FATIGUE LIFE ASSESSMENT IN CRUCIFORM JOINTS

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Results of fatigue life assessment in cruciform joints using Fracture Mechanics are reported. Fatigue life was computed as a function of initial flaw size, weld geometry, plate thickness and loading mode. The Bouckner weight function method was used for stress intensity factor computation.

Plate thickness and loading mode (tension and bending) were the main parameters of fatigue life and their influence was quantified for design purposes, introducing in the S-N curve equation a correction term. The ratio attachment thickness over main plate thickness has little influence on fatigue strength. The influence of weld toe radius is critical and will be studied in detail in further work.

INTRODUCTION

Design against fatigue is one of the most important points to consider in welded joints subjected to dynamic loadings. Design codes such as BS 5400 for metallic bridges (1), BS 6235 for offshore structures (2) and the recently developed Eurocode (3) provide a series of S-N curves and design procedures applicable to a wide range of weld details and loading conditions. The designer can use this data to define the appropriate dimensions of the joint (i.e. thickness, width, weld size, etc.) or characterize the allowable stress spectra in the joint.

The present fatigue design curves are usually applicable to a specific set of weld details and are normally independent of thickness and loading mode. One exception is the BS6235 code for offshore structures, who assumes that fatigue life decreases with increasing thickness of the main plate. However, this assumption is still

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open to question since the experimental evidence of the thickness and geometry effect is contradictory (4). For example, one of the important factors to take into account is the residual stress pattern which depends on thickness and can be reduced by a stress relieving treatment as pointed out by Maddox (5). Also the loading mode (i.e. tension or bending) has a different influencing factor on thickness as reported recently in preliminary work carried out by the authors (6, 7).

The fatigue design curves of the codes were derived from test data obtained in medium to high thickness specimens and under predominantly tensile loading. In practice most welded structures are fabricated from thin sections as is the case of chassis, frames, some attachments in metallic bridges, cranes and in general metal construction work.

In thin sections bending loading is predominant due to local stiffening and geometric effects in the weld metal who may create significant bending stresses. Recent results published by the authors (7, 8) have shown that in non load carrying fillet joints the bending stress vary with thickness. For low thickness values fatigue life could be considerably higher than the values given by the fatigue design curves in the codes (7, 8, 9). Hence this indication is particularly important in the design of thin walled structures where the use of existing design curves may lead to very conservative designs.

In previous work fatigue life was assessed applying a fracture mechanics model based on 2D stress intensity computation by the Albrecht method (10). Only a limited range of weld geometrics was tested varying the weld angle, δ, weld size, LG, plate thickness, B, and attachment plate thickness, B. The results were plotted as a function of initial flaw size and a range of S-N crack propagation curves were obtained. Correlation with available experimental data was good (9).

The results obtained in the preliminary investigations referred above have emphasized the need for a more comprehensive computational procedure applying other methods such as the weight function and J integral. Also a fatigue life assessment is required in a more wide range of connection types and main plate thickness. The influence of weld toe radius must also be taken into account in the analysis.

The present paper reports results of a fatigue life assessment carried out on cruciform joints and using...
Fracture Mechanics. Stress intensity factors were computed with the Bueckner method function of the main plate thickness and loading mode. Fatigue life data was obtained as a function of initial flaw size and main plate thickness keeping both the toe to toe distance and weld toe radius constant.

COMPUTATIONAL ANALYSIS

For stress intensity factor computation the stresses were computed using the typical 2D finite element mesh illustrated in Fig. 1. Eight node isoparametric elements were used with two degrees of freedom in each node (displacements along the two coordinate axis). Tension loading was simulated by a uniformly distributed load along the line 286-294 (Fig. 1). Cantilever loading was represented by a vertical load applied at node 294 and at the built-in end along the far left line 1-9. The attachment was assumed both free to rotate or fixed (built-in) along the top line 117-193 (Fig. 1). The stresses and the geometrical factor Y for the stress intensity computations were calculated along the vertical sections starting at nodes 82 and 223 where fatigue cracks propagate along the plate thickness and from the weld toe. The results have indicated that the values of Y were very close for both sections either in tension or cantilever bending.

The geometric variables studied are indicated in Fig. 2. The joint is cruciform, non-load carrying with a full penetration weld as represented in the finite element mesh (Fig. 1). Besides the variables referred in the figure the computational studies include an extensive number of connection types, with different values of weld toe radii, plate length A, and ratio L/B where L is the toe to toe distance (B1+2L).

Stress intensity factors were obtained with the weight function method (11). In this method the values of Y are calculated from a weight function m(x,a) given by the general equation

\[ m(x,a) = \frac{E}{2K_0} \frac{3v(x,a)}{3a} \]  

where v are the displacements in the crack line and in the y direction of any point x, for a crack of length a, K0 is \( \sqrt{a} \), the basic stress intensity solution for a infinite plate with a central crack, and E is the Young's modulus. Y is obtained from the following equation:

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\[ Y = \int_{0}^{\frac{p(x)}{a}} m(x, a) \, dx \]  

where \( p(x) \) is the real stress distribution in the joint along the crack line for a particular crack length, \( a \), and loading mode. The values of \( p(x) \) were given by the finite element program (Fig. 1) and \( m(x, a) \) is the weight function equation for a plain bar with a lateral crack without the weldment. The equation for \( m(x, a) \) was taken from the compendium solutions presented by Parker (12).

Before the values of \( Y \) were obtained for all the cases referred in Fig. 2 the method was tested in some simpler geometries for which accurate solutions in the literature are available. For example the present solutions gave errors below 0.5% compared with the Brown (13) solutions for bars in tension and bending.

At this stage of the analysis no correction was taken in the program to account for crack tip ellipticity. Work is now in progress to take into account this effect. Hence the stress intensity results are only valid for straight-through cracks and therefore will represent the maximum values of \( K \) in the joint.

For all the cases in Fig. 2, \( Y \) was plotted against the main geometrical variables. Thus, Fig. 3 shows a typical plot \( Y \) against crack length \( a/B \) in a joint with 150 mm plate thickness. The same curve was fitted for the results obtained considering the attachment free or fixed. Near the crack tip \( (a/B < 0.04) \) \( Y \) is slightly greater in bending than in tension but outside the stress gradient created by the weldment, the equation for \( Y \) in tension lies above the one in bending (Fig. 4) as expected from the well known base stress intensity factor solutions (13, 14).

However, it is likely that if the plate length \( A \) is increased \( Y \) could vary with the stiffness of the attachment since the bending stress distribution in the joint may change considerably.

The combined influence of \( B_1/B \) and stiffness of the attachment is shown in Fig. 4, both in tension and bending, for main plate thicknesses of 2 and 4 mm and \( LG/B = 0.5 \).

Again no significant variation in the results was obtained in the range of \( B_1/B \) values between 0.5 and 5 both for the free and fixed attachment. These results
confirm the trend reported in previous work (6,9) that $K$ only increases slightly with $B_1/B$ and has a considerable variation with $LG/B$.

For 150 mm plate thickness (Fig.3) the values of $Y$ are very much above the results for 2 and 4 mm plate thickness (Fig.4). In both cases near the crack tip the variation of $Y$ is very steep. $Y$ decreases with the ratio $a/B$ and approach the values given by the basic plate solutions outside the stress gradient at the weld toe. The selected basic solutions were

\[
\text{Tension} \quad (13) \\
Y = 1.12 -0.23a + 10.6a^2 - 21.7a^3 + 30.4a^4 \quad 0.12 < a < 0.65 \quad (3a)
\]

\[
\text{Bending} \quad (14) \\
Y = 0.97 - 0.955a + 6.786a^2 - 6.667a^3 \quad 0.18 < a < 0.65 \quad (3b)
\]

where $a = a/B$

The geometrical factor $Y$ was correlated with the main plate thickness. For each bending mode (tension and bending) it was possible to obtain a single correlation curve valid in the entire thickness range (2 to 150 mm). The equations are:

- **Bending with fixed attachment ($B_1/B = 1$)**
  \[
  \log Y = -0.2104 - 0.2658 \log (a/B) + 0.0352 \log(a/B)^2 \quad (4a)
  \]
  for $a/B < 0.12$ and a correlation coefficient, $r = 0.990$

- **Tension with fixed attachment ($B_1/B = 1$)**
  \[
  \log Y = 0.0206 + 0.0245 \log (a/B) + 0.101 \log(a/B)^2 \quad (4b)
  \]
  for $a/B < 0.18$ and a correlation coefficient, $r = 0.9923$

Similar equations were obtained for the other cases (free attachment and values of $B_1/B = 0.5$ and 5). Since the difference in the values of $Y$ in relation to equations (4a) was less than 1%, and also the best correlation factors were obtained with equations (4a) and (4b), these were used in the fatigue life assessment whose results will be presented in the next section.

The life in fatigue crack propagation was calculated by integration of Paris law

\[
N = \frac{1}{C \Delta e B} \left( \frac{m}{2} \right)^{-1} \quad (5)
\]

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where \( I \) is the crack propagation integral and \( C \) and \( m \) are both the constant and exponent of Paris law. The crack propagation integral is given by the equation

\[
I = \int_{a_i}^{a} (Y \sqrt{a})^{-m} \, da
\]

(6)

where \( a = a/B \) and \( a_i = a_i/B \) where \( a_i \) is the initial crack size assumed in the calculation.

Equations (4a,b) were substituted in equation (6) in their range of validity of values of \( a = \) 0.12 in tension and 0.18 in bending. For higher values of \( a \) the basic solutions for \( Y \) were given by equations (3a,b).

The values of \( C \) and \( m \) in equation (5) were the usual values quoted in the literature for structural carbon steels in plane strain (\( m = 3.1 \) and \( C = 1.33 \times 10^{-13} \) mm/cycle; \( N_{\text{mm}} = 3/2 \) ) (13).

Numerical integration of equation (6) was carried out by Simpson's rule and then substituted in equation (5) to obtain fatigue life function of nominal stress and thickness.

RESULTS AND DISCUSSION

Two plots nominal stress (fatigue strength) \( \sigma \) against thickness \( B \) are shown in Figs. 5 and 6. These curves are the solutions of equation (5) for \( N = 10^6 \) cycles (Fig.5) and \( 2 \times 10^6 \) cycles (Fig.6) typical fatigue life values in welded structures. The influence of the initial flaw size can also be seen in these plots since the results were obtained for \( a_i \) values of 0.1; 0.2; 1 and 2 mm.

The results show that for the same initial flaw size and fatigue life the fatigue strength is lower in tension than in bending and decreases with increasing thickness, when the initial flaw size is below 0.2 mm. The reduction in nominal stress varies between 20 to 40% both in tension and bending.

When the initial flaw size is above 1 mm and hence the crack tip is near the limit of the stress gradient at the weld toe, fatigue strength increases with thickness, up to the medium thickness limit, and decrease slightly after. These results indicate that when the initial flaw size is outside the stress gradient at the weld toe, fatigue life increases with thickness.

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For every flaw size analysed fatigue life in bending is greater than in tension and for the smaller flaw sizes the difference in fatigue life increases when the thickness decreases. Hence the stress gradient at the weld toe, steeper in the lower thickness values, is the controlling parameter of crack propagation for small fatigue cracks.

The stress gradient at the weld toe depends on the radius of curvature, \( \rho \). In this work, \( \rho \) was not taken constant but is defined by the distance from the weld toe to the nearest point of stress calculation as defined in the finite element mesh. In this mesh type (Fig. 1) the equivalent \( \rho \) value varied between 0.012 to 0.070 mm depending on the thickness of the main plate. Work is now in progress to assess the influence of \( \rho \) on fatigue life assuming typical values measured in welded joints.

For the crack size of 0.2 mm, the most common flaw size found usually in welded joints, (16) the S-N crack propagation curves for the thicknesses of 2, 20 and 150 mm both in tension and bending are plotted in Fig. 7. These curves were obtained from curves \( \sigma \) vs. \( B \) such as Figs. 5 and 6. It is seen that fatigue life decreases in tension compared to bending and also decreases with increasing thickness. In the range of thicknesses from 2 to 150 mm the decrease in fatigue strength is more than 50\% thus covering several class details design curves in the codes (1-3). Therefore for lower thicknesses and bending loading the designer should use a higher design curve.

To decide upon the acceptability for design purposes of curves such as plotted in Fig. 7 an extensive experimental programme is now in progress in collaboration with the WI. In this project a detailed study of the thickness effect in fatigue strength is to be carried out.

With the results obtained in Figs. 5 and 6 a correction factor for the thickness effect could be included in the S-N curve equation. Similar correction factors were proposed before by Gurney (17) and Berge (18). For the crack size of 0.2 mm the correction factors \( C_T \) in the S-N curve are

\[
\Delta \sigma^N = C_T
\]

where

\[
C_T = 10^{11} (40.0886 - 2.2735 \times 10^{-2} B - 5.92 \times 10^{-4} B^2 + 1.906 \times 10^{-6} B^3)
\]

for bending and with a correlation factor \( r = 0.995 \) and

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\[ C_T = 10^{11} (24.5148 - 0.93618B + 2.006 \times 10^{-2}B^2 - 1.83 \times 10^{-4}B^3 + 5.63 \times 10^{-7}B^4) \]

for tension, with a correlation factor, \( r = 0.999 \). These equations were derived for \( B = B_0 \), \( L / B = 0.5 \) and a fixed attachment. However, very close equations were obtained for the other cases since the differences in fatigue life were negligible.

**CONCLUSIONS**

1- Calculations of fatigue strength in cruciform joints using Fracture Mechanics have shown that fatigue life is greater in bending than in tension for a range of plate thicknesses between 2 and 150 mm.

2- The ratio attachment thickness over main plate thickness has very little influence on the fatigue strength of non load carrying cruciform joints.

3- For a plate length and attachment length of 10 times the main plate thickness no significant variation in fatigue strength was obtained when the stiffness of the attachment was changed.

4- Fatigue strength has decreased with increasing plate thickness when the initial crack size is below 0.2 mm. For greater values of initial crack size fatigue life has increased with plate thickness up to the medium thickness range and then decreased for very high thicknesses.

5- The stress gradient at the weld toe is the controlling parameter of crack propagation and detailed studies are in progress to study its variation with weld geometry and toe radius.

6- Correction factors for the thickness effect on fatigue life were derived. These factors can be included in the S-N design curve and are valid both for tension and bending in the thickness range 2 to 150 mm.
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Fig. 1—Finite element mesh used for stress intensity factor computation

\[ A = 10B \]
\[ B_1/B = 0.5; 1.5 \]
\[ B = 2, 4, 6, 12, 20, 40, 100, 150 \]
\[ LG/B = 0.5 \]
\[ \theta = 45^\circ \]
\[ P \to 0 \]

Fig. 2—Geometrical variables of the cruciform joint

Fig. 3—Geometrical factor \( Y \) against \( a/B = 150 \) mm.

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Fig. 4 - Geometrical factor $Y$ against $a$. $B = 12$ mm. Free and fixed attachment.

Fig. 5 - Nominal stress $\sigma$ against plate thickness $B$. $N = 10^3$ cycles. Carbon steels. Cruciform joint.
Fig. 6 - Nominal stress $\sigma$ against plate thickness $B$. $N=2\times10^6$ cycles. Carbon steels.

Fig. 7 - S-N curves as a function of plate thickness $B$. $a=0.2$ mm, $R=0$. Carbon steels.