A method to obtain intrinsic failure curve in dynamic loading condition is presented. It consists of thin tube with an arbitrary oriented crack which is submitted to a pressure pulse generated by a shock wave. This tube is inserted between the incident and the transmitted rod of a split Hopkinson bar. An example of a dynamic intrinsic failure curve of a Magnesium alloy is presented and discussed.

INTRODUCTION

Magnesium is used under multiple forms: as an alloying element in aluminium alloys, in nodularisation of iron, desulfuration of steel, chemical and reduction processes, cathodic protection. Magnesium alloys can be produced also in all wrought and cast forms, but this use in 1985 represented 45,000 metric tons over a total of primary consumption in Western World of 226,000 tons of magnesium.

Castability, productivity and versatility are in favour of pressure die casting with magnesium alloys. The widespread acceptance of light metal castings in cars would mean that magnesium alloys can be used for many parts, if some progress can be made on corrosion resistance, deep drawing and mechanical properties.

Because of its hexagonal structure, magnesium is relatively sensitive to strain rate. Experimental results given by Davies and Hunter /1/ and Senseney et al. /2/ indicate that the strain rate sensitivity coefficient $m_{12}$ of magnesium is in the range of 2-4.

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This coefficient is defined by the following expression:

\[ \nu_{ij} = \frac{\sigma_{ij}}{\sigma} \left( \frac{\gamma}{\eta} \right) \]  

where \( \sigma_{ij} \) and \( \gamma \) are the respective values of shearing stress obtained in torsion tests at constant strain-rates \( \eta \) and \( \eta' \).

Very few data on the fracture toughness of magnesium exist in literature and there is no data on dynamic fracture toughness. A frequently encountered problem in engineering applications of fracture mechanics is the mixed mode loading of a crack subjected to biaxial tension. To obtain a conservative estimate of the load required for crack initiation it is only necessary in many cases to perform calculations according to mode I type of loading. This underestimate the fail safe load and holds very well for isotropic media.

In this paper, a method for determination of mixed mode dynamic fracture toughness of magnesium alloy tubes is described and dynamic intrinsic curve of failure of this material is presented.

MATERIAL AND EXPERIMENTAL DEVICE

Dynamic fracture toughness tests have been performed on an aluminium-magnesium alloy GA3Z1 containing 3% aluminium and 1% zinc. The mechanical properties of this alloy are presented in Table I.

<table>
<thead>
<tr>
<th>( \dot{e} ) (s(^{-1}))</th>
<th>Re (MPa)</th>
<th>Rm (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-3} )</td>
<td>143.5 MPa</td>
<td>220.7 MPa</td>
</tr>
<tr>
<td>( 10^{1} )</td>
<td>200 MPa</td>
<td>290 MPa</td>
</tr>
</tbody>
</table>

Table I

Static tests are performed at low strain rate (\( \dot{e} = 10^{-3} \) s\(^{-1}\)) while dynamic tests (\( \dot{e} = 10^{3} \) s\(^{-1}\)) are performed using a split Hopkinson pressure bar. The stress-strain curves in the two cases are shown in Figure 1.

The material was obtained as round bars of 30 mm diameter. Wedge loading CT (WLC7) samples of 20 mm thickness and tube of 22 mm inner diameter, 1 mm thickness and 25 mm length were issued of these bars. The geometry and the details of the slots of these two kind of samples is described in figures 2 and 3.

The depth \( c \), the angle \( \theta \) with the longitudinal direction the length \( 2a \) of the slots in tube samples are given in table II.

262
The WLCT samples were fatigue precracked with ratio \( a/W = 0.5 \). Dynamic fracture toughness was obtained using a split Hopkinson pressure bar.

The dynamic loading of the WLCT is given by a stress wave produced by the impact of a projectile on the incident road of a split Hopkinson pressure bar device. This method is shown schematically in Figure 4 and described in reference /5/.

The recorded transmitted pulse provides the critical applied load for fracture \( P_C \). The friction between the wedge and the specimen has to be taken into account. The relation between critical load \( P_C \) and the applied load \( P_c \) is given by:

\[
P_c = P_C \cdot \frac{1}{2 \tan \left( \frac{\theta}{2} + \lambda \right)}
\]

\( \theta \) is the wedge angle, \( \lambda = \arctan \mu \) is the friction coefficient. For magnesium-steel contact a value of \( \mu = 0.17 \) has been measured by the authors according to Klepacko method described in reference /16/.

The small size of the specimen and the relatively long time for fracture (20 \( \mu s \)), compared to the travel time of the wave (150 \( \mu s \)), leads to the conclusion that the dynamic applied stress intensity factor is only slightly different to the static applied stress intensity factor and the quasistatic assumption can be used to calculate the critical stress intensity factor (5).

A further development of the dynamic fracture toughness in mode I was achieved to procedure mixed mode of rupture. It consists of the same split Hopkinson bar apparatus, but the specimen is now a tube, the interior of which is filled with oil. A pressure pulse from the incident rod builds up an hydrostatic pressure in the tube. The device gives necessary fluid tightness using "O" rings between bar and device (1) and tube and device (2) as can be seen in figure 5.

The free ends of the tube are in contact with teflon rings (3) in
order to prevent tube buckling. The oil is filled by the holes no 4 in figure 5.

This method is similar to the one used by Ohlson /6/ /7/. The transmitted wave produced by impact is registered with strain gages glued on the rod. The electrical signal is amplified and recorded on a digital oscilloscope. A typical example of the registered transmitted wave can be seen in figure 6. Due to the fact that oil is not absolutely incompressible, radial inertia causes transversal waves which interfere with the signal. The critical pressure is then a average value.

ANALYSIS OF THE RESULT

The inclined crack in such a tube is submitted to a biaxial loading. The pressure provides membrane stress, but since the symmetry of the tube is disturbed by the crack, bending stress must be taken into account. In addition the loading rate being relatively high, time to fracture (40-80 μs), and fracture process can be considered as dynamic. This will mean that the mechanical properties are modified by strain rate. As previously mentioned the total travel time of the wave is relatively long compared to travel time in specimen and the inertial effect on stress history can be neglected.

This procedure is called quasi-static approximation and can be used if the following criteria is respected.

\[ \frac{C_w \cdot T_r}{2a} \gg 10 \]  
(3)

This is the case in our tests. \( C_w \) celerity of wave, \( T_r \) time to rupture, \( 2a \) crack length.

The defect in the tube is a surface defect, but, as the tube is relatively thin (1 mm), the ligament between the bottom of the defect and the inner surface is easily plastified. In this case the surface defect can be considered as a "through the thickness" defect. This procedure is usually used for recategorisation of defects. Although, the tube fails in the ductile range, the analysis is based on apparent fracture toughness \( K_C \).

This apparent fracture toughness is defined as the value of the equivalent elastic and critical stress intensity factor multiplied by a plasticity-Irwin type correction \( \rho_0 \).

\[ K_C^* = K_C \sqrt{\rho_0} \]

The stress intensity solutions for an arbitrary oriented "through the thickness" crack in a long cylindrical shell have been obtained by Lakshminarayana and Murthy /8/ /9/ using a analytical-numerical procedure.
The method of analysis involved obtains series solutions to the governing thin shell equation:
\[ \nabla^2 R + 8i \alpha^2 \frac{\partial^2 R}{\partial x^2} = 0 \]  
(5)
where \( \alpha^2 = (a^2/8Rt) \int 12 \left(1 - v^2 \right) \frac{J^3}{J^3} \) with \( R \) radius of curvature of the shell, \( t \) the thickness and a half the crack length.

The solutions of equation (5) keep the following criteria:
- periodicity of solutions
- symmetry considerations
- stress and displacement vanish when \( x \rightarrow \infty \)

This method leads to a stress function \( f(r, \theta) \) and real displacement \( W(r, \theta) \) of the form:
\[
\begin{align*}
\Theta &= \Theta_0 + \Theta_1 r^3 \cos \frac{\theta}{2} + \Theta_2 r \Theta_3 3/2 \sin \frac{\theta}{2} + \Theta_4 r^{3/2} \cos \frac{\theta}{2} + \Theta_5 r^{3/2} \sin \frac{\theta}{2} + O(r^2) \\
W &= \Theta_6 + \Theta_7 r^3 \cos \frac{\theta}{2} + \Theta_8 r \Theta_9 3/2 \sin \frac{\theta}{2} + \Theta_{10} r^{3/2} \cos \frac{\theta}{2} \\
&\quad + \Theta_{11} r^{3/2} \sin \frac{\theta}{2} + O(r^2)
\end{align*}
\]

The \( \Theta_i, W_i \) functions are numerically tabulated and the stress intensity factors in mode I and II are given by the superposition of the membrane and bending stress intensity factors:
\[
\begin{align*}
k_{11}^{(m)} &= 3 \left( \Theta_3 + \Theta_4 \right) / 2\sqrt{2} \\
k_{11}^{(b)} &= 3 \left( \Theta_6 + 3 \Theta_7 \right) / 2\sqrt{2} \\
k_{11}^{(m)} &= 3 \left( 3 - v \right) W_4 - (5 + 3v) W_3 J / (24 \left(1 - v^2 \right) J^3) \\
k_{11}^{(b)} &= 3 \left( 1 - v \right) W_7 + 3W_8 J / 24 \left(1 - v^2 \right) J^3
\end{align*}
\]

Here the subscripts \( m \) or \( b \) refers to membrane or bending stress and the subscripts \( I \) for \( I \) refers to mode I or II. In the original paper of Lakshmikumar and Murthy, the non-dimensional stress intensity factors with respect to \( \sigma \sqrt{a} \) are given as a graph function of the \( \Theta \) angle and for different values of the parameter. These non-dimensional stress intensity factors are called respectively \( C_{m1}, C_{m2}, C_b, \) and \( C_{b1} \) the subscripts having the same meaning as previously.
PRESENTATION OF RESULTS

A serie of tests with five values of have been performed in order to build an intrinsic failure curve \( K_{le} / K_c^* = f \left( K_{le} / K_c^* \right) \) where \( K_{le} \) and \( K_{le} \) are the apparent fracture toughness in mode I and mode II and \( K_c^* \) is the apparent fracture toughness measured with the same test but with \( \theta = 0^\circ \).

The results are presented in table III, where are listed for the different values: the membrane stress \( \sigma_m \), the halfcrack length \( a \) and the coefficients \( f_I \) and \( f_{II} \) which are defined by the following relation-ship:

\[
\frac{K_{le}}{\sigma_m a} = f_I, \quad \frac{K_{lle}}{\sigma_m a} = f_{II}
\]  

(8)

These coefficients include Irwin's plasticity correction \( C_p \), finite length correction \( C_o \) according to / 10 /, and Lakshminarayana bending and membrane coefficients:

\[
f_I = \sqrt{\sigma_p}, \quad C_p \cdot \left( C_{mI} + C_{bII} \right)
\]

\[
f_{II} = \sqrt{\sigma_p}, \quad C_o \cdot \left( C_{mII} + C_{bII} \right)
\]  

(9)

The intrinsic failure curve is presented in Figure 7. In the same figure is presented an other failure curve relative to aluminium alloy AS6 for comparison.

For an isotropic and brittle material in which the failure in mixed mode is governed by the SIH criteria / 14 /, the intrinsic failure curve can be described by the following relationship:

\[
\left( \frac{K_I}{K_{IC}} \right)^2 + \left( \frac{K_{II}}{K_{IC}} \right)^2 = 1
\]

The effects of anisotropy, geometry and ductility effects modifies the shape of intrinsic failure curve which then can be more generally described by:

\[
\left( \frac{K_{le}}{K_{c}^*} \right)^u + \left( \frac{K_{lle}}{K_{c}^*} \right)^v = 1
\]

where \( u \) and \( v \) are empirical coefficients, \( K_c^* \) is the apparent fracture toughness and \( K_{le} \), \( K_{lle} \) are the equivalent elastic stress intensity factors. We notice that the intrinsic failure curve of the magnesium alloy shows a small influence of the \( K_{le} \) value in a large portion of the curve. This shape is inherent to anisotropic materials and has been observed by Jodin / 15 /.

266
### MAGNESIUM ALLOY

<table>
<thead>
<tr>
<th>No</th>
<th>ø</th>
<th>$\sigma_{tm}$</th>
<th>2a</th>
<th>2c</th>
<th>$f_I$</th>
<th>$f_{II}$</th>
<th>$K_{le}$</th>
<th>$K_{lle}$</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>MPA</td>
<td>mm</td>
<td>mm</td>
<td></td>
<td></td>
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<tr>
<td>1-00-G</td>
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<td>116.6</td>
<td>6.75</td>
<td>0.6</td>
<td>3,375</td>
<td>0</td>
<td>32.3</td>
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<td>6.75</td>
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<td>6</td>
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<td>3,042</td>
<td>0.5265</td>
<td>27.5</td>
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<td>0.6050</td>
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<td>146.4</td>
<td>3.75</td>
<td>0.5</td>
<td>0.4613</td>
<td>0.5125</td>
<td>4.1</td>
<td>4.6</td>
</tr>
</tbody>
</table>

**Table III**

### DISCUSSION

As mentioned previously, the fracture toughness data of magnesium alloys are very scarce. Hickerson / [4] / gave a value of $K_C = 48.6$ MPA m for GA321 alloy using a 4 mm CT sample, using $\sigma_{tm}$ 20 MPA m and dynamic value of $K_C = 18.50$ MPA m. Using a tube of 1 mm thick an apparent dynamic fracture toughness $K_e = 32.5$ MPA m is obtained. This value is in the same order of magnitude as $K_C$ or $K_e$ values in respect to the increasing loading rate and decreasing thickness.

These parameters affect the fracture toughness in an opposite manner: high loading rate decreases the fracture toughness, while decreasing thickness increases the fracture toughness.

The concept of intrinsic failure curve is a particular form of Mohr-Caquot criteria / [11] / [12] / [13]. A modification of this criteria using a description of normal and shearing stresses based on the values of the mode I and mode II stress intensity factors has been proposed by Mandel / [13] /.
A fractography investigation of the failure surface of magnesium alloy WLCT sample indicates that a typical fibrous fracture appears (Figure 8). For comparison an aluminium alloy intrinsic curve is shown in Figure 7. The aluminium alloy (cubic centered crystallographic system) is generally considered as an isotropic material, which is not the case of magnesium alloy (hexagonal crystallographic system). The mutual effect of approximatively orthogonal to the principal maximum stress direction. The geometrical influence including mode III friction effects gives an underestimation of $K_I$ values for a angle higher than 45°. This phenomena appears on the two intrinsic failure curves.

CONCLUSION

Dynamic fracture toughness of a magnesium alloy, in mixed mode of loading has been measured using a split Hopkinson pressure bar apparatus and thin tubes. The intrinsic failure curve presents a large portion which is independent of the mode I equivalent elastic stress intensity factor. This phenomenon can be explain by the orthotropy and the ductility of the material.

LIST OF SYMBOLS

- $\mu_{12}$ strain rate sensitivity coefficient
- $\sigma_x$, $\sigma_y$ shearing stresses in torsion test at constant shearing strain rates
- $\dot{\varepsilon}_1$, $\dot{\varepsilon}_2$ shearing strain rates
- $K$ stress intensity factor rate
- $\dot{e}$ strain rate
- $R_{e}$ yield stress
- $R_{m}$ ultimate stress
- $\theta$ crack angle with longitudinal direction of the tube
- $a$ half crack length
- $c$ crack depth
$F_C$  critical applied load (WLCT)

$P_C$  critical load (WLCT)

$\theta$  wedge angle

$\lambda$  friction coefficient

$C_L$  celerity of the wave

$T_r$  time to rupture

$K_C^*$  apparent fracture toughness

$K_{le}^C$  critical equivalent elastic stress intensity factor (mode I)

$\rho_o$  Irwin plasticity coefficient

$\nu$  poisson's coefficient

$K^{(m)}_I$  membrane, mode I stress intensity factor

$K^{(m)}_{II}$  membrane, mode II stress intensity factor

$K^{(b)}_I$  bending, mode I stress intensity factor

$K^{(m)}_{II}$  bending, mode II stress intensity factor

$G_m$  membrane stress

$C_{ml}$  non dimensional stress intensity factors (subscripts refers to membrane, I to mode I)

$C_{mlII}$  refers to membrane, to mode II

$C_{bl}$  refers to bending, to mode I

$C_{blII}$  refers to bending, to mode II

$f_{1,II}$  coefficient stress intensity factors correction coefficients

$C_e$  finite length correction
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Figure 1: Stress-strain deflection curves for dynamic and static loading rates.

Figure 2: Thin tube sample.

Figure 3: WLCT 20 SAMPLE.

Figure 4: Fractography of dynamically loaded Mg specimen (WLCT).

272
Figure 4: Split Hopkinson pressure bar

Figure 5: Pressure chamber
Figure 6: Typical examples of registered transmitted wave

a) the critical value of the pressure can be easily determined
b) Inertial effects lead to an average value

Figure 7: Intrinsic failure curve for Magnesium and aluminium alloys