A short review of the concept of ASPEF (absorbed specific energy till fracture) and its relation to other fracture properties is given. A model is elaborated for fatigue crack initiation and the process is related to monotonic loading based on energetical considerations. Different structural steels have been fatigued in the low cycle range and the results were used to find the expected correlations. For the present a totally analytical procedure seems to be questionable, and an experimental support is recommended.

INTRODUCTION

In the last decades of this century the interest of many engineers and scientists has been focused on the problem of fracture. Most of them dealt with the brittle kind of fracture and the validity of the developed theory - the so called fracture mechanics - has been shifted only later towards more plastic materials. L. Gillemot [1] on the contrary introduced a concept - the absorbed specific energy till fracture (ASPEF) - to characterize plastic fracture. This quantity has been mentioned by some other scientists too [2,3], but a systematic research to investigate the specific features of it and the relations with other material characteristics has been performed at first at the Technical University of Budapest. The results were summarized recently by Czoboly et al. [4].

ASPEF is a specific energy per volume concerning an infinitely small element of the specimen, where the crack nucleates after a given amount of plastic deformation. Since energy criteria became general also in fracture mechanics, it was evident that the two fracture theories - although based on different principles - can be related to each other. This was done by Radon and Czoboly [5], who calculated the usual fracture mechanics parameters by using ASPEF of the material and a characteristic dimension of the plastic zone ahead of the crack. The method has been improved and extended.
by Havas et al. [8] and its validity verified by extensive

tests. [7-9]

F.Gillemot [10] found a relation between fatigue crack

propagation data and ASPEP in the case of constant strain

amplitude loading. Similarly, a strong correlation has been
demonstrated between ASPEP and the material constants of
the Paris equation by Romvári and Tóth [11]. It seems therefore that
- although after simplifications - a quantitative relationship can
be found among the different types of fractures, which can
help to solve engineering problems.

Less attention has been devoted to the question of
fatigue crack nucleation in respect of ASPEP, however, Halford
[12] presented a compilation of a great amount of experimental
data and Havas [13] examined the possibility of any corre-
lation between ASPEP and fatigue life in a former work. Aim of
the present paper is a review of the former results and a comple-
tion of them with new data to find relations suitable
for numerical calculations.

THE APPLIED MODEL

Fatigue cracks generally nucleate at some kind of geometrical
or metallurgical stress- and strain concentrator. This critical
part of the specimen or structure can be modeled by the tip
of a notch (Fig.1.). The local stress here exceeds very often
the yield stress of the material and a loading cycle produces
local plastic strain. A fatigue crack will nucleate at this
point, if the plasticity of the material will be exhausted.

Across the plastic zone a small specimen suitable for
studying the exhaustion of plasticity can be imagined. Such
modeling experiments are well known as "low cycle fatigue
tests", where alternating plastic strain is applied. A typical
hysteresis loop can be seen on Fig.2., representative for the
loading cycle and the meaning of the main parameters of the
test as stress and strain amplitude, strain and stress range,
total-, plastic- and elastic strain can be recognised. The area
of the loop represents a given amount of energy accomplished
by the loading force. This "input energy" increases with in-
creasing strain amplitude.

It is supposed, furthermore, that some part of the "input
energy" absorbed in the specimen will damage the micro
structure resulting finally a fatigue failure. This consider-
ation is very similar to the philosophy of ASPEP. Here too
the "input energy" is regarded as a characteristic quantity
concerned to a unit volume. If ASPEP has to be measured by a

tensile test, the energy absorbed in the necking part of
the specimen is of interest. In the close neighbourhood of the
least cross section the absorbed specific energy during the
test is equal to

\[ \varepsilon_f = \int \sigma \, d\varepsilon \]

(1)

where \( \sigma \) and \( \varepsilon \) are true stress and true strain, resp. and
\( \varepsilon_f \) is fracture strain.

It is obvious that only one fragment of the absorbed
energy is used for damaging processes, while the greater part
is dissipated as heat. However, the total absorbed energy, \( \varepsilon_f \)
has proved to be a material characteristic [14].

Taking into account that a tensile test can be regarded
as a low cycle fatigue test with a very high strain range,
and a very short fatigue life, \( \varepsilon_f = 0,5 \) a relation between
the input energies in the case of monotonic and cyclic load-
ing is to be expected.

TESTING METHODS AND MATERIALS

A great variety of structural steels have been involved in the
experiments, which are listed in Table 1. Some of the materials

<table>
<thead>
<tr>
<th>Original notation</th>
<th>Corresponding DIN Mat. No.</th>
<th>Test temp. °C</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 10</td>
<td>1.1121</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>C 35</td>
<td>1.1181</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>CrMo 125</td>
<td>1.7218</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>11523</td>
<td>1.0562</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>12HNMF (charge 1)</td>
<td>-</td>
<td>20</td>
<td>In service for 500000 hours</td>
</tr>
<tr>
<td>(charge 2)</td>
<td>-</td>
<td>20</td>
<td>In service for 1000000 hours</td>
</tr>
<tr>
<td>(charge 3)</td>
<td>-</td>
<td>540</td>
<td></td>
</tr>
<tr>
<td>(charge 4)</td>
<td>-</td>
<td>20</td>
<td>Tempered</td>
</tr>
<tr>
<td>13CrMo 44</td>
<td>1.7335</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>(charge 1)</td>
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<td></td>
</tr>
<tr>
<td>(charge 2)</td>
<td>-</td>
<td>350</td>
<td>In service for 1000000 hours</td>
</tr>
</tbody>
</table>

Table 1. The list of materials tested.

originated in structures (pressure vessels, power station drums,
etc.) used for many years, therefore the given data should be
considered only as specific values of the charge investigated.
The purpose of including these data in this compilation was

to increase the range of validity of the correlations, if there
are any.

Tensile- and low cycle fatigue specimens were machined
with a diameter of 8 and 6 mm, resp. The tests were performed
at 20, 350, 500 and 540 °C, depending on the service temperature
of the given structure. Both to the tensile as well to the
low cycle fatigue tests the same electro-mechanical machine of
100 kN capacity was used. The cross head speed varied
between 0.6 and 2 mm/min, but it was kept constant for each
material.

Low cycle fatigue tests were carried out at constant
strain amplitude. The hysteresis loops have been plotted periodically to monitor cyclic softening or hardening. Although this technique provides the possibility to observe macro crack initiation, the tests were continued until complete failure.

The number of cycles to crack initiation were not registered. Only the total fatigue lives are given. However, it may be argued that this procedure has many advantages, although they are not discussed here for shortness.

Tensile properties, as yield stress $R_y$, ultimate tensile stress $R_m$, true fracture stress $R_f$, and true fracture strain $\varepsilon_f$ have been determined and the values of ASPF and have been calculated. Instead of the more exact equation given in Ref. [1], a simplified formula was used:

$$W_C = \frac{R_y + R_f}{2} \varepsilon_f$$

which approximates the real value within ±10 percent.

The low cycle fatigue tests furnish the data to construct the Manson-Coffin diagram and to determine the exponent, $m$ of the equation 3.

$$\Delta \varepsilon_f \cdot N_m = C_f$$

According to the considerations above, $\Delta \varepsilon_f$ is equal to $\varepsilon_f$, if $N$ is equal to 0.3. Therefore

$$C_f = \frac{\varepsilon_f}{m}$$

Furthermore, the input energy of an average load cycle was determined. That means that - the area of the loops was measured and the energy was calculated by equation 5.

$$\Delta E = \frac{\delta F}{\pi \delta d} \int F \cdot d$$

$F$ is load $d$ is the varying diameter and $d_0$ is the original diameter of the specimen. This equation is an approximation and is in our case less than 3 percent.

Using the input energy of the half life cycle, $E_f$, the total input energy was calculated as a product of $\Delta E$ and the number of cycles till fracture

$$E_f = N_f \Delta E$$

Plotting the $E_f$ values in a log-log scale as a function of the fatigue life, a linear relation is found [12, 13, 15]. Accordingly

$$E_f N_f^m = C_2$$

where $n$ is a material constant and

$$C_2 = \frac{W_C}{2^n}$$

because $E_f = W_C$, if $N_f = 0.5$.

TEST RESULTS

Some typical examples of the results are illustrated on Figs. 3 and 4. It can be seen that for the materials tested - also not shown on the Figure - the measured values fit very well on the lines according to equations 3 and 7. The values of the exponents $m$ and $n$ are summarized in Table 2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Temp.</th>
<th>$C_f$</th>
<th>$n$</th>
<th>$m$</th>
<th>$n+1$</th>
<th>$m$</th>
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</thead>
<tbody>
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<td>C 10</td>
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<td>-36</td>
<td>1.49</td>
<td>1.19</td>
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<tr>
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<td>0.460</td>
<td>-19</td>
<td>1.18</td>
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<tr>
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<td>-37</td>
<td>1.61</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>12H1MP (charge 1)</td>
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<td>0.650</td>
<td>-22</td>
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<tr>
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<tr>
<td>12H1MP (charge 3)</td>
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<td>1.11</td>
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</tr>
<tr>
<td>12H1MP (charge 4)</td>
<td>20</td>
<td>0.670</td>
<td>-29</td>
<td>2.33</td>
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<td>13 Cr Mo 44</td>
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<td>1.87</td>
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<td>500</td>
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<td>BHW 38 (charge 1)</td>
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<td>0.709</td>
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<td>0.790</td>
<td>-31</td>
<td>2.55</td>
<td>0.87</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. The values of the exponents $m$ and $n$ and their combinations

It is worth mentioning that although the values of $m$ and $n$ are near to 0.5 and -0.33 resp., as it is given in the literature, their deviations of these average values has a strong influence on the followings.

DISCUSSION

As mentioned earlier, one - unknown - part of the input energy damages the structure of the specimen during the test and causes fracture. This damaging part includes the
energy to increase dislocation density, number of vacancies, porosity, micro cracks, etc. and also the energy to raise the local temperature. The other part of the input energy is dissipated by heat transfer and will not play a role in the fracture process.

In the case of static tests the ratio of the damaging part and the total input energy has been proved to be constant and therefore the total input energy (ASPEF) can be used as a convenient specific value of the material in engineering.

On the supposition that a given material will fail after absorbing a characteristic quantity of energy depending only on the circumstances of loading, as temperature and loading rate, but not on the kind of loading, a constant ratio of damaging and total input energies was expected in the case of cyclic loading too. If so, then $E_f$ should be equal to $W_r$ and should be independent of the strain amplitude. The positive slope of the $E_f/N_f$ curve shows, however, that this is not true, but the ratio decreases with decreasing strain amplitude, or with other words: with decreasing strain amplitude the total input energy increases.

The rate of increment can be calculated by using equations 3 and 7 as follows:

$$\frac{\Delta E_p}{\varepsilon_f} = \left(\frac{W_c}{E_f}\right) \frac{m}{n}$$  \hspace{1cm} (9)

In Table 2 also the exponent $-(m/n)$ is given and it can be noticed that this value varies over a wide range. Any energetic calculations can be misleading, if average values for $m$ and $n$ are used, that is 2/3 for $-(m/n)$, while calculations with the actual $m$ and $n$ exponents provide satisfactory results. (See Fig.3.) Considering the great scatter of individual fatigue data, the points fit well on the theoretical 45° line.

The input energy in one load cycle can be expressed too by the help of equations 3, 4, 6 and 9.

$$\Delta E = 2 W_c \left(\frac{\Delta E_p}{\varepsilon_f}\right)^{\frac{n+1}{m}}$$  \hspace{1cm} (10)

Here again, application of average values for $m$ and $n$ recommended in the literature can be misleading. However, the variation of exponent $(n+1)/m$ is not so great as that of $-(m/n)$ (See Table 2).

Finally, the variation of the damaging part of the input energy can be assessed. As mentioned earlier, we suppose that only one part of the input energy is responsible for the damaging process. This should be denoted by $W_d$, where

$$W_d = C_3 \cdot W_c$$  \hspace{1cm} (11)

$C_3$ is less but unknown. Assuming a uniform energy absorption during the fatigue life, as a first approximation, $\Delta W_d$ energy is used for the damaging process in one cycle.

$$\Delta W_d = \frac{C_3 \cdot W_c}{N_f}$$  \hspace{1cm} (12)

Using equations 3 and 4, it follows

$$\frac{\Delta W_d}{W_d} = 2 \left(\frac{\Delta E_p}{\varepsilon_f}\right)^{\frac{1}{m}}$$  \hspace{1cm} (13)

CONCLUSIONS

The energy concept provides the possibility to relate fatigue crack initiation to static material properties like ASPEF. For numerical calculations, however, the constants of the material in question should be used, since average values recommended in the literature may be misleading. Determination of the constants is possible for the present only by experiments. Further research work is carried on in the hope that an analytical solution will be elaborated.

LIST OF SYMBOLS

$C_1$, $C_2$, $C_3$ = material constants
$d$ = diameter of specimen (mm)
$d_0$ = original diameter (mm)
$\Delta d$ = variation of diameter during one load cycle (mm)
$\varepsilon$ = true strain
$\varepsilon_f$ = true fracture strain
$\Delta \varepsilon_{p_d}$ = plastic strain range
$E_f$ = total specific input energy in the course of a fatigue test (MJ/m³)
$F$ = specific input energy in one load cycle (MJ/m³)
$m, n$ = exponents
$N_f$ = number of cycles till fracture
$R_e$ = yield stress (MPa)
$R_{t_f}$ = ultimate tensile stress (MPa)
$R_{t_m}$ = true fracture stress (MPa)
$\sigma$ = true stress
$W_c$ = absorbed specific energy till fracture (ASPEF) (MJ/m³)
$W_d$ = damaging part of input energy (MJ/m³)
$\Delta W_d$ = damaging energy in one load cycle (MJ/m³)
LIST OF REFERENCES


FIG.1. Plastic zone and imagined small specimen at a notch tip.

FIG.2. Typical hysteresis loop.
FIG. 3. Manson-Coffin diagrams of some material tested

\[ \Delta \varepsilon_{pl} \text{ vs. } N_t \text{ number of cycles} \]

\[ \Delta \varepsilon_{pl} = \begin{cases} \Delta \varepsilon_{pl} & \text{CrMo125} \\ \Delta \varepsilon_{pl, 3} & 12 \text{HMFe (charge 3)} \\ \Delta \varepsilon_{pl, 44} & 13 \text{CrMo44} \\ \Delta \varepsilon_{pl, 38} & \text{BHV 38 (charge 1)} \end{cases} \]

\[ N_t = \begin{cases} 500 & 1000 \end{cases} \]

\[ \Delta \varepsilon_{pl} = 0.05 \]

FIG. 4. The total input energy of some material tested as a function of load cycle

\[ E_t \text{ vs. } N_t \text{ number of cycles} \]

\[ E_t = \begin{cases} E_t & \text{CrMo125} \\ E_t, 3 & 12 \text{HMFe (charge 3)} \\ E_t, 44 & 13 \text{CrMo44} \\ E_t, 38 & \text{BHV 38 (charge 1)} \end{cases} \]

\[ N_t = \begin{cases} 500 & 1000 \end{cases} \]

\[ E_t = 10^3 \]

FIG. 5. Comparison of strain ratio and energy ratio values according to equation 9

\[ (W_c/E_t)^{1/3} \text{ vs. strain range and total fracture} \]

\[ A_{pl} = \begin{cases} A_{pl} & \text{CrMo125} \\ A_{pl, 35} & \text{CrMo35} \\ A_{pl, 44} & \text{CrMo44} \\ A_{pl, 38} & \text{BHV 38 (charge 1)} \end{cases} \]

\[ N_f = \begin{cases} 500 & 1000 \end{cases} \]

\[ W_c/E_t = \begin{cases} 80 & 120 \end{cases} \]

490

491