EFFECTS OF CRACK LENGTH AND PRE-STRAIN ON DUCTILE FRACTURE

H.E. Thompson* and J.F. Knott*

The paper describes research carried out to investigate effects of \((a/W)\) ratio on critical C.O.D. values measured for free-cutting mild steel, in normalised and uniformly prestrained conditions. It is found that a decrease in \((a/W)\) from 0.5 to 0.2 increases the initiation C.O.D. by a factor of 2.5 in normalised material, but has no effect in pre-strained material. Differences in values of C.O.D. between the normalised and pre-strained conditions are related to differences in work-hardening capacities and the effects of \((a/W)\) ratio are discussed in terms of relaxation of hydrostatic stress in SEN-bend test-pieces.

INTRODUCTION

Measurements of a material's fracture toughness in the elastic/plastic regime are conventionally made as critical values of the J-integral (\(J\)) or the crack-(tip)-opening-displacement, C.(T.)O.D. (\(\delta\)). The most commonly quoted values are those at initiation (\(J_i\), \(\delta_i\)), although there is interest in the determination of values for increments of crack growth (\(\Delta a\)) if stability analyses are to be carried out. Graphs of \(J\) vs \(\Delta a\), or \(\delta\) vs \(\Delta a\), are referred to as resistance-curves or R-curves. If \(\Delta a\) is measured from the position of the original crack tip (i.e. to include a stretch-zone-width, smw), initiation is defined as the intersection of the R-curve with a blunting line. For a ductile fracture process, involving the coalescence of voids formed around non-metallic inclusions, \(\delta_i\) may be interpreted as the amount of crack-tip opening required to cause the blunting fatigue-crack tip just to coalesce with the closest void which is expanding ahead of the tip in a hydrostatic tensile stress field (1). In the present paper, analysis is confined primarily to effects on C.O.D. because the relationship to the micromechanisms of fracture is more direct, but interpretation in terms of \(J\) is equally feasible, if use is made of the expression

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\[ J = m \sigma_y \delta \]  \hspace{1cm} \ldots \ldots \  (1) \]

where \( m \) is approximately 2 in small-scale yielding, but has been calculated as 3 for generally-yielded bend specimens (McMeeking and Parks \( (2) \)). Experimental values tend to give \( 1 \leq m \leq 3 \) (Willoughby \( (3) \), Slatcher and Knott \( (4) \) and Dawes \( (5) \)).

The Standard C.O.D. test (BS5762) \( (6) \) specifies a specimen thickness \( (B) \) equal to that used in the service application and also requires a precrack grown to a length-to-width \( (a/W) \) ratio of approximately 0.5. For this testpiece geometry, it is possible to calculate two components to the C.O.D. a) an elastic component, \( \delta_{el} \), obtained from \( (a/W) \) and the load from which the testpiece was unloaded, b) a plastic component, \( \delta_{pl} \), obtained from a displacement, \( V_g \), measured using a clip-gauge mounted on knife-edges across the crack mouth, at a height, \( z \), above the top surface of the specimen. Evaluation of \( \delta_{pl} \) from \( V_g \) assumes rigid body rotation around plastic "hinges" in the ligament \( (W-a) \) and a rotation constant, \( r = 0.4 \), is appropriate for the standard geometry.

Whilst values of C.O.D. obtained using the standard procedure have been used successfully in a large number of failure assessments (Harrison \( (7) \), Dawes \( (8) \)), it has become clear that, for some ductile materials, the initiation values of \( \delta_i \), or \( J_i \), are unrealistically over-conservative. One route employed to overcome this conservatism has been a concession that up to 2 mm stable crack growth could be tolerated in some thick-section material, provided that the material is clearly on the "upper shelf" and that no reversion to cleavage could occur. Thus, the critical value of \( \delta \) could be enhanced to a level:

\[ \delta_{crit(mm)} = \delta_i + (\Delta \delta/\Delta a) \times 2 \]  \hspace{1cm} \ldots \ldots \  (2) \]

A precisely equivalent route is adopted with respect to values of \( J \) \( (9) \).

Although such procedures may be perfectly satisfactory in an empirical sense for the purposes of engineering design, it is unlikely that they represent the physical situation in service. This is rather more likely to be the existence of an initially small defect (of size determined by quality control of processing or fabrication, or by NDT inspection limits) in a structure, for which the normal operating stresses are of order two-thirds of the yield stress. Under such circumstances, it is difficult to see how the displacements required to promote ductile crack growth in a tough material can be generated and it is necessary to seek alternative explanations for the over-conservatism of standard \( \delta_i \) values, when compared with service performance.

One main point is that, if initial defects are, indeed, small compared with structural dimensions, it may not be possible to
generate sufficient hydrostatic stress fields around the defects to produce microvoid expansion and linkage before the structure itself experiences general plastic collapse. An experimental method of studying this effect is to measure values of $\delta_1$ and study crack growth behaviour in testpieces with (a/W) ratios ranging from the standard (deep-crack) (a/W) = 0.5 to ratios more typical of those found in service applications. A useful figure for the latter could be taken as (a/W) $\geq$ 0.2. It might be noted that the 25 mm NDT inspection limit specified by the CEGB for PWR applications (10) would correspond to (a/W) = 0.125 in 200 mm thick pressure-vessel shell; if a $+10$ mm tolerance were allowed on the 25 mm figure, the (a/W) ratio would become 0.175.

Work by Dawes (11) on C/Mn steel and You and Knott (12) on high-strength structural steels has, indeed, indicated that values of $\delta_1$ increase as (a/W) decrease and an attempt was made to relate this effect in a quantitative manner to details of the local stress field ahead of the crack tip at initiation (13). One inference was that the changes in local crack-tip ductility might be reflected by the aspect (depth-to-height) ratios of the microvoids, but experimental observations in the high-strength steel were confined by the occurrence of "fast-shear" fracture, which does not conform to the simple void coalescence mechanism. An interesting result was that, for the higher strength steels (i.e. those with lower $\delta_1$ values) in which deeply cracked testpieces the critical value of (a/W) for which the changeover from deep-crack to shallow-crack behaviour occurred was smaller, i.e. relatively shallow cracks still gave low $\delta_1$ values.

Thus observation was rationalised in terms of the overall amount of testpiece plasticity required to accommodate a given value of $\delta_1$ and is discussed later in this paper.

The present work was undertaken to extend previous observations on effects of (a/W) on $\delta_1$ and ($\Delta\delta/\Delta a$) as part of a more general study of effects of variables on upper-shelf toughness. To model the simple void-expansion mechanism and to avoid the onset of fast shear, it was decided to employ a free-cutting steel, which has a high volume fraction of easily-debonded, manganese sulphide inclusions. Changes in the material's stress-strain characteristics and local crack-tip ductility were effected by applying uniform, cold pre-strain prior to machining and pre-cracking the testpieces. In this way, the variation of critical (a/W) with the testpieces. Additionally, it is of interest to determine effects of pre-strain on values of C.O.D., since many engineering components have been subjected to cold work, prior to entering service.

**Experimental Methods**

The free-cutting steel had the composition 0.07C, 0.96Mn, 0.28Si, < 0.02Si, 0.013P, 0.005S, and, in all cases, testpieces were machine so that the length and width dimensions lay in a plane normal to the rolling direction. Fig. 1. The MnS stringers were there-
fore sectioned effectively to provide circular inclusions, to enable void growth and coalescence to be treated approximately in terms of in-plane, two-dimensional deformation. Specimen blanks were normalised (1 h 950 °C, followed by air-cooling) and half were then subjected to a uniform pre-strain of 20% at room temperature prior to machining and pre-cracking. The specimen width of all pre-strained specimens was 15.5 mm, and that of normalised specimens 20 mm. In all cases, the specimen thickness was 10 mm. The grain diameter after normalising was 25 μm, and the mechanical properties for the two conditions are given in Table I. The mean inclusion spacing was 100 μm, and the mean inclusion radius was 5 μm.

TABLE I - Mechanical Properties

<table>
<thead>
<tr>
<th></th>
<th>Yield Stress, σ_Y (MPa)</th>
<th>Ultimate Tensile Strength (MPa)</th>
<th>σ_Y / UTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised</td>
<td>240</td>
<td>400</td>
<td>0.60</td>
</tr>
<tr>
<td>Pre-strained</td>
<td>630</td>
<td>660</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Two crack-length ratios, (a/W) = 0.5 and (a/W) = 0.2, were employed. The low (a/W) ratio was obtained by growing cracks in over-size (large W) bars and then machining off sufficient of the surface behind the crack to produce the desired value. Specimens were deformed in four-point bending at a constant displacement-rate of 0.15 mm/min to produce pure bending over a distance 20 mm on either side of the crack. Individual testpieces were unloaded from different positions along the load-clip-gauge displacement curve and were then either broken open (by cleavage) in liquid nitrogen or infiltrated with araldite and sectioned metallographically to determine the extent of fibrous crack growth.

Values of δ for the deep-crack geometry were calculated using the BS5762 procedures. The elastic component, δ^el, is calculated from the expression:

\[ \delta^{el} = \frac{K^2(1-ν^2)}{2σ_Y E} \]  \hspace{1cm} (3)

where E is Young's modulus, ν is Poisson's ratio, and K is calculated using the load from which the testpiece was unloaded and a value of compliance, Y(a/W), appropriate to the total crack length. The plastic component, δ^pl, is calculated using the expression:

\[ \delta^{pl} = \frac{VP}{(1 + (a+z)/(N-a))} \]  \hspace{1cm} (4)
where the rotation constant, r, is 0.4 for (a/W) = 0.5. For the shallow-crack specimens, the elastic component can be calculated in a similar manner, using compliance values appropriate to (a/W) ≤ 0.2 (14), but there is more ambiguity with respect to the plastic component because values of r for low (a/W) ratios are not well-established. In the present paper, we use experimental values determined by You and Knott (15) in the range r = 0.18 → 0.25, which are supported by the finite-element analyses of Kikuchi (16), Fig. 2.

Metallographic specimens were etched in 2% nital and examined using an Olympus Microscope and fracture surfaces were examined using a Camscan scanning-electron-microscope (SEM) operated at 30 kV.

Experimental Results

Values of C.O.D. are plotted vs crack length in Figs. 3 and 4 for the normalised and pre-strained conditions, respectively. In each case, the initiation point is estimated by fitting a least-mean-squares line to the experimental points and finding the point of intersection of this line with a theoretical blunting line, given by Δ = 2a. The four values of Δ thus obtained are given in Table II and Fig. 5 shows a comparison of the results for both conditions.

<table>
<thead>
<tr>
<th>TABLE II - Values of Δ_1</th>
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<tbody>
<tr>
<td>(a/W) = 0.5</td>
</tr>
<tr>
<td>normalised</td>
</tr>
<tr>
<td>pre-strained</td>
</tr>
</tbody>
</table>

It is clear that the effect of reducing (a/W) from 0.5 to 0.2 in the normalised condition is to increase Δ_1 by a factor of approximately 2.5, although there is negligible effect on (ΔΔ/Δa).

The effect of pre-strain is substantially to reduce the value of Δ_1 (from 0.068 mm to 0.02 mm in deep-cracked testpieces) and essentially to remove any differences between deep-crack and shallow-crack values. The value of (ΔΔ/Δa) is higher for (a/W) = 0.2 in pre-strained material, but the scale in Fig. 4 is relatively expanded, and the differences in slope between the two (a/W) ratios are of much less significance than differences in slope between both (a/W) ratios in normalised material and both ratios in pre-strained material (see Fig. 5).

Crack profiles are shown in figs. 6-9. Here, it can be seen that there is a great difference between long and short cracks.
for the normalised condition, but that long and short cracks in
pre-strained material are closely similar. Figure 8 also shows
clearly that, even though the micromechanism of crack growth in
pre-strained material involves void coalescence, the crack follows
macroscopically a "zig-zag" path, which corresponds to directions
of maximum shear stress.

Discussion

In the normalised condition, mild steel has a high work-hardening
capacity (as evidenced by the low Y.S./U.T.S. ratio, Table I. When
such material is deformed in a pre-cracked testpiece, yielding
first occurs ahead of the crack tip on planes corresponding to di-
rections of maximum shear stress. The yielded regions then harden
and it becomes easier to produce plastic deformation on planes
where the resolved shear stress is not so high, but where the re-
stance to flow is still the initial yield stress, rather than the
(higher) flow stress of yielded material. The effect is to spread
yielding in a wide region ahead of the crack tip, so that the crack
blunts substantially before high, localised strains can be develop-
ed. For a microvoid coalescence mechanism, the value of \( \delta_1 \) is a
combination of two factors: a) the geometrical displacement which
would be required to produce internal necking between inclusions
in rigid/plastic material; b) the accumulation of plastic strain
throughout the plastic zone. If the inclusion spacing is \( X_0 \), the
calculations of Rice and Johnson (17) suggest values \( 1 < \delta_1/X_0 < 3 \).
For pre-strained material, the work-hardening capacity is much
less (cf. the high Y.S./U.T.S. ratio in Table I) and so the second
contribution to \( \delta_1 \) is small. The values obtained in the present
work are \( \delta_1 = 0.068 \) mm for normalised material and \( \delta_1 = 0.02 \) mm in
pre-strained material. The ratio between these two values cor-
responds (within experimental scatter) to the (upper-bound) factor
of 3 that might be expected from the Rice and Johnson theory.

Smith (18) found a \( \delta_1 \) value of 0.037 mm for a free-cutting
mild steel with an inclusion spacing of 0.045 mm, giving a \( \delta_1/X_0 \)
ratio of 0.82, in agreement with the present work.

The reduction in \( \delta_1 \) due to pre-strain in deep-cracked speci-
mens compares favourably with results obtained by Willoughby (3)
in En32 steel, for which 20% pre-strain caused the value of \( \delta_1 \) to
decrease from 0.09 to 0.02 mm.

In a pre-cracked, SEN bend testpiece, account must also be
taken of the relaxation of hydrostatic tensile stresses ahead of
the crack tip as a result of gross-section yielding. Using slip-
line field analysis as a guide, it may be deduced that deeply
cracked testpieces deform (at general yield) by a "hinge" mecha-
nism which retains high constraint, but that shallow cracks (at
general yield) permit gross yielding by the onset of 45° slip-
lines emanating from the top surface, Fig. 10a.
The specimen in Fig. 10b with \((a/w) = 0.2\), shows signs of these slip lines developing prior to general yield. Figures 11a shows a specimen of \((a/w) = 0.5\) just prior to general yield and Fig. 11b shows the development of gross plastic yielding in a testpiece with \((a/w) = 0.2\). The details of these deformation models will not necessarily apply to materials which work-harden, but it is entirely plausible that, once a given value of C.O.D. has been attained in a shallow-cracked specimen, gross yielding will occur, the hydrostatic component will be relaxed, and much higher values of C.O.D. will need to be generated to produce fracture. This is the explanation for behaviour in the normalised material. The deep-crack value for \(\delta_1\) is 0.068 mm. Presumably, when this value (or a value between 0.02 and 0.068 mm) is generated in a testpiece with \((a/w) = 0.2\), gross yielding occurs), there is a degree of stress relaxation and it becomes necessary to generate a C.O.D. of 0.16 mm to initiate fracture. In pre-strained material, on the other hand, the deep-crack value \(\delta_1\) is only 0.02 mm.

This value of C.O.D. at a crack tip in a testpiece with \((a/w) = 0.2\) does not require gross surface deformation and so fracture initiates at the same C.O.D. as that in a testpiece with \((a/w) = 0.5\).

An indication of behaviour in testpieces with \((a/w) = 0.2\) can be obtained by examining the bending moments required for different stages of yielding. The three successive stages are a) initial top surface yielding, b) general yielding, when 45° slip planes first break through from the top surface to the "hinge" region, as in Fig. 11a, and c) the point at which yielding has fully occurred to a depth 0.2W from the top surface when full relaxation of hydrostatic stresses must have been achieved. The bending moments in any perfectly plastic material are given by:

\[
\begin{align*}
   M_{1Y} &= 0.16 \sigma_y BW^2 \\
   M_{GY(\text{net section})} &= 0.18 \sigma_y BW^2 \\
   M_{GY(0.2W)} &= 0.22 \sigma_y BW^2
\end{align*}
\]

For the normalised case, these correspond to loads of 16KN, 18.12KN, and 21.12KN, respectively. The normalised short-crack specimens actually initiated at a load of 21KN, suggesting that at this point, interaction between the top surface plasticity and the plastic region ahead of the crack tip had taken place. The specimen clearly has relaxed by \(\delta = 0.16\) mm but it can also be deduced that when the short crack has developed a C.O.D. of 0.068 mm (for which sufficient constraint is present to initiate a long crack), relaxation must have occurred because fracture would have initiated had there been no relaxation. The load at this C.O.D. is approximately 21KN which is roughly the load for yielding to a depth 0.2W.
For the pre-strained case (with a much higher yield stress of 630 MPa), when (a/W)=0.2, the load corresponding to initiation is 25kN, which is at approximately the same load as the start of top surface yielding (25.2kN) and is well below the load for net section yield (28.7kN) and for yield to a depth of 0.2W (33.3kN). This evidence supports the contention that full constraint is maintained at initiation: even if a small amount of top-surface yielding has occurred at 25kN, fig. 10b suggests that this does not interact with the plasticity emanating from the crack tip.

CONCLUSIONS

The effect on fibrous fracture initiation of reducing the (a/W) ratio from 0.5 to 0.2 has been examined for a free cutting mild steel in the normalised condition and after application of 20% uniform pre-strain. The main conclusions are as follows:

a) the general effect of pre-strain is to reduce δ1 substantially: from 0.068 mm to 0.02 mm for (a/W) = 0.5 and from 0.16 mm to 0.02 mm for (a/W) = 0.2. The results for (a/W) = 0.5 is related directly to work-hardening characteristics: that for (a/W) = 0.2 involves also features of relaxation of stress in the testpiece, as summarised below:

b) the increase in δ1 for normalised material from 0.068 mm for (a/W) = 0.5 to 0.16 mm for (a/W) = 0.2 is attributed to the fact that when δ = 0.068 mm (or even a lower value) in a testpiece of (a/W) = 0.2, relaxation of the hydrostatic stresses around the crack tip is achieved by gross plastic deformation of the testpiece. More applied C.O.D. is then required to initiate fibrous fracture, and so δ1 increases;

c) for pre-strained material δ1 is 0.02 mm for both (a/W) = 0.5 and (a/W) = 0.2. At such a small value of C.O.D., gross plastic deformation of the testpiece has not occurred, whether (a/W) = 0.5 or 0.2;

d) bounds have been established, in terms of the applied load required to relax the hydrostatic stresses. The lower bound in testpieces with (a/W) = 0.2 is considered to be general yield, rather than the first onset of top-surface yield: the general yield load may be calculated using Ewing's slip-line-field analysis for shallow cracks. The upper bound is taken as the load at which gross yielding would extend to a depth of 0.2W in a plain bend bar.

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**ADDITIONAL SYMBOLS**

\[ a = \text{Crack Length} \]
\[ \Delta a = \text{Fibrous Crack Growth} \]
\[ B = \text{Specimen Thickness} \]
\[ J = \text{J-Integral} \]
\[ \varepsilon = \text{Young's Modulus} \]
\[ K = \text{Stress Intensity} \]
\[ m = \text{Constant Relating J to } \sigma_y \delta \]
\[ P = \text{Load} \]
\[ r = \text{Rotation Constant} \]
\[ V = \text{Clip Gauge Displacement} \]
\[ W = \text{Specimen Width} \]
\[ Y = \text{Compliance Function} \]
\[ Z = \text{Knife-Edge Thickness} \]
\[ \delta = \text{Crack Opening Displacement (C.O.D.)} \]
\[ v = \text{Poisson's Ratio} \]
\[ \sigma_y = \text{Uniaxial Yield Stress} \]

**REFERENCES**


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Figure 1 Testpiece extraction.

Figure 2 Comparison of values for \( r \), rotation constant.

Figure 3 R-curve for normalised testpieces.

Figure 4 R-curve for pre-strained testpieces.
Figure 5 Comparison of normalised and pre-strained results.

Figure 6 Normalised crack profiles etched in Nital.

Figure 7 Etched crack profile, 20% Pre-strain, \( a/W = 0.2 \).

Figure 8 Unetched crack profiles.

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Figure 9 Unetched crack profiles.

Figure 10 Deformation patterns a) (a/W) < 0.3, b) (a/W) = 0.2.

Figure 11 Fry's etch for a) (a/W) = 0.5, b) (a/W) = 0.2.