.4). The calculation for the final crack length is at present being performed considering also the geometric nonlinearity.

**TABLE 1 - Numbering of solutions in 2-dimensional parameter studies.**

<table>
<thead>
<tr>
<th>sol. no.</th>
<th>integr. order</th>
<th>type of singularity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2·2</td>
<td>1/r</td>
</tr>
<tr>
<td>2</td>
<td>3·3</td>
<td>1/r</td>
</tr>
<tr>
<td>3</td>
<td>2·2</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>2·2</td>
<td>1/r &amp; 1/r</td>
</tr>
</tbody>
</table>

610
### TABLE 2 - The accuracy of J-values calculated using the method introduced by Parks (6).

\[ J_{av} = \frac{J_2 + J_3 + J_4}{3}; \quad i, j = 2, 3, 4. \]

<table>
<thead>
<tr>
<th>sol. no.</th>
<th>appl. displ.</th>
<th>( J_{av} )</th>
<th>( \frac{(J_i - J_{av})<em>{max}}{J</em>{av}} )</th>
<th>( \frac{J_i - J_{av}}{J_{av}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.981</td>
<td>0.28</td>
<td>45.9</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>1.002</td>
<td>0.05</td>
<td>30.6</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>1.001</td>
<td>0.43</td>
<td>-0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.992</td>
<td>0.30</td>
<td>-1.6</td>
</tr>
<tr>
<td>1</td>
<td>2.1</td>
<td>234.4</td>
<td>1.22</td>
<td>58.7</td>
</tr>
<tr>
<td>2</td>
<td>2.1</td>
<td>237.6</td>
<td>1.18</td>
<td>40.4</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>240.0</td>
<td>1.37</td>
<td>22.2</td>
</tr>
<tr>
<td>4</td>
<td>2.1</td>
<td>236.3</td>
<td>0.78</td>
<td>13.9</td>
</tr>
</tbody>
</table>

### TABLE 3 - The accuracy of J-values calculated using the method by deLorenzi (3).

\[ J_{av} = \frac{J_2 + J_3 + J_4}{3}; \quad i, j = 2, 3, 4. \]

<table>
<thead>
<tr>
<th>sol. no.</th>
<th>appl. displ.</th>
<th>( J_{av} )</th>
<th>( \frac{(J_i - J_{av})<em>{max}}{J</em>{av}} )</th>
<th>( \frac{J_i - J_{av}}{J_{av}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.980</td>
<td>0.29</td>
<td>-4.4</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>1.001</td>
<td>0.03</td>
<td>-1.5</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>1.000</td>
<td>0.45</td>
<td>-3.9</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.992</td>
<td>0.32</td>
<td>-0.6</td>
</tr>
<tr>
<td>1</td>
<td>2.1</td>
<td>234.4</td>
<td>1.17</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>2.1</td>
<td>237.6</td>
<td>1.15</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>240.0</td>
<td>1.33</td>
<td>-14.1</td>
</tr>
<tr>
<td>4</td>
<td>2.1</td>
<td>236.3</td>
<td>0.76</td>
<td>3.5</td>
</tr>
</tbody>
</table>
PRESENT AND FUTURE RESEARCH

At present we are participating in the German HDR-program and in the Nordic LBB-program. In the LBB-program the 'Leak Before Break' criterion for pipe and pressure vessel geometries is being assessed. Experiments using semi-scale specimens, pipes and full-scale pressure vessels will be made and simulated with numerical models.

SYMBOLS USED

\[ a = \text{Crack length (mm).} \]
\[ \Delta A_c = \text{Increase in cracked area (mm}^2\text{).} \]
\[ [B] = \text{Strain displacement matrix.} \]
\[ [I] = \text{Identity matrix.} \]
\[ J = \text{J-integral (kJ/m}^2\text{).} \]
\[ |J| = \text{Determinant of Jacobian matrix.} \]
\[ \varepsilon_e 1 = \text{Length of crack tip element.} \]
\[ [U] = \text{Nodal displacement matrix.} \]
\[ V = \text{Volume of the structure.} \]
\[ W = \text{Strain density function.} \]
\[ [X] = \text{Nodal coordinate matrix.} \]
\[ [\Delta X] = \text{Change in [X] due to the virtual crack extension.} \]
\[ \delta = \text{Amount of virtual crack extension.} \]
\[ \{\sigma\} = \text{Stress vector.} \]
REFERENCES

(1) ASTM E 813-81. Standard Test for \( J_{IC} \), a Measure of Fracture Toughness, ASTM 1981.


Figure 1. VTT: S FE-system.

Figure 2. FE-mesh for 3-dimensional calculation of a CT-specimen.
Figure 3. Error in $J$, calculated using the method by deLorenzi.

Figure 4. Load plotted against load line displacement in the NRC round robin.