FATIGUE LIFE PREDICTION: CALCULATION OR EXPERIMENT?

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The fatigue life of components and structures can be predicted by calculation or by experiment. The state of the art for both types of procedures is described and their problems, advantages and disadvantages are discussed.

Several hypotheses for the crack initiation as well as for the crack propagation period are specifically mentioned and discussed in more detail.

1. INTRODUCTION

The service fatigue life of a component or structure under random loading can be predicted by calculation or by experiment. Both procedures will be explained step by step in the following, because fatigue life prediction is quite a new subject for most industries, although it has quite a long history in, for example, the aircraft or automobile field.

For a fatigue prediction we need, on one hand, information on the service stresses that occur in a component, and on the other hand, we must know the stresses that the component can withstand. If the service stresses are of variable amplitude - as they certainly are in all vehicles, like cars, trains, planes, ships and in many structures, like offshore rigs (Fig. 1), and the stresses the component has withstood in test are of constant amplitude, we further need a damage accumulation hypothesis to bring the two sides together to obtain the required result, the predicted fatigue life. This, however, is a very much simplified presentation of a very complex subject.

2. FATIGUE LIFE PREDICTION BY CALCULATION

If the component can be tested in full scale, say an axle spindle of an automobile, the procedure is as shown in Figure 2:

The stress sequence must be measured in service. It then has to be evaluated statistically to obtain the stress spectrum. There are many different evaluation or counting methods available (1), resulting in different stress spectra as shown in Fig. 3, which represents the result of three different counting procedures of an aircraft stress sequence (1). While it is not at all clear which method best reproduces the various damaging portions of a stress sequence, it is clear, that the predicted

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fatigue life will be different for these three spectra, whichever damage accumulation hypothesis one chooses later on.

Because a small component with a simple shape is assumed in this example, the measured stress spectrum for nominal stress is now available. However, before its utilization in the following steps, it should be considered what has actually been measured:

- Is the measured spectrum representative of actual usage?
- What is the variability of the service stress spectra of nominally identical components?

Both questions actually refer to the most severe problem industry has to face in fatigue life prediction. An automobile maker, for example, does not know with any exactness how his products will be driven by customers. Some automobile makers (la) therefore try to define the so-called 1-percent driver, that is the most severe driver out of one hundred for whom the car must be designed fatigue-wise.

In the next step the measured stress spectrum must be "manipulated", because in practically all cases the measurement period is too short:

Assuming that it contains $10^6$ cycles, see Fig. 4, (a very large measuring and evaluation effort, corresponding to about 2,000 and 4,000 km for automobiles or about 10 weeks measurement time in an offshore oil drilling rig) this is still only about 1-percent of the required service life of 200,000 to 400,000 km for an automobile or 20 years for the oil rig, corresponding to about $10^7$ cycles in both cases. The obvious solution is to increase the number of the measured stress amplitudes of all sizes by a factor of 100.

The next question to be solved is what to do with those events with a probability of occurrence of $P < 10^{-9}$, which are not contained in the measured sequence.

There may be an upper physical limit which cannot be exceeded, for example when a helical suspension spring "goes to block". Assuming this happened once during the measurement period of $10^6$ cycles, this maximum amplitude occurs 100 times in $10^8$ cycles, but no higher amplitudes occur (dashed line in right half of Fig. 4). If there is no such limit the higher stress amplitudes will have to be added to the service stress spectrum. (The "truncation dilemma" mentioned in Fig. 4 will be treated in chapter 4). We now have arrived at the service stress spectrum (for nominal stress).

Instead of measurements in service quite often load assumptions have to be used, for example during the development phase, when the component in question is not yet available. These assumptions usually consist of stress or load spectra. Some fatigue standards also contain such stress spectra.

The other element of the procedure consists of fatigue tests under constant stress amplitudes (right side of Fig. 2). Quite a large number of such tests have to be carried out, especially near the fatigue limit, to define the S-N curve even if a probability of survival of 50 per cent is accurately enough.

The two elements are now brought together by the "damage accumulation hypothesis". Let us assume for the moment that an exact hypothesis were available: it would still only yield the mean life, that is the life for a probability of survival of 50 per cent, a life which is of no practical value whatever, because a much higher probability is required (between 90 and 99, 999, per cent, depending on the application). In order to calculate the allowable fatigue life for these higher probabilities, the scatter (standard deviation) of fatigue lives under service-like stress sequences must be known; that is we must calculate a safety factor by statistical means or we must define it from experience. We finally arrive at an allowable fatigue life, from which we can calculate the corresponding allowable stresses for the component.

If the fatigue life thus obtained is too short, the component's fatigue strength has to be improved, for example by better detail design, and the test procedure, shown in the right half of Fig. 2, has to be repeated.

Let us skip for the moment the problem of the damage accumulation hypothesis and continue to the fatigue life prediction of a large structure which cannot be tested full scale, say an offshore structure or ship. Here the procedure is still more complex, see Fig. 5, as several additional parameters have to be accounted for. The most important one probably is the local stress at the point of failure. In offshore structures this is called the "hot spot stress". As the structure cannot be tested full scale, at best models can be used. It is obviously better if these models (right side of Fig. 5) can be tested the S-N-curves for the local stress or hot spot stress (right side of Fig. 5) and compare them through the damage accumulation hypothesis with the hot spot stress spectrum of the structure (left side of Fig. 5).

Even then, it must be assumed that equal hot spot stresses of model and full scale structure result in equal fatigue lives; besides the problem of calculating these hot spot stresses, this assumption means that not only the local stresses due to the external loads are identical but also the allowable stresses for the relatively small, thin wall model and the large, thick wall structure are equal. This assumption might be especially wrong for the crack propagation phase and or for welded structures.

To prove or disprove it, it is therefore necessary to fatigue test at least a few big, almost full scale models and - by comparison with smaller models - determine what "size effect" must be accounted for. If the basic data are acquired from specimens (middle column of Fig. 5), the actual wall thickness of a welded structure (containing the identical number of the weld passes) can sometimes be simulated, but not the extremely high stress concentration factors of, for example, tubular joints or ship super structures and most probably not the magnitude of the residual tensile welding stresses present in the actual structure. So here again assumptions have to be made, namely: equal nominal-stresses in the (low stress concentration) specimens and the hot spot stress in the (high stress concentration) structure result in equal lives and the differences in residual welding stresses do not influence the fatigue life either. If they do, they must be accounted for.
A further factor in some structures and vehicles is corrosion. Although many corrosion fatigue tests have been carried out, for example in the offshore steel program of the European Community of Coal and Steel (2), the time in the environment can obviously not be simulated correctly in test.

Finally, so-called design SN-curves shown in certain standards, like the AWS-x-curve for offshore structures or the British standard 5400 for bridges can be used. Some of these curves are defined for hot spot stresses, others for nominal stresses and for probabilities of survival much higher than 50%, so no additional safety factors are necessary (extreme right of Fig. 5).

What constitutes failure is a further question with models of structures: First crack found by non-destructive testing or through-crack or complete failure?

3. DAMAGE ACCUMULATION HYPOTHESIS

3.1 General Remarks

The problem of the damage accumulation hypothesis will now be discussed:

Any current and foreseeable fatigue life prediction method must make an assumption on the accumulation of damage caused by the cyclic amplitudes of the stress sequence. In spite of decades of research on this subject, "damage" cannot yet be correctly described in the quantitative way necessary for a fatigue life prediction method by calculation. Even the modern fatigue life prediction methods for crack initiation and for crack propagation therefore assume linear accumulation of damage, although this is known to be incorrect. Even the length of a fatigue crack is not a measure of the damage accumulated: Depending on the previous stress history, the residual lives of two identical specimens or components with identical crack lengths can differ by at least one order of magnitude, that is, both have suffered widely different damage. The many unsolved problems of damage and its accumulation can probably best be understood by describing the macroscopic events occurring in the most highly stressed volume of material around the crack tip, when a high tensile stress peak occurs during an irregular stress sequence:

- The crack length increases, the stress intensity therefore also grows, and the crack rate after the stress peak will be higher.

- By plastic deformation of the material at the crack tip a plastic zone is produced, whose size depends on the maximum tensile stress of the cycle, on the crack length, on the momentary yield strength of the material at the crack tip, which is most certainly not the original yield strength, and on the multiaxial stress state.

- The plastic deformation around the crack tip also introduces residual compressive stresses which result in lower effective mean stresses and, therefore, in a reduced crack propagation rate of the following cycles.

- At the same time the crack geometry changes: The crack tip is blunted and therefore the crack propagation rate decreases.

- The crack also closes when the loads are still tensile (crack closure) (3); succeeding smaller cycles therefore need a portion of their amplitude before the crack opens again; the effective stress intensity amplitude therefore has decreased, as has the crack growth rate.

- The plastic deformation of the material at the crack tip has strain-hardened or softened the material. This has altered important material properties and the crack growth rate may henceforth increase or decrease.

Thus all parameters influencing crack propagation have been altered just one high stress cycle and all the above events would quantitatively have to be accounted for in their synergistic effects on crack propagation in a scientific solution of damage accumulation.

To make matters worse the above example is still very simple: In a service-like stress sequence unknown effects caused by previous peak stresses will still be present when a new stress peak occurs.

One look at Figures 2 and 5 tells us, furthermore, that the hypothesis utilised is only one of several problems of fatigue life prediction. Even if damage accumulation were solved, all the other problems shown would remain and the end result, the predicted fatigue life in service, would probably not be much improved. Some of the above problems for example the variability of the service stress spectra, elude an scientific solution and may lead to much larger errors in the predicted life.

It is therefore not surprising that even in the aircraft field, where fatigue life prediction is extremely important, the simplest linear methods (Miner's rule or, for crack propagation, Forman's equation) are widely used.

That is to say: Scientific solutions may not be available yet, but engineering approximations are available. All of them fall into one of the following groups:

- Simple methods which do not account for any of the complex events taking place in the most highly stressed area around the notch root or crack tip. The assumption is that the various factors accelerating or decelerating damage accumulation cancel each other. Miner's rule for the crack initiation period (4) and a simple linear addition of the crack extension due to each individual cycle using Forman's (5) or Paris' (6) equations for the crack propagation period are examples.

- Methods, which do account for some of the complex events taking place at the notch root or crack tip, using assumptions, for example about the size and shape of the plastic zone and its effect on fatigue life or crack propagation, for example the notch root hypotheses (7-13) for the crack initiation phase and Willenborg's model (14) for the crack propagation phase.
- Methods which try to measure what actually happens at the notch root or crack tip, for example the crack closure concept (3) for crack propagation.
- Methods having a restricted general applicability using variable amplitude test data and assumptions on how to read across from these data to the actual load sequence. The so-called "relative" Miner (15) rule is an example for the crack initiation period, while for the crack propagation period there is the Wheeler model (16).

3.2 Miner's Rule

Miner's rule (4) states that the damage caused by a cycle in a variable amplitude sequence is equal to that of a cycle of the same size under constant amplitudes. Failure occurs as soon as the sum of the damages caused by the individual cycles in the variable amplitude sequence has reached unity

$$D = \sum \frac{n_i}{N_i} = 1.0$$

(1)

In the original paper Miner states many restrictions, for example:
- all cycles must be above the fatigue limit
- valid only for aluminum alloys
- gives life to crack initiation only, and
- valid only for \( R = -1 \)

which would have severely restricted the applicability of his rule - had they not been ignored over the years by the users. It has also been employed indiscriminately for predicting the life to crack initiation or for the complete fatigue life (crack initiation plus propagation). Although damage sums at failure of 0.01 as well as 10 have been recorded in special, but realistic, cases, for welded structures quite often the predictions were not very far off (for example between 0.5 and 2.0).

3.3 The "Relative" Miner Rule

If the results of realistic, that is variable amplitude tests are used as basic data, an improved prediction may be possible. If the test spectrum is identical with the one for which the fatigue life prediction is required, no hypothesis is needed. Usually however, there will be a difference in spectrum shape and some sort of damage accumulation hypothesis is still required.

Employing Miner's rule only as a "transfer function", the life under the different spectrum can be calculated.

- It is not necessary in this case that the damage sum at failure be unity, as postulated by Miner, but only that the damage sum at failure be the same for the different spectra.

This is the basic idea of the relative Miner rule (15):

$$N_A = \frac{N_B}{N_A} \frac{(\Sigma n_i/N_i)_B}{(\Sigma n_i/N_i)_A}$$

(2)

where \( N_A \) is the fatigue life to be predicted for spectrum A; \( N_B \), the fatigue life in test under similar spectrum B; \( (\Sigma n_i/N_i)_B \), the damage sum in test with spectrum B and \( (\Sigma n_i/N_i)_A \) the damage sum under similar spectrum A.

As long as the spectra are similar this hypothesis certainly holds true, the problem boils down to the question: What, in this respect, is a similar spectrum?

The considerable number of checks on the accuracy of this method have in most cases shown an improvement compared to Miner's rule (15, 17 - 23).

3.4 Paris' and Forman's Equations

From 1960 onward many crack propagation models appeared in the literature, mostly based on fracture mechanics principles. The first still best known was published by Paris and coworkers. Paris (16) postulated that the cyclic change in the stress field surrounding the crack tip, that is the range of the stress intensity factor \( K \), determined crack propagation according to the formula

$$\frac{da}{dN} = \frac{C}{(1-R)} \Delta K^n$$

(3)

This formula is still being used in design, in spite of its obvious shortcomings: It accounts neither for mean stress effects, nor for the threshold of crack propagation at \( \Delta K_{th} \), nor for static failure when \( K_c \) is reached. In 1967 Forman and coworkers (5) published an improved Paris equation

$$\frac{da}{dN} = \frac{C}{(1-R)} \frac{\Delta K^n}{K_c - \Delta K}$$

(4)

in which at least mean stress effects and \( K_c \) were incorporated. This formula has been proved by many laboratories, also in France by the CEAT in Toulouse, to give a reasonable approximation to crack propagation test results for many different materials.

The above equation still does not account for \( \Delta K_{th} \) and therefore implies fatigue crack propagation even at infinitely small \( \Delta K \). Kiesmuller and Lukas (24) modified the original Paris equation in the following way

$$\frac{da}{dN} = C \cdot (\Delta K^n - \Delta K_{ch}^n)$$

(5)
Finally the author and coworkers (25) incorporated this proposal into the Forman equation

$$\frac{da}{dN} = \frac{C \cdot (\Delta K^n - \Delta K_{th}^n)}{(1 - R)K_e - \Delta K} \quad (6)$$

and found it to give a good fit to the experimental data available. However, the numerical values for $\Delta K_{th}$ must be determined by test.

Other formulas for calculating crack propagation have appeared in large numbers; none, has, however, been so thoroughly investigated as the Forman equation and judged against the criterion - how well did it predict crack propagation under different stress amplitudes and mean stresses.

The above discussion dealt exclusively with crack propagation under constant stress amplitudes. The Paris or Forman equations can also be used for variable stress amplitudes by calculating the crack propagation per individual cycle, neglecting interaction effects. This method has also been called a Miner-approach because the damage increments in the form of crack growth increments are linearly added.

Checks on the accuracy of Forman's equation under variable amplitudes usually gave conservative predictions. It is also much simpler than more modern methods accounting for interaction effects without giving more inaccurate results. This might be especially true for stress sequences with irregularity factors $I = 0.99$. As long as no mean stress effects exist, and for the "middle" region of the crack propagation curve, even Paris' equation might suffice.

### 1.5 The RMS-method

The basic idea of this method is to find a constant stress amplitude $\sigma_{eq}$, or for crack propagation, a stress intensity factor range $\Delta K_{eq}$ which is equivalent (with regard to fatigue life or crack propagation) to the variable stress amplitudes.

The root mean square (r.m.s.) of the variable sequence was thought to be that equivalent by some researchers. It is clear that no interaction effect can be accounted for in this way. For example rare high stress peaks will not influence the r.m.s.-value appreciably, but they do influence fatigue life or crack growth.

In effect the r.m.s.-approach presupposes that the Miner rule is valid and that in addition the corresponding S-N-curve has the (very steep) slope of two, especially the latter assumption is demonstrably wrong. This leads to the following conclusion: In those few cases where the rms-method gave good results, the errors in both above assumptions cancelled each other.

4. **FATIGUE LIFE PREDICTION BY EXPERIMENT**

At first sight this seems quite simple: Duplicate what happens in service.

According to the little available evidence (36, 27) the fatigue life in test is indeed identical, in a statistical sense, to the life in service, if the stress sequence and the components or specimens are identical in both cases. Such tests are usually called service-duplication tests; they have, however, problems of their own and are not considered to be the answer to fatigue life prediction by experiment. Rather, the stresses in service are simulated, and the corresponding tests are called service simulation tests.

The corresponding procedure is shown overall in Fig. 6. As before, a stress sequence must be measured in service, which is statistically evaluated to obtain the measured stress spectrum.

The questions of "variability" and "manipulation" can be solved as before to arrive at the service stress spectrum. From then on, however, the procedure is different and the many engineering decisions necessary before a useful fatigue test under variable amplitudes can be carried out are explained in detail in the following:

Large but infrequent stress amplitudes may actually prolong fatigue life due to the beneficial residual stresses they cause. Thus, if the test is carried out with too high infrequent stress amplitudes the fatigue life prediction by experiment will most probably be unconservative, at least for those structures which do not see the high stress amplitudes in service. So the correct choice of the maximum stress amplitude to be applied in test, the so-called "truncation dilemma" (28), is an important decision. Some experts (29) have suggested that the maximum stress amplitudes in the test spectrum should occur not less than 10 times.

Longlife structures, like offshore structures, ships, trucks, automobiles etc. see about $10^6$ cycles during their required life; that is the service stress spectrum contains $10^6$ cycles, too many for an economically feasible fatigue test because at 10 Hz this means 100 days testing time. So the next question ("manipulation II" in Fig. 6) is how best to simulate this large number of cycles. $10^7$ test cycles results in a reasonable test time (10 days at 10 Hz), Fig. 7 shows schematically some of the solutions utilised in various industries. It is easily seen that the best compromise to reach the goal of $10^7$ test cycles is omitting cycles (option 2) because this shortens testing time without an increase of the maximum stresses. Unrealistically high residual stresses and their possible effect on fatigue life in test are thereby avoided. In a typical straight-line spectrum, this reduction of the number of cycles of one order of magnitude means that all stress amplitudes lower than about 15% of the maximum amplitude are omitted; they are far below the fatigue limit.

If the number of test cycles has to reduced still further, for example if a low test frequency is thought to be necessary, as in some corrosion fatigue tests, further omission may run into the problem of the "omission dilemma": the stress amplitudes left out may be near or even above the fatigue limit and the resulting fatigue life in test will be different.
"Scaling up" of all stress amplitudes, a further option, applies higher stress amplitudes than occur in service, with the attendant problems mentioned above.

Using a more severe test spectrum than occurs in service is typically the option the automobile makers utilize when they test drive cars on race tracks (301) in order to shorten development time.10,000 km on a new race track at high speed by test drivers is supposed to be equivalent to more than 500,000 kilometers as driven by the normal customer. In this case, neither the shape of the test spectrum nor its maximum stress amplitude agree with those of the service stress spectrum (option 4 in Fig. 7) and we have compounded the problems of too high stresses and of reading across from one spectrum to another.

Most service simulation tests have a fixed sequence which is repeated after a certain number of cycles. The length of this so-called return period is critical: On one hand it has to be repeated at least several times; otherwise the various stress amplitudes do not occur in their correct percentages. On the other hand a too short return period means that high infrequent stress amplitudes are not contained in the test sequence, while they do occur, if rarely, in service and might affect fatigue life. Thus the load spectrum applied in test is quite different from that in service. The effect is shown in Fig. 8, left side. Again assuming the service stress spectrum of $10^8$ cycles, a return period of, say $10^8$ cycles has to be repeated $10^4$ times and the test spectrum will be applied in which all stress amplitudes above 50 per cent of the maximum stress occurring in service have been truncated and the test will most certainly not give the correct answer.

A similar effect is caused, if unwittingly, by using random generators with too low crest factors for controlling the servohydraulic test machine (see right side of Fig. 8). So the selection of the correct return period size is an important question.

If all the above problems have been solved successfully, then, and only then, the irregularity of the stress sequence in test must be decided; however, according to a lot of evidence (31 - 35) for many metallic materials, this is of secondary importance and will not influence the life obtained to any degree, at least not between inequality factors of $i = 0.99$ ("narrow band") and 0.7; even at $i = 0.3$ ("very broad band") the life obtained was not very much different, if the number of zero crossings were compared. So the choice of the irregularity factor for the subsequent test is not very critical.

We now have arrived at the final step: The synthesis of the sequence to be applied in test from the test spectrum decided upon. The Markovian matrix (36) is a very useful tool for this, which will, however, not be discussed in any detail. Some decisions still have to be taken, for example: When should the maximum stress amplitude occur in the sequence? In the standardized Gaussian load sequence developed by LBF and IABG (37), they are applied at the middle of the return period of $10^8$ cycles, that is after about $5 \times 10^7$ cycles.

After all the steps described above have been taken the fatigue test can finally be carried out (Fig. 6). As with fatigue life prediction by calculation, sometimes load assumptions have to be used. However, if the stresses in service cannot be measured, the relevant standardized load sequence, for tactical aircraft for example the "Falstaff"-sequence (38) may also be used.

If the service stress spectrum of the component whose life is to be predicted is different from the test spectrum, for example if assumptions had been used for the test spectrum and later shown to be incorrect by measurements in service, an additional fatigue life prediction by calculation must be carried out in order to read across from the test spectrum to the service spectrum. This is usually done employing one of the relative damage accumulation hypotheses, see Fig. 6. As before, the allowable fatigue life for high probabilities of survival is determined using a statistically derived scatter factor, which can be taken directly from the tests, as they are of variable and not of constant amplitude. This is one advantage of this type of fatigue life prediction.

Obviously the case where we can test the component full scale is shown in Fig. 6. For a large structure all the different parameters like hot spot stress, size effect, corrosion etc., as mentioned for fatigue life prediction by calculation, would also have to be accounted for here.

REFERENCES


Fig 1: Typical Load Sequences (Schematic)

Fig 2: Fatigue life prediction of a small component by calculation
Fig 4: Manipulation of the measured stress spectrum

![Graph showing manipulation of stress spectrum](image)

Stressamplitude $a$

\[ \log N \]

Number of Cycles due to Van Dijk, due to different counting procedures.

\[ 10^6 \]

\[ 1 \]

\[ 10^2 \]

\[ 10^4 \]

\[ 10^6 \]

Stress sequence measured in service

- Physical limit?
- Manipulation?

Stress sequence obtained by three different counting procedures.

- Higher peak stresses (truncation dilemma)
- Number of cycles for 20 years expected in service.

Fig 5: Fatigue life prediction of a large structure by calculation

![Diagram showing fatigue life prediction](image)

Stress sequence measured in service

- Load assumptions
- Relevant standardized load sequence

Statistical evaluation

- Measured stress spectrum
- Service stress spectrum
- Test spectrum

Fatigue test under realistic sequence

- Predicted fatigue life $P_e \geq 50\%$
- Safety factor from the test themselves

Allowable fatigue life $P_e \geq 50\%$

Fig 6: Fatigue life prediction of a small component by experiment

![Diagram showing fatigue life prediction](image)