THE T-CRITERION FOR FAILURE OF ENGINEERING MATERIALS

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The T-criterion, based on the extrema principles of the components of SED, considers separately the particular character of the dilatational, T_V, and distortional, T_D, components of the total SED, T. This was achieved by calculating the variation of T_V along contour lines of constant T_D represented by contours of constant equivalent stress, σ_e. The maximum of T_V calculated along a conveniently selected contour of σ_c-stress defines the eventual crack initiation angle, θ_0, for the particular contour, where the $(T_{V_{max}}+T_D)$ attains a critical value for impending fracture. For the accurate prediction of fracture characteristics (angle θ_0 of fracture and fracture stress, $\sigma_{c\infty}$) in ductile materials exact elastic-plastic stress fields are needed at the vicinity of the crack tip. A comparison of theory and experiments indicated that the T-criterion may be formulated as well by LEFM for cleavage-type cracks.

INTRODUCTION

Most of the developed theories concerning the mechanisms of fracture are based naturally on the microscopic structure of materials. Those dealing primarily with ductile fracture imply an influence of the voids created at the process zone around the crack tip on fracture onset. According to these theories microscopic cavities are developed in front of the crack tip by particle cracking or by interfacial decohesion. This process is induced by the favourable triaxial state of stress developed ahead of the crack tip. Subsequently these cavities or voids grow, because of the intense plastic deformation developed at the vicinity of the crack tip, i.e., the region which is influenced by the blunting of the tip, and the larger among them start a process of coalescence, as soon as the local void-volume fraction at this zone exceeds some critical value [1-6]. Then, the ligaments between neighboring and pre-

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ferentially oriented voids are thinned down by concentration and intensification of the plastic zones between these ligaments and prepare the way of the first step of crack advance.

The macroscopic effect of this mechanism of nucleation, growth and coalescence of voids is a plastic dilatancy, which is apparent in various degrees in all materials from the quasi-brittle to the quasi-plastic. Then, the nucleation and development of voids in the neighbourhood of the crack tip is intimately interrelated with the dilatational component of the strain energy density, henceforth denoted SED, absorbed by the fracturing body.

Among the different criteria predicting fracture from the macroscopic point of view and which are based on energy principles, as the extrema of the SED, or of its components, the only one preserving the conception of the above-mentioned theories is the T-criterion [7,8]. The simple reason for this distinction is that this criterion is the only one, which takes into consideration the particular effects of the two components of the SED, that is the distortional one, which changes the shape of an element and the dilatational one, which is the main cause of cavity nucleation around the crack tip.

All fracture criteria which have been introduced up-to-now are of limited applicability, because of their perseverance in some conservative ideas concerning the microscopic mechanisms of fracture, appropriate only for special cases and certainly not of a universal character. Thus, the maximum tangential stress criterion [9] is an oversimplified consideration of fracture processes, since it takes into account only one component of the stress tensor. The oversimplification induced by this fracture criterion can be compared to that of a yielding criterion formulated in the one-dimension stress space.

The maximum energy release rate (G-criterion) [10,11] is a self-contradicting criterion, since it assumes a progressive self-similar expansion of the crack, a fact which violates reality. Finally, the formulation of the minimum elastic strain-energy density criterion (S-criterion) [12], was based on the existence of the artificially defined circular boundary of the so called *"core region"*, in the inside of which non-linear phenomena take place. Although the existence of such a region is evident, this border could be better approximated by an elastic-plastic boundary instead of the proposed artificial circle. The criterion uses for its formulation only the singular term of the elastic solution for the stress and strain field around the crack tip, fact that restricts the extent of its applicability. Anyway, a possible formulation of this criterion by means of elastic-plastic solutions seems not encouraging in the first glance.

The T-criterion, based also on strain energy extrema considerations postulates that the crack propagates along a direction defined by a maximum of the total SED, which is also a maximum for the dilatational SED when this distribution is evaluated along a contour of constant distortional SED, as is for example the Mises elastic-plastic boundary. Up to now the formulation of the T-criterion was restricted either on the linear elastic singular solution [7,8], or recently on the two-term linear elastic solution [13].

In brittle or quasi-brittle materials, the developed plastic enclaves before the final fracture remain rather small with respect to the crack length. For ductile materials under plane strain conditions, exhibiting a discontinuous mode of fracture, i.e., some of the grains cleave, ahead of the main crack and the bridging regions between them then break in a ductile manner [14], there is also a very limited extent of the plastic zones in front of the crack tip. Then, for this type of fractures the elastic solutions may be considered as a faithful representation of the stress-and-strain field outside the plastic zone.

In the case of a material which yields on a large scale and presents a continuous development of the successive elastic-plastic boundaries before its final ductile fracture, the formulation of any criterion by means even of the exact elastic solution, becomes a crude approximation. In these cases it is imperative to make use of the elastic-plastic solutions in formulating a fracture criterion intended to predict ductile fracture.

In a recent publication, the T-criterion was extended to encompass ductile fracture [15]. In this reference an application concerning fracture initiation, was also presented in which use was made of the HRR field [16-18] and the mixed-mode plastic singular solutions [19-20]. In this recent application only the case of "low stress" fractures ocurring under small scale yielding was formulated.

In this paper the T-criterion, as resulted from recent developments [15], is presented, together with considerations concerning the domain of validity of its early versions. A review of the results about the angle of fracture and the fracture load, as theoretically predicted by the different versions of the T-criterion, is also presented here.

STATEMENT OF THE T-CRITERION

The T-criterion is based on energy concepts and states that fracture occurs, around a crack tip, in the direction where the SED, T, has its maximum value when calculated in an appropriate contour.

For isotropic bodies the SED, T, can be decomposed to a dilatational component, T_V, and a distortional one, T_d, and it is valid that:

$$T = T_V + T_d \tag{1}$$

The T-criterion emphasizes the conceptual difference between these two energy components, by evaluating the T_V-component of the SED along a contour where the other component, T_d, remains constant. As such a contour, in the early versions of the T-criterion, the initial or neighbouring Mises elastic-plastic boundaries were used, provided that the material fracture in a brittle manner and the calculation of the elastic-plastic border by means of the elastic solution was still a good approximation.

When an elastic-plastic analysis, instead of the linear elastic solution, is used for the formulation of the fracture criterion, and the evolution of critical quantities as the extrema of the SED at a constant point near the tip, is of interest, it becomes clear that the existing versions of the T-criterion [7,8,13] need some modifications, in order to properly describe such types of failure.

In the case in which the region of interest is well contained in the plastic zone, the Mises elastic-plastic boundary cannot be used anymore for the evaluation of the T_V-component of the SED, which is now given by relation:

$$T = (T_V + T_d)_{el} + (T_V + T_d)_{pl} \tag{2}$$

The dilatational component of the plastic SED, $(T_V)_{pl}$, is taken equal to zero, in what follows, by accepting the plastic incompressibility-assumption, which is a doubtful hypothesis generally accepted in elastic-plastic problems in practice.

It must be noted here that, by the term plastic SED, the density of the irreversibly absorbed by the material plastic stress working is meant but for reasons of consistency the term SED was also conserved. However, for the application of the T-criterion by means of the HRR field, the use of the term plastic SED is still legitimate since in the HRR derivation, total deformation theory of plasticity has being used.

Consider now, a cracked plate with an internal slant crack at an angle β with the direction of the applied tension at infinity of the plate. The T-criterion may be stated as follows:
(i) Contours of constant effective stress, σ_e, i.e., contours where the distortional SED, $(T_d)_{el}+(T_d)_{pl}$, remains constant, lying on a region well contained into the plastic zone, can be used for the evaluation of T_V. The angular position of $T_{V_{max}}$ on these contours indicates the angle θ_0 of crack propagation.
(ii) The crack starts to propagate when the value of the SED, T, at an appropriately defined point in the near vicinity of the crack tip, depending on the angle of inclination, β, and at the above-mentioned angle, θ_0, of fracture, reaches a critical value, T_{cr}. In other words, we are concerned with the evolution of the quantity T_{max}, calculated at an appropriate polar distance r_j from the tip for each angle of inclination, β.

This critical value, T_{cr}, must be a material constant and, of course, independent of the particular geometric configuration of the problem. Then, it is valid that at fracture initiation the following relation holds:

$$[(T_v+T_d)_{el}+(T_d)_{pl}]_f^{\beta} = [(T_v+T_d)_{el}+(T_p)_{pl}]_f^{90^o} = T_{cr} \quad (3)$$

where the index f, stands for fracture and $\beta=90^o$ is the Mode-I crack loading.

In the proposed version, the T-criterion can be formulated by means of any elastic-plastic solution and for the case of small-scale, as well as for the case of large-scale yielding, provided that the stress-field is known for the cracked configuration. In this paper the case for which the HRR and the mixed-mode plastic singular fields were applied [15] will be developed for the formulation of the T-criterion.

FORMULATION OF THE T-CRITERION BY STRAIN HARDENING PLASTICITY SOLUTIONS

The HRR stress field, for the symmetric (Mode I), anti-symmetric (Mode II), and the mixed-mode loading yields the plastic solution valid at the close vicinity of a crack-tip for a cracked plate of a material obeying the generalized Ramberg - Osgood power hardening relation [16-20]:

$$\varepsilon_{ij} = \frac{3}{2}\alpha\left[\frac{\sigma_e}{\sigma_0}\right]^{n-1}\frac{\sigma_{ij}}{E} \tag{4}$$

in which the J_2-deformation theory of plasticity was assumed. In this relation ε_{ij} and s_{ij} are the strain- and deviatoric-stress components, σ_e and σ_0 the equivalent stress and the yield stress in simple tension and α the Ramberg-Osgood parameter given in ref.[16] and considered as a material constant with a value of 0.02 for the usual definition of yielding. Finally, n is the strain-hardening exponent.

For the mixed-mode loading problem of the cracked plate it may be shown with reference to polar coordinates, r and θ, centered at the crack tip that the components of stress and strain may be expressed by [19-20]:

$$\begin{aligned} \sigma_{ij} &= \sigma_0 k_j^p r^{-\frac{1}{n+1}}\,\bar{\sigma}_{ij}(\theta, M^p) \\ \sigma_e &= \sigma_0 k_j^p r^{-\frac{1}{n+1}}\,\bar{\sigma}_e(\theta, M^p) \\ \varepsilon^p_{ij} &= \frac{\alpha\sigma_0}{E}(k_j^p)^n r^{-\frac{n}{1+n}}\,\bar{\varepsilon}^p_{ij}(\theta, M^p) \end{aligned} \tag{5}$$

where the dimensionless functions $\bar{\sigma}_{ij}, \bar{\sigma}_e$ and $\bar{\varepsilon}^p_{ij}$ depend only on the angle θ and the near-field twisting parameter, M^p, expressed by:

$$M^p = \frac{2}{\pi}\tan^{-1}\left(\frac{\bar{\sigma}_{\theta\theta}}{\bar{\sigma}_{r\theta}}\right), \quad \text{for} \quad \theta = 0^o \tag{6}$$

Moreover, k_j^p is the plastic stress intensity factor which for j=1 corresponds to mode-I deformation and for j=m it is related to the mixed-mode case.

Now following refs.[19-20] we simplify our calculations by considering only the plastic parts of the stress-strain relations in the dominant singularity analysis used. Consequently, the elastic parts of the SED will be calculated by means of the displayed values for the elastic components of stress in the above-cited references and the $(T_V)_{pl.}$ will be considered as insignificant.

Then, the components of the SED are expressed by [15]:

$$(T_v)_{el.} = \frac{t}{18K} c_j^2 \left(\frac{\bar{\sigma}_{rr} + \bar{\sigma}_{\theta\theta}}{\bar{\sigma}_e}\right)^2$$

$$(T_d)_{el.} = \frac{1}{6G} c_j^2 \tag{7}$$

$$(T_d)_{pl.} = \left(\frac{n}{n+1}\right) c_j^2 \frac{\alpha}{E} \left(\frac{c_j}{\sigma_0}\right)^{n-1}$$

In these relations t is a loading constant taking the value t=1 for plane stress and $t=(1+\nu)^{2}$ for plane strain.

Moreover, since the T-criterion implies that the strain energy density must be calculated along contours of constant effective stress, σ_e, the radial distance r_e of these contours may be expressed as a function of the polar angle θ and the mixed mode twisting parameter M^p and is expressed by:

$$\sigma_0 k_j^p r_j^{-\frac{1}{n+1}} \bar{\sigma}_e(\theta, M^p) = \sigma_e = c_j \tag{8}$$

In this relation the angle θ is meant to be $\theta=\theta_0$ that is the angle for which a $(T_v)_{max}$ is attained.

It can be seen from relation (8), that the constant c_j is load-dependent. By setting an arbitrary initial value for r_1 in relation (8), close enough to the crack tip and along the fracture direction 0^o, for Mode I conditions and for a first loading step, the constant c_1 could be calculated. Then, for an arbitrary angle of inclination β and for the same value of external load the distance $r_m(\theta_0)$ for the new geometrical configuration of the crack was calculated by taking $c_m=c_1$, i.e., the same level of distortional SED, and was given by the following relation:

$$\frac{r_m}{r_1} = \left(\frac{\bar{\sigma}_{e_m}}{\bar{\sigma}_{e_1}}\right)^{n+1} \frac{I_n(1)\sin^2\beta}{I_n(m)} \tag{9}$$

At the points defined along the radial distances from the crack tips for the fracture angles 0^o and θ_0 respectively and the constant effective stress contours $\sigma_e = c_1 = c_m$, the variation of the respective $T_{V_{max_j}}$ may be evaluated. The index j now refers to the corresponding angle of inclination, β, of the crack.

The respective values of T_{D_j} on the same points can be evaluated and found to be approximately the same for the fracture loads, that is for $\sigma_\infty^{90^o}$ and σ_∞^{β}. Small differences in the respective values of T_{D_j} were taken into account by means of relation (3).

It must be noted, that if, in the elastic-plastic analysis, a yielding criterion is used which takes into account not only the contribution of dictortion but also the contribution of dilation in yielding mechanisms, the proportion of dilatational and distortional SED considered in the existing versions of the T-criterion, must be re-examined since this procedure now is implied by the yielding criterion.

By considering now relations (7), it can be seen that only the elastic dilatational component of the SED is dependent on the θ-polar coordinate, the other two remaining constant when calculated on the contour $\sigma_e = c_j$. Then, following the statement of the T-criterion, the angle θ_0 of crack propagation coincides with the angular direction for which the right-hand side ratio of equation (4) has its maximum value.

For the correlation of the fracture loads between the transverse ($\beta=\pi/2$) and the inclined crack at an angle β, with respect to the direction of loading, relation (3), after introducing in it relations (7) and (9), becomes:

$$(k_m^p)^2\left[\frac{\sin^2\beta I_n(1)}{I_n(m)}\right]^{-\frac{2}{n+1}} r_1^{\frac{n-1}{n+1}}\left\{\frac{t(1-2v)}{6}\left(\frac{\bar{\sigma}_{rr}+\bar{\sigma}_{\theta\theta}}{\bar{\sigma}_e}\right)_m^2+\left(\frac{1+v}{3}\right)\right\}+$$

$$+(k_m^p)^{n+1}(\bar{\sigma}_{e_1})^{n-1}n\alpha/\frac{\sin^2\beta I_n(1)}{I_n(m)}(n+1) = (k_1^p)^2 r_1^{\frac{n-1}{n+1}}\left\{\frac{t(1-2v)}{6}\left(\frac{\bar{\sigma}_{rr}+\bar{\sigma}_{\theta\theta}}{\bar{\sigma}_e}\right)_1^2+\right.$$

$$\left.+\frac{1+v}{3}\right\}+(k_1^p)^{n+1}(\bar{\sigma}_{e_1})^{n-1}n\alpha/(n+1) \tag{10}$$

For the case of small scale yielding, the plastic stress intensity factor can be related to the far-field elastic intensity factors, and equation (10) can provide the variation of the ratio $(\sigma_\infty^\beta/\sigma_\infty^{90})$, expressing the fracture stress of a plate containing an oblique crack, normalized to the fracture stress for a crack perpendicular to the loading axis.

By considering the length parameter, J_c/σ_0, that is the ratio of the critical value of the J-integral over the yield stress, σ_0 in simple tension, to be equal to the characteristic radius r_1 of equation (10), one has:

$$r_1 = \frac{J_c}{\sigma_0} = \frac{\alpha\sigma_0}{E}I_n(1)[k_I^p]^{n+1} \tag{11}$$

and as can be seen, the J_c-value refers to Mode-I conditions, i.e., to the angle $\beta=\pi/2$. The quantity $I_n(j)$ is expressed by the definite integral given in refs. [19-20]:

$$I_n = \int_{-\pi}^{\pi}\left\{\frac{n}{(n+1)}\bar{\sigma}_e^{n+1}\cos\theta-\left[\sin\theta(\bar{\sigma}_{rr}(\bar{u}_\theta-\bar{u}_r^{\cdot})-\bar{\sigma}_{r\theta}(\bar{u}_r+\bar{u}_\theta^{\cdot}))+\right.\right.$$
$$\left.\left.+\frac{1}{(1+n)}(\bar{\sigma}_{rr}\bar{u}_r+\bar{\sigma}_{r\theta}\bar{u}_\theta)\cos\theta\right]\right\}d\theta$$

Values of the I_n-integral for n ranging between unity (elastic case) up to $n=\infty$, for perfect plasticity, and M^p extending between zero for mode-II deformation, to unity, for mode-I conditions in the near field are given in ref. [19].

The integral $I_n(M^p)$ may be determined from the analysis of the singular stress field and it depends on n and M^p. Since any effort up-to-now did not succeed to find a method to directly connect the near field (defined by k_j^p and M^p) to the far field (defined by K_I and K_{II}) without an intervention of the in-between field [18], it is reasonable to make recourse to a numerical analysis, based on the singular finite-element approach, which has calculated satisfactorily the relationship between M^p and M^e. Therefore the near field for small scale yielding may be considered as completely determined. The variation of $M^p=f(M^e)$ and $I_n=f(M^p)$ for plane-stress and plane-strain conditions were plotted in ref. [19] for n varying between unity and infinity and for values of Poisson's ratio $\nu=0.30$. However, it has been shown in ref.[19] that the function $M^p=f(M^e)$ is insensitive to variations of ν. Then, the ratio of the plastic stress intensity factors for mixed mode k_M^p and for mode-I conditions are given by:

$$\frac{k_M^p}{k_I^p} = \left[\left(\frac{\sigma_\infty^\beta}{\sigma_\infty^{90^o}}\right)^2 \frac{\sin^2\beta I_n(1)}{I_n(m)}\right]^{1/n+1} \tag{12}$$

Introducing now relations (11) and (12) into relation (10), one has:

$$\left(\frac{\sigma_\infty^\beta}{\sigma_\infty^{90^o}}\right)^{\frac{4}{n+1}}\left[\frac{\alpha\sigma_0 I_n(1)}{E}\right]^{\frac{n-1}{n+1}}\left\{\frac{t(1-2\nu)}{6}\left(\frac{\bar{\sigma}_{rr}+\bar{\sigma}_{\theta\theta}}{\bar{\sigma}_e}\right)_m^2+\left(\frac{1+\nu}{3}\right)\right\}+$$
$$+\left(\frac{\sigma_\infty^\beta}{\sigma_\infty^{90^o}}\right)^2 \frac{(\sigma_{e_1})^{n-1} n\alpha}{(n+1)} = \left[\frac{\alpha\sigma_0 I_n(1)}{E}\right]^{\frac{n-1}{n+1}}\left\{\frac{t(1-2\nu)}{6}\left(\frac{\bar{\sigma}_{rr}+\bar{\sigma}_{\theta\theta}}{\bar{\sigma}_e}\right)_1^2+\left(\frac{1+\nu}{3}\right)\right\} +$$
$$+ \frac{(\bar{\sigma}_{e_1})^{n-1} n\alpha}{(n+1)} \tag{13}$$

A numerical solution of the non-linear equation (13) yields the variation of the ratio $(\sigma_\infty^\beta/\sigma_\infty^{90})$ with respect to the crack angle β. As it can be seen, the ratio of the elastic modulus to the yield stress, E/σ_0, the material constant α, the hardening exponent, n, the Poisson ratio, ν, and the state of stress, t, figure as parameters in the above cited relation.

DOMAIN OF VALIDITY OF THE ELASTIC VERSIONS OF THE T-CRITERION

In the preceding, the T-criterion was extended in order to cover the fracture behavior of materials presenting a considerable degree of yielding before their final fracture. In this last version the fracture criterion can as well be applied to fractures preceded by small scale yielding, and such an example was the formulation by means of the plastic singular fields. Of course these solutions retain their validity for large scale yielding also, although a new relation between the plastic SIF's and the far field elastic S.I.F'.s should be established.

In the case of materials which fracture by cleavage or in a discontinuous mode, i.e., cleavage and yielding, and where regions of plastic deformation in the crack-tip are of limited extent, the formulation of the T-criterion by LEFM solutions constitutes a very reasonable approximation.

Indeed, it has been shown in references [7,8,21,22] that, calculating the T_V-component of the SED along the Mises elastic-plastic boundary, where the other component, T_d, remains constant, by means of the singular elastic solution, or the two-term elastic solution [13], the theoretical predictions of the T-criterion were in satisfactory agreement with extensive experimental data.

The calculation of the Mises elastic-plastic boundary by means of the singular elastic solution is also an acceptable situation often found in LEFM applications, provided that this locus is the final one before fracture, i.e., there is not a significant evolution of the elastic-plastic boundaries.

Among the above-mentioned references concerning the formulation of the T-criterion by means of elastic solutions, reference [13], where use was made of the two-term elastic solution, is the more accurate, due to the fact that a more exact representation of the stress-field around the crack tip was considered. In this version of the T-criterion, as also in the others, the angle θ_0 of crack propagation was found by defining the angular position of the maximum value of the dilatational component, T_V, of the SED when calculated along the Mises elastic-plastic boundary.

For the calculation of the ratio $\sigma_\infty^\beta/\sigma_\infty^{90^o}$, that is the fracture stress at infinity for an inclined crack at an angle β, versus the same quantity for a crack with $\beta=90^o$, an assumption concerning the bounded extent of the plastic zones was made in order to take into account the limitation imposed by the use of the elastic solution.

According to this idea the yield locus in its subsequent steps reduces its rate of expansion with loading and finally ceases to expand at a limiting stress, $\sigma_{i,\beta}$, less than the fracture stress σ_∞^β. The physical meaning of this assumption is that at some loading step the supplied mechanical energy is absorbed, to a higher rate, by the already yielded regions. Of course, this intermediate stress, $\sigma_{i,\beta}$ must be very close, quantitatively, to the fracture stress, that is, the failure process zone attains a critical condition, at which the rate of void growth or the degree of cleavage, are restricted only to a very narrow zone round the fracture-initiation point.

The above-described mechanism recognises the fact, which is based on physical considerations, that the antagonistic processes of strain hardening and material softening with loading regulate the mode of deformation of the body. The assumption rests only on the fact that this intermediate stress $\sigma_{i,\beta}$ is independent of the geometry and orientation of the crack. Then, as it can be readily proved, the application of the relation:

$$\sigma_{i,\beta} = \sigma_{i,90^o}$$

to the normalized fracture stress at infinity yields:

$$\frac{\sigma_\infty^\beta}{\sigma_\infty^{90^o}} = \frac{\sqrt{\frac{\alpha}{2r_{90^o}}} + (k-1)}{\sqrt{\frac{\alpha}{2r_\beta}}\left\{[(1+k)-(1-k)\cos2\beta]\cos\frac{\theta_0}{2} - (1-k)\sin2\beta\sin\frac{\theta_0}{2}\right\}+(1-k)\cos2\beta} \tag{14}$$

provided that the stress-field of ref.[13] is used. In this relation, r_{90} and r_β are the radii of the Mises yield loci in the directions of fracture, 0^o and θ_0 respectively, k is the load biaxiality factor, and α the crack semi-length.

It must be pointed out that, as soon as the constant term for the stress representation is used [13], the universality of relations concerning fracture characteristics as in [7,8,21] is lost, and the components of the SED depend on the particular mechanical properties of the material, as is also the case for the "ductile" formulation of the T-criterion [15] presented at the begining of this paper.

RESULTS AND DISCUSSION

Details of calculations, concerning the angle of crack propagation θ_0 as well as the non-dimensional fracture stress σ_∞^β are given in reference [15]. Here we limit ourselves solely to a presentation of these results. Plane-stress conditions were considered, and in addition to the calculation of θ_0, which is equal to the angular direction for which the term

$$\left[\frac{\bar{\sigma}_{rr}(\theta,M^p)+\bar{\sigma}_{\theta\theta}(\theta,M^p)}{\bar{\sigma}_e(\theta,M^p)}\right]^2$$

has its maximum value, the variation of the ratio $(\sigma_\infty^\beta/\sigma_\infty^{90^o})_f$ with respect to the inclination angle β is also given.

All these quantities are plotted in figures 1 and 2. Theoretical results displayed in figure 2 were provided by numerical solution of the non-linear equation (13) with the following values of the characterizing parameters, $\nu=0.34$ for Poisson's ratio, $\alpha=0.02$, $E/\sigma_0=300$ for plane-stress conditions.

A numerical investigation concerning the influence of the variation of the above-cited parameters in the solution of equation (13) showed showed that the plane stress case is not very sensitive to any parameter change.

In Figures 3 and 2 presenting the angle of fracture θ_0 and the ratio $(\sigma_\infty^\beta/\sigma_\infty^{90})$ for plane-stress conditions also, T_I means the T-criterion when formulated by means of the singular elastic solution, T_{II} its version with the two-term solution, and in addition T_p means the prediction of the criterion according to relation (13) presented here.

The theoretical curves for n = 3 and n = 13 of the Tp-criterion provided by numerical solution of equation (13) were influenced by relation (9), i.e., the choice of the appropriate polar radii r_l for each angle of inclination β . However, as is discussed in reference [15] another possible choice is to take the radii r_m and r_l of relation (9) equal. Then, the resulting theoretical curves constitute the upper bound of a zone containing all the experimental points, whose lower bounds are the Tp-curves of figure 2. Both alternatives are equivalent in the sense that they satisfy the basic requirements for the application of the T-criterion. According to this criterion the dilatational component, Tv, of the SED must be evaluated on points lying on constant effective-stress contours for different β's, which has almost the same levels of distortional energy densities at the fracture loads.

Comparison of the theoretical results suggests that the assumptions made in the "elastic versions" of the T-criterion were reasonable, since the theoretical predictions of all versions lie very close to the experimental data which corroborate well enough with these theoretical predictions.

CONCLUSIONS

The T-criterion of fracture, as extended recently in ref.[15] in order to predict also ductile fractures, was reviewed in this paper. According to this criterion, although the modelling of the influence of triaxiality in the void development and expansion was made by the dilatational SED-component, the plastic distortional SED was also taken into account. It is logical to expect that even if void nucleation is influenced mainly by triaxiality, the extent of plastic deformation near the crack tip must be catalytic to the void coalescence rate.

It must be emphasized once again, for the T-criterion as is presented here and in ref.[15], that it can be formulated by means of any numerical elastic-plastic solution or possibly any experimental one. Then as the accuracy of the solution used increases, it is believed that the accuracy of the predictions of this fracture criterion also must increase.

For the early versions of the T-criterion, where elastic stress- and strain-fields were used, one has to conclude, by comparing the theoretical predictions of all versions, that the assumptions made in their formulations were not unrealistic and certainly not arbitrary.

The introduction by the criterion of the maximum value of the SED, when calculated along a contour of constant effective stress, σ_e, very near to the crack tip as characterizing toughness of the material, seems to be an interesting feature, which may fill the existing gap in suitably defining this very important property of materials and structures.

Finally, it is expected that the introduction of a pressure-sensitive yielding criterion taking into account the plastic anisotropy in the elastic-plastic analysis will provide an even more exact fracture criterion, better adaptable to the whole hostory of loading of the material.

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REFERENCES

(1) McClintock, F.A., J. Appl. Mech., Trans. ASME, Vol.35, No.2, 1968, pp.363-371.

(2) Rice, J.R., and Tracey, D.M., J. Mech. Phys. Solids Vol.17, No.2, 1969, pp.201-217.

(3) Gurson, A.L., J. Engng. Mat. Techn., Trans. ASME, Vol.99, No.1, 1977, pp.2-15.

(4) Hahn, G.T., and Rosenfield, A.R., Metall. Trans. A, Vol.6A, 1975, pp.653-670.

(5) Argon, A.S., Im, J., and Safoglu, R., Metall. Trans. A, Vol.6A, 1975, pp.825-837.

(6) Tvergaard, V., J. Mech. Phys. Solids, Vol.30, No.4, 1982, pp.265-286.

(7) Theocaris, P.S., Proc. VII Int. Conf. Exp. Stress Anal. Haifa, Israel, Vol.1, 1982, pp.161-171.

(8) Theocaris, P.S., and Andrianopoulos, N.P., Engng. Fracture Mech., Vol.16, No.3, 1982, pp.425-432.

(9) Erdogan, F., and Sih, G.C., ASME, J. Basic Engng., Vol.85, 1963, pp.519-527.

(10) Palaniswamy, K., and Knauss, E.G., Int. J. Fracture Mech., Vol.8, 1972, pp.114-117.

(11) Hussain, M.A., Pu, S.L., and Underwood, J., ASTM STP, Vol.560, 1974, pp.2-28.

(12) Sih, G.S., Engng. Fracture Mech., Vol.5, 1973, pp.365-377.

(13) Theocaris, P.S., Engng. Fracture Mech., Vol.19, No.6, 1984, pp.975-991.

(14) Cottrell, A.H., Proc. Roy. Soc. London, Vol.A285, 1967, pp.10-21.

(15) Theocaris, P.S., and Philippidis, T.P., submitted for publication in the J. Mech. Phys. Solids.

(16) Hutchinson, J.W., J. Mech. Phys. Solids, Vol.16, 1968, pp.13-31.

(17) Hutchinson, J.W., ibid, Vol.16, 1968, pp.337-347.

(18) Rice, J.R., and Rosengren, G.F., ibid, Vol.16, 1968, pp.1-12.

(19) Shih, C.F., Ph. D. Thesis, Harvard University, 1973.

(20) C.F. Shih, ASTM STP, Vol.560, 1974, pp.187-210.

(21) Theocaris, P.S., and Andrianopoulos, N.P., Int. J. Fracture, Rep. Curr. Res., Vol.20, 1982, pp.R125-130.

(22) Theocaris, P.S., Kardomateas, G., and Andrianopoulos, N.P., Engng. Fracture Mech., Vol.17, No.5, 1983, pp.439-447.

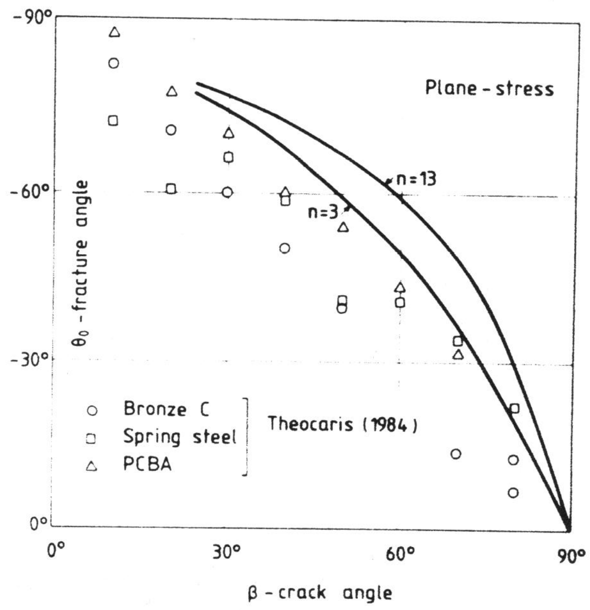

Fig.1. Variation of the predicted angle of crack propagation, θ_0, versus the crack inclination angle, β^o, for plane-stress conditions.

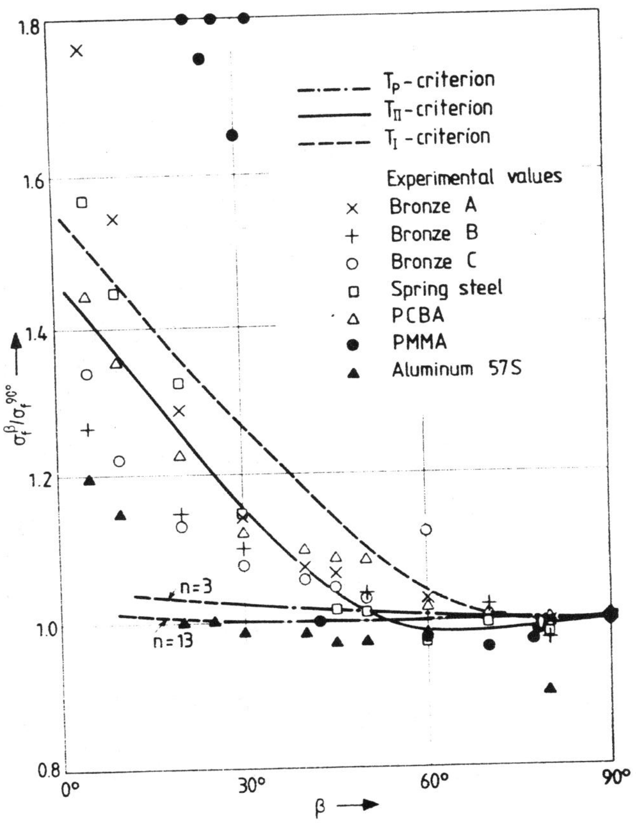

Fig.2. Variation of the normalized fracture stress at infinity, $(\sigma_f^\beta/\sigma_f^{90°})$, versus the crack inclination angle, $\beta°$, for different materials in the range $0° \leq \beta \leq 90°$.

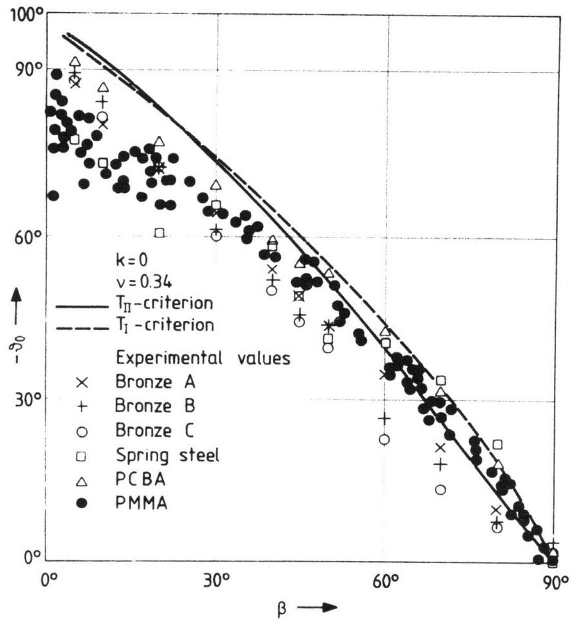

Fig.3. The initiation angle of the crack θ_0, versus the crack inclination angle, β^o, for plates of different materials, under conditions of plane stress, submitted to simple tension at infinity.