SLOW THERMAL CRACK GROWTH IN THERMALLY LOADED TWO-PHASE COMPOSITE STRUCTURES CONTAINING INNER STRESS CONCENTRATORS

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Curved thermal cracks are considered running along special principal stress trajectories of self-stress fields existing in different shaped thermally loaded brittle two-phase solids containing inner stress concentrators. The resulting boundary-value problems of the plane thermoelasticity are solved by use of the finite element method. Further, stress distributions for uncracked and cracked circular two-phase solids have been calculated. Finally, a series of cooling experiments have been performed in order to study the influence of the location of inner stress concentrators (small circular holes) in self-stressed brittle bimaterial specimens (optical glasses) on the shape of the crack path of slowly propagating thermal cracks.

INTRODUCTION

Thermal fracture in multiphase materials represents a very important problem in the failure analysis of modern composite structures. Thereby very often the appearance of curved crack paths in thermally loaded composites has been observed. The reason for this phenomenon should be the non-uniform thermal stress fields existing in the corresponding compound materials. Besides, curved or kinked cracks were investigated in the past either as interface cracks along circular inclusions, England (1), Toya (2), Herrmann (3) or in connection with the assessment of existing crack propagation criteria, Cotterell and Rice (4), Bergkvist and Guex (5) and Nemat-Nasser (6). Finally, the crack path prediction of extending thermal cracks as functions of the geometrical configuration of a self-stressed solid as well as on the applied thermal load distribution has been studied by Grebner and Herrmann (7) and Herrmann and Grebner (8-9).

In this paper, curved thermal cracks are considered running along special principal stress trajectories of thermal stress fields existing in brittle two-phase solids with inner stress concentrators (small circular holes) due to applied well-defined temperature distributions. The two-phase composite structures consist of homogeneous, isotropic and linearly elastic materials with differing thermoelastic properties varying discontinuously at the straight interface Γ from the values E_I, ν_I, α_I of the region I to the values E_{II}, ν_{II}, α_{II} of the region II. Moreover, the conditions of perfect contact at the material interface Γ are assumed. Fig. 1 shows as an example, the circular cross section of an uncracked doubly connected compound cylinder which has been submitted to a constant temperature distribution $T = T_I = T_{II} \neq T_o$ where T_o represents the temperature of the unstressed

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initial state. The resulting thermal stress problem can be treated as a plane strain state by assuming temperature-independent thermoelastic properties. Thereby the material properties of the two composite structures BK4/BK12 and ZK5/BK7 (optical glasses) used in the numerical calculations are given in Table 1.

Table 1 - Material Properties of Different Optical Glasses.

Notation	BK4	BK12	ZK5	BK7
E	69474	84033	68180	80148
ν	0.210	0.206	0.241	0.211
α	$7.14 \cdot 10^{-6}$	$6.75 \cdot 10^{-6}$	$8.7 \cdot 10^{-6}$	$7.1 \cdot 10^{-6}$

Furthermore, the cracked two-phase composite structures, having for $t = 0$ the constant temperature $T = T_o$ of the unstressed initial state are subjected at $t > 0$ to a homogeneous temperature distribution causing the temperature $T = T_o + \Delta T$ with $\Delta T = 560$ deg C in the cross sections of those two-phase solids.

Formulation and Solution of Boundary-Value Problems

The evaluation of fracture mechanical data governing the quasistatic growth of a curved thermal crack has been performed by applying the concepts of linear elastic fracture mechanics. By using the basic equations of the continuum mechanics from reference (3) and by assuming the existence of a plane strain state a mixed boundary-value problem of the thermoelasticity has to be solved. By introducing Airy's stress function F this boundary-value problem reads:

$$\nabla^4 F_i(x,y) + \frac{E_i \alpha_i}{1-\nu_i} \nabla^2 T_i(x,y) = 0 \quad ; \quad (i = I, II) \tag{1}$$

with the following boundary conditions

$$\sigma_{ij} n_j = 0 \quad ; \quad (i,j = x,y) \tag{2}$$

where $\underline{n}$ means the unit normal vector with respect to the outer and inner boundaries S_o and S_i, respectively, and also to the two crack surfaces $S_c^+ \cup S_c^-$, respectively. Moreover, at the material interface Γ the continuity conditions

$$[\sigma_{xx}(x,y)]_{x=x_o} = [\sigma_{xy}(x,y)]_{x=x_o} = 0 \tag{3}$$

$$[u_x(x,y)]_{x=x_o} = [u_y(x,y)]_{x=x_o} = 0 \tag{4}$$

have to be fulfilled where x_o corresponds to the distance of the discontinuity area Γ from the center of the circular cross section of the two-phase composite structure. Besides, the jump relations at the interface Γ are defined as follows

$$[\lambda(x,y)]_{x=x_o} = \lambda^{I}(x_o,y) - \lambda^{II}(x_o,y) \tag{5}$$

with $\lambda = \sigma_{xx}, \sigma_{xy}, u_x, u_y$.

The boundary-value problems (1)-(4) for the two composite structures mentioned above have been solved under the assumption of plane strain conditions using the finite element method.

Fig. 2 shows the field of the principal stress trajectories of the associated uncracked two-phase solid (composite structure ZK5 (region II)/ BK7 (region I)) gained by a graphical integration procedure of the following ordinary differential equation

$$2\sigma_{xy} dy + \{(\sigma_{xx} - \sigma_{yy}) \mp \sqrt{(\sigma_{yy} - \sigma_{xx})^2 + 4\sigma_{xy}^2}\} dx = 0 \tag{6}$$

Thereby the stress components have been taken from the corresponding solution of the boundary-value problem for the self-stressed uncracked two-phase solid. Further, the geometrical parameters were chosen to $r_1 = 16.5$ mm, $r_2 = r_1/10$, $\phi_o = 70^o$ and the applied temperature distribution reads $\Delta T = T_I = T_{II} = 560$ deg C. Moreover, Fig. 2 shows the existence of two orthogonal sets of principal stress trajectories which embrace two singular points on the symmetry line of the cross section. Thereby the positions of these singular points of the plane self-stress field are given by the following relations valid in those points

$$\sigma_{xx} - \sigma_{yy} = 0 \quad , \quad \sigma_{xy} = 0 \tag{7}$$

Furthermore, for the cases $2\phi_o \neq \pi$ there exists only one principal stress trajectory which is running from one intersection point of the interface Γ with the external boundary S_o to the opposite intersection point. Besides, cooling experiments performed in our laboratory at Paderborn University with disk-like different shaped bimaterial specimens have shown that a curved thermal crack starts with relatively high initial velocity from one of the two intersection points and is running with decreasing velocity for the first time rather smooth to this special principal stress trajectory located entirely in one of both glass segments (for the cases $2\phi_o \neq \pi$ in the larger segment) to the opposite intersection point.

Fig. 3 - 5 show the results of such cooling experiments performed for two different bimaterial specimens (composite structure ZK5/BK7) with the thickness $d = 3$ mm. For a central hole with a diameter of $2r_2 = 2.5$ mm (cf. Fig. 3) the crack path deviates in the vicinity of this hole from the original principal stress trajectory of the associated uncracked specimen (cf. Fig. 2) due to the existence of this inner stress concentrator. Besides, Fig. 3 contains the temporal development of a slow thermal crack growth. Thereby the numbers between the markings along the crack path show the minutes needed for further crack extension. Furthermore, it can be seen that after a long time a secondary crack originates in the opposite intersection point of the interface Γ with the external surface S_o and is creeping with very low velocity into the larger segment of the two-phase solid up to its arresting point. Further, Fig. 4 shows the result of another cooling experiment where the path of the dominating curved thermal crack is not influenced by the inner stress concentrator due to its eccentric position with respect to the center of the bimaterial specimen. Finally, Fig. 5 gives the result of a cooling experiment performed for a two-phase solid with a hexagonal cross section.

Calculation of Stress Distributions and Fracture Mechanical Data

The numerical calculations concerning the stress distributions inside of a cracked thermally loaded bimaterial specimen as well as the strain energy release rates at the tip of a quasistatically extending curved thermal crack were carried out on the computer PRIME 550 at the University of Paderborn by the aid of the standard finite element program ASKA and by applying the substructure technique as well as by using triangular linear strain six-node elements. Fig. 6 shows a typical finite element mesh which consists of about 2800 nodal points, focused essentially in the neighborhood of the prospective crack path. Further, the figures 7 - 9 show the stress distributions for the stresses σ_{xx}, σ_{yy} and σ_{xy}, respectively, in the cross section of a cracked two-phase solid for a certain crack length. Moreover, stress distributions in dependence on crack length were calculated for the cracked composite structure BK4/BK12 too. But the results must be omitted here because of space limitations.

Furthermore, the evaluation of the strain energy release rates G_j ($j=I, II$) at the tip of a quasistatic extending curved thermal crack has been performed by using a modified crack closure integral, Rybicki and Kanninen (10). Thereby for small crack extensions $\Delta a \ll a$ the desired strain energy release rates G_j ($j=I, II$) are given by the formulae

$$G_I = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} F_{rc}(u_{rc} - u_{rd}) \tag{8}$$

$$G_{II} = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} F_{\theta c}(u_{\theta c} - u_{\theta d}) \tag{9}$$

Finally, Fig. 10 gives the strain energy release rates G_j ($j=I, II$) and $G = G_I + G_{II}$, respectively, in dependence on crack length a for the cracked composite structure ZK5/BK7. As can be seen from the graph the G_I-curve decreases monotonously with increasing crack length. But it should be noticed that valids: $G_I \to 0$ for $a \to 0$. Therefore, the G_I-curve should have a maximum value for a very small crack length, that means near the starting point of the thermal crack at the external surface S_o of the two-phase solid. Further, the G_{II}-curve shows that the thermal crack propagation in bonded brittle two-phase solids with inner stress concentrators takes place up to crack lengths of about 12 mm mostly under mode I-loading. Then the G_{II}-curve increases remarkably with increasing crack length. Therefore, the total strain energy release rate G increases again with increasing crack length. This result differs from investigations performed in reference (9) concerning the thermal crack propagation in self-stressed bimaterial specimens without inner stress concentrators. The reason should be the deviation of the crack path due to the existence of inner stress concentrators from the original principal stress trajectory belonging to the thermal stress field of the associated uncracked specimen.

CONCLUSION

Curved thermal crack propagation in two-phase solids with inner stress concentrators subjected to well-defined thermal stress fields has been investigated theoretically and experimentally. Cooling experiments performed with disk-like bimaterial specimens show a sufficient coincidence of the experimentally gained crack paths with special principal stress trajectories of the associated uncracked specimens. But there exists a remarkable influence of the location of inner stress concentrators (small circular holes)

on the shape of the crack path. Further, up to a certain crack length the thermal crack propagation takes place mostly under mode I-loading. The G_I-values decrease with increasing crack length whereas the G_{II}-values show the opposite behavior.

SYMBOLS USED

a	=	crack length (mm)
ν_i	=	Poisson's ratios
α_i	=	linear coefficients of thermal expansion ($10^{-6}\,K^{-1}$)
E_i	=	Young's moduli (Nmm^{-2})
σ_{ij}	=	components of the stress tensor (Nmm^{-2})
u_{rc}, u_{rd}	=	crack surface displacements (mm)
$F_{rc}, F_{\theta c}$	=	nodal point forces (Nmm^{-1})
G_i	=	strain energy release rates (Nmm^{-1})

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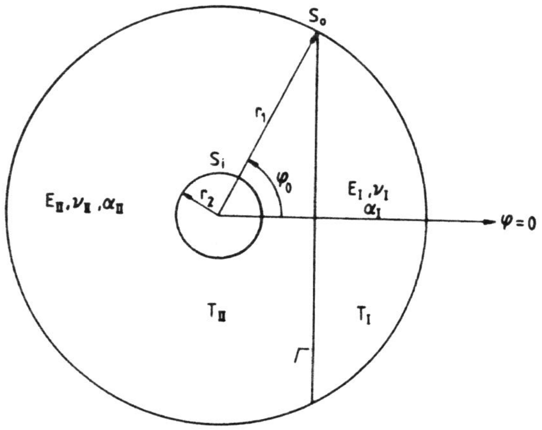

Fig. 1. Circular cross section of an uncracked doubly connected two-phase solid.

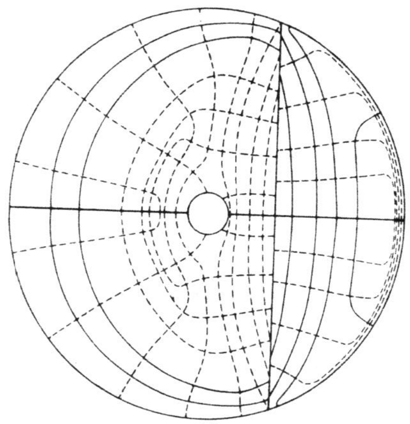

Fig. 2. Principal stress trajectories in the cross section of a thermally loaded uncracked two-phase composite structure (solid lines: tension stresses, dotted lines: pressure stresses).

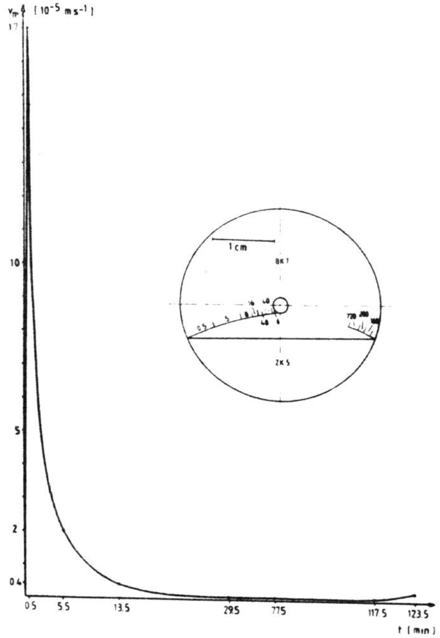

Fig. 3.

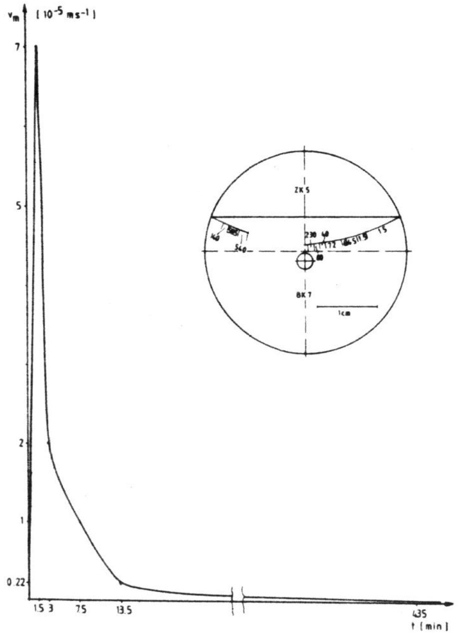

Fig. 4.

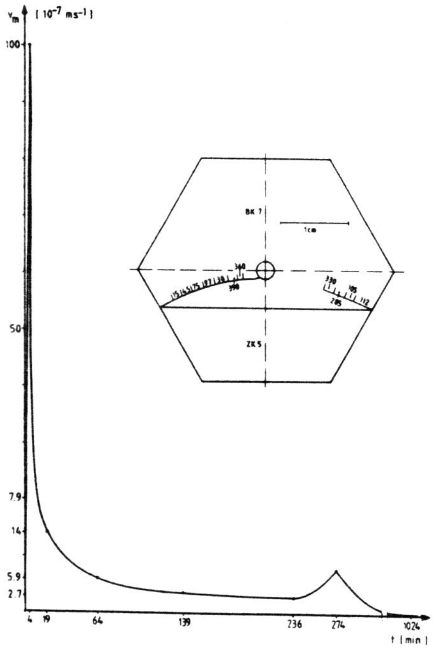

Fig. 5.

Fig. 3 - 5. Temporal development of slow thermal cracks in different shaped bimaterial specimens with inner stress concentrators obtained from cooling experiments.

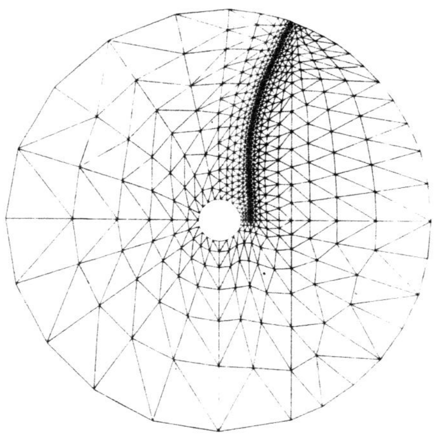

Fig. 6. Finite element discretization of a doubly connected two-phase solid with a circular cross section.

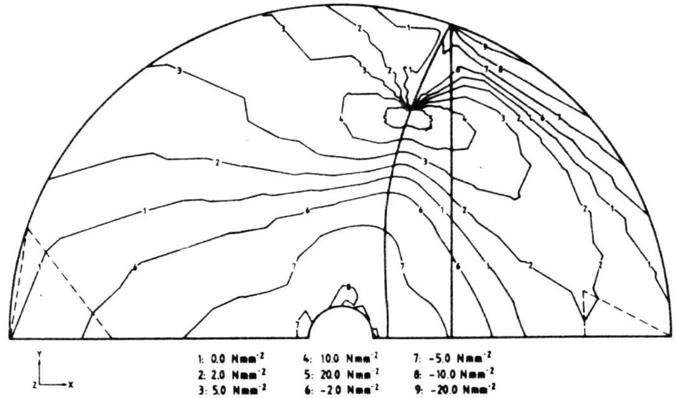

Fig. 7. σ_{xx}-stresses

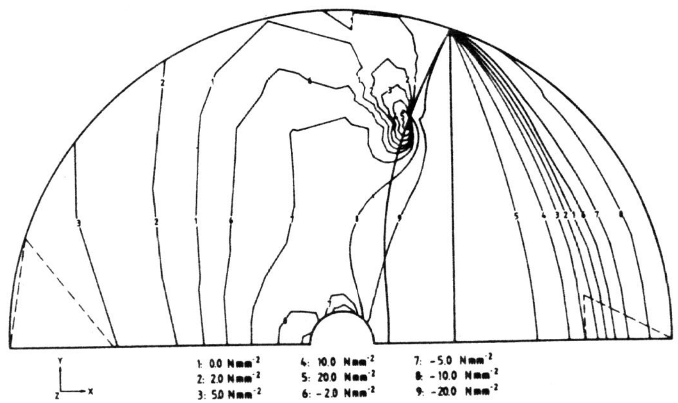

Fig. 8. σ_{yy}-stresses

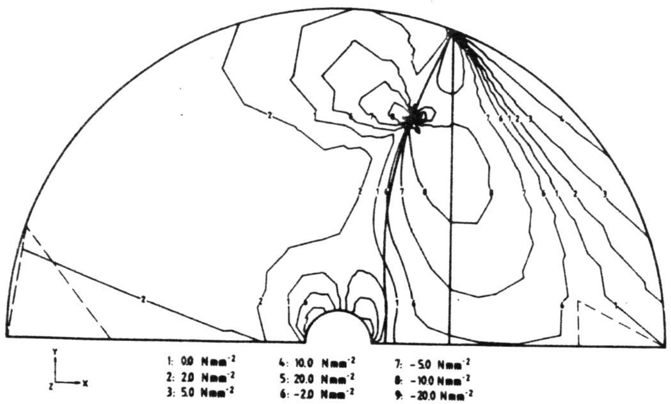

Fig. 9. σ_{xy}-stresses

Fig. 7 - 9. Equi-stress lines concerning the stresses σ_{xx}, σ_{yy} and σ_{xy}, respectively, in the cross section of a cracked two-phase composite structure (material combination ZK5/BK7).

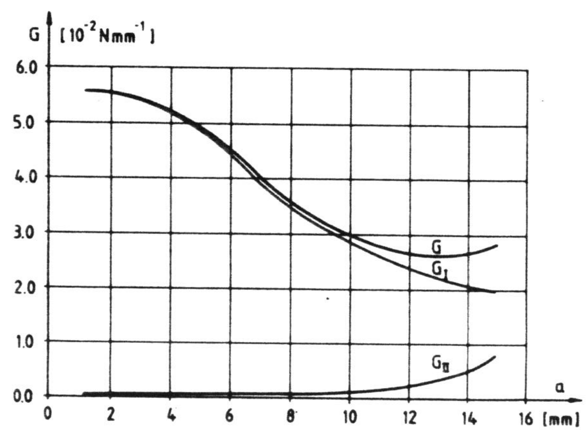

Fig. 10. Strain energy release rates as function of crack length a for a two-phase solid (composite structure ZK5/BK7).