INTRODUCTION TO SESSION NINE: THIN SHEET FRACTURE MECHANICS

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ABSTRACT

Test methods for the determination of R-curves on centre cracked tension specimens are briefly described. Special emphasis is given to the compliance method and the potential method.

KEYWORDS

R-curve; compliance method; potential method; centre cracked tension specimen; size effect; minimum size requirements; buckling.

BACKGROUND

The fracture mechanics of thin-walled structures is characterized by two specific areas of problems:

- Problems related to *large amounts of crack growth*.
- *Buckling* under tensile loads, when width and crack length are large and when thickness and Young's modulus are small.

Sections 2 and 3, respectively, are dedicated to both of these subjects.

In thin-walled structures often high-strength materials are used. Initiation of crack growth may occur at relatively low load levels, e.g. under conditions which can be characterized by LEFM. As thickness is small plane stress conditions prevail. Consequently, the material's resistance (K_R if expressed in terms of LEFM) against crack growth may increase with increasing crack growth far beyond the initiation value, Fig. 1.
This strong rise of K_R has the following consequences:

- Designing against initiation K_o is extremely conservative; it is worth exploiting the true load carrying capacity K_c.

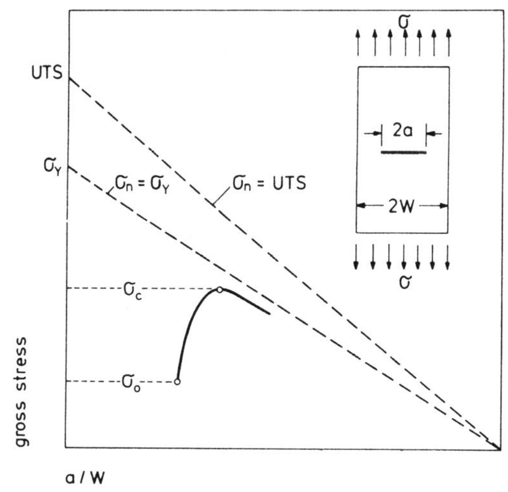

Fig. 1: Possible location of R-curve with respect to general yield.

- When doing so one faces a pronounced variation of K_c with the geometry of the structural part (including crack length) and the geometry of loading (homogeneous or inhomogeneous stress distribution, point forces ...). Fig. 2 shows the variation of K_c with width, crack length, and thickness for a given geometry.

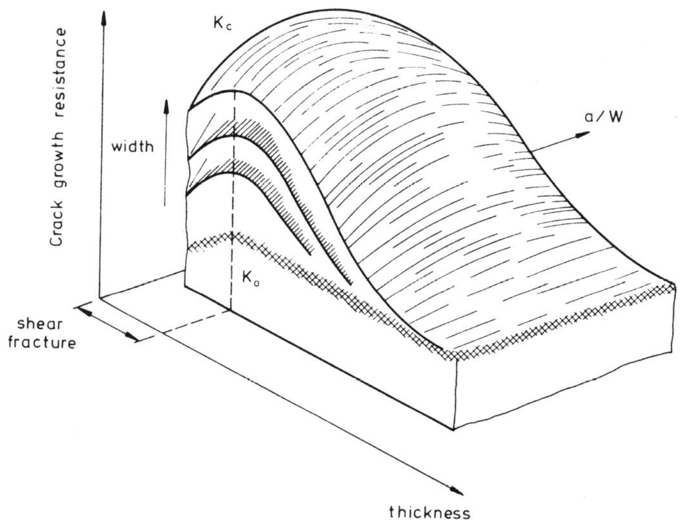

Fig. 2: Variation of K_c with thickness, width, and crack length.

- The aforementioned geometry dependence of K_c requires a prediction technique to avoid excessive testing. The R-curve concept is one such method.
- Whereas the initial part of crack growth is usually well in the regime of LEFM the later stages of crack growth may reach or pass over the limits of applicability of LEFM. It is therefore desirable to know up to which degree of plasticity ($\triangleq F/F_L$; F: actual load, F_L: plastic limit load) the stress intensity can serve as a correlation parameter for crack growth. Two other papers of this conference deal with this problem (Kaiser and Hagedorn (1980); Schwalbe and Setz (1980)).

THE R-CURVE METHOD

Test Techniques

A standard test procedure for the R-curve determination was developped by ASTM (1979). In the present paper some details of a more recent proposal will be described. The complete test procedure is being published elsewhere in a German and an English version (Schwalbe and co-workers (1980); NN).

Thin sections are usually tested as centre cracked tension specimens. The buckling occuring due to negative transverse stresses must be restrained by anti-buckling guides. This problem is briefly treated in the third chapter.

The size requirements are such that general yield of the net section is excluded. For this purpose, only data for which the net section stress, σ_n meets the condition

$$\sigma_n \leq 0.9\sigma_{0.2} \tag{1}$$

should be used for the generation of the R-curve. The net section stress has to be calculated using the *actual* net section $2B(W-a_{phys})$. If during the test only effective crack lengthsare determined the physical crack length can be obtained through

$$\begin{aligned} a_{phys} &= a_{eff} - r_y \\ r_y &= \frac{1}{2\pi}\left[\frac{K}{\sigma_{0.2}}\right]^2 \end{aligned} \tag{2}$$

The factor 0.9 in Eq (1) is used to keep some distance from the general yield condition. If K_{max} is the maximum stress intensity which has to be evaluated the minimum width is

$$2W_{min} = 5.3\left[\frac{K_{max}}{\sigma_{0.2}}\right]^2 \tag{3}$$

or alternatively, the maximum K which can be evaluated on a specimen of a given width 2W is

$$K_{max} = 0.43\sigma_{0.2}\sqrt{2W} \tag{4}$$

Here it is presumed that $K = \sigma\sqrt{\pi a_{phys}}$.

For the crack length determination two alternative methods are recommended:

Most convenient is the *compliance method* as it doesn't require additional instrumentation. From the load-COD-record the effective crack length is obtained by drawing secants through the origin, Fig. 3. Of one needs information about the actual physical crack length the partial unloading technique can be applied: the unloading lines characterize the elastic compliance of the specimen, Fig. 3. From the compliance the physical or the effective crack length can be obtained via an experimental calibration curve or the relation (Eftis and Liebowitz (1972))

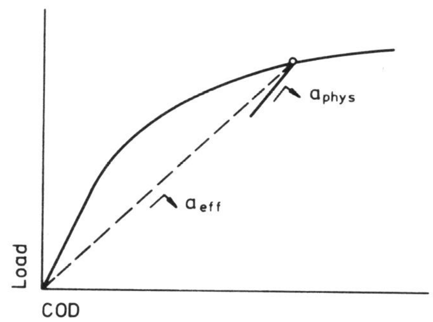

Fig. 3: Determination of effective and physical crack length from load-COD-record.

$$\frac{EB(2v)}{F} = \sqrt{\frac{\frac{\pi a}{2W}}{\sin\frac{\pi a}{2W}}}\left\{\frac{4W}{\pi y}\,\mathrm{arcosh}\left[\frac{\cosh\frac{\pi y}{2W}}{\cos\frac{\pi a}{2W}}\right] - \frac{1+\nu}{\left[1+\left\{\frac{\sin\frac{\pi a}{2W}}{\sinh\frac{\pi y}{2W}}\right\}^{2}\right]^{0,5}} + \nu\right\}\frac{y}{W} \qquad (6)$$

which is valid for the crack length range $0.2 \leq a/W \leq 0.8$. Theoretical and experimental calibration curves exhibit a certain offset (Fig. 4) which can be determined and hence corrected by determining the compliance from the initial straight line portion of the load-COD-record; this is the compliance for the fatigue pre crack length $a = a_o$.

As centre cracked tension specimens are not very compliant the sensitivity of the compliance method is rather limited. In the author's experience the *potential method* yields more consistent results. Fig. 5 shows a set up for potential measurements which has the advantage that a theoretical calibration formula by Johnson (1965) can be used for the crack length determination:

$$a = \frac{2W}{\pi} \arccos \left[\frac{\cosh \frac{\pi y}{2W}}{\cosh \left\{ \frac{U}{U_o} \operatorname{arcosh} \frac{\cos \frac{\pi y}{2W}}{\cos \frac{\pi a_o}{2W}} \right\}} \right] \qquad (7)$$

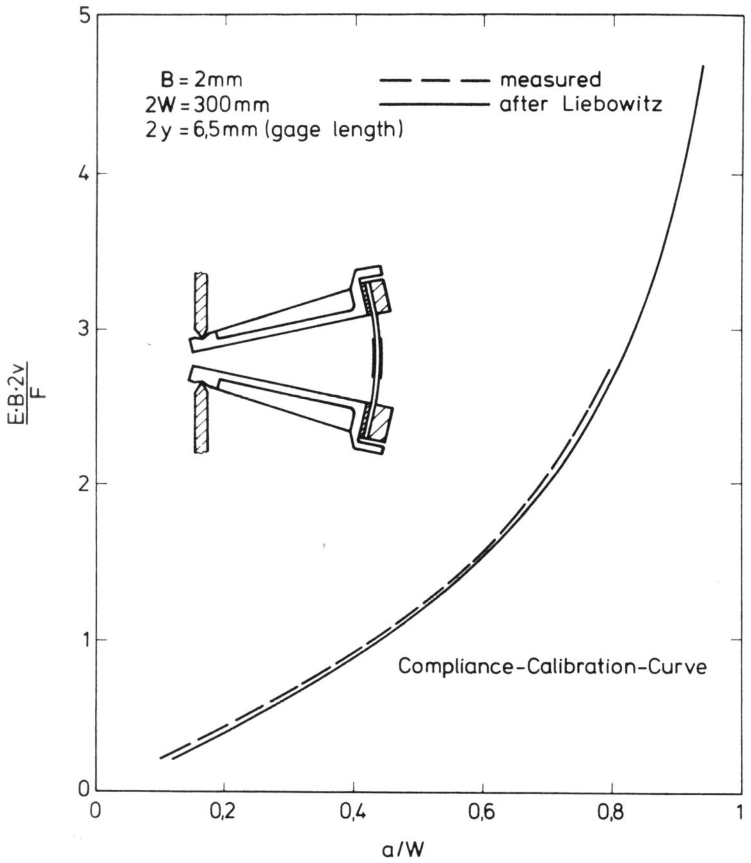

Fig. 4: Theoretical and experimental compliance calibration curve.

In Fig. 5 Eq. (7) is plotted along with an experimental calibration curve. Eq. (7) has the advantage that it can be used for single edge notched and bend specimens as well. Furthermore, due to the normalisation (U/U_o) Eq. (7) is exact for $a \to a_o$.

By the potential method the physical crack length is determined; the effective crack length is given by

$$a_{eff} = a_{phys} + r_y \qquad (8)$$

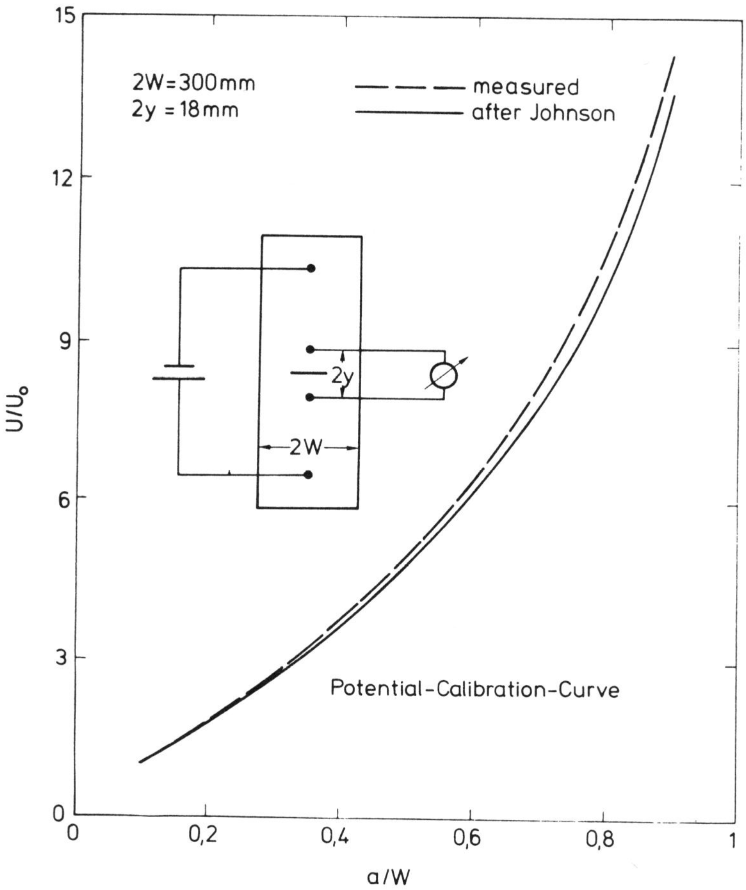

Fig. 5: Theoretical and experimental potential calibration curve.

In the case of CCT specimens the Irwin correction works up to the load level given by Eq. (1). With the effective crack length the effective stress intensity

$$K_{eff} = \sigma\sqrt{\pi a_{eff}}\sqrt{\sec\frac{\pi a_{eff}}{2W}} \tag{9}$$

is calculated, and the plot $\Delta a_{eff} = a_{eff} - a_o$ versus K_{eff} is the R-curve.

In Fig. 6 the potential based and the compliance based R-curve for a high-strength aluminum alloy are compared. The larger scatter of the compliance method is obvious.

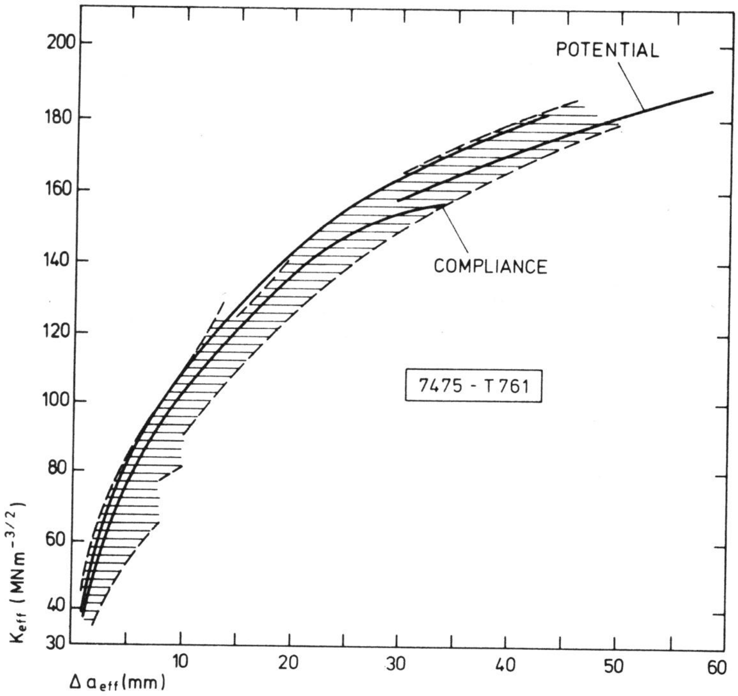

Fig. 6: Scatterbands of the compliance based and potential based R-curve of an aluminum alloy.

Application

The R-curve is believed to serve as a tool for the prediction of instability; for this purpose load controlled situations are often considered. Instability occurs when the energy release rate increases more rapidly with increasing a_{eff} than the crack growth resistance does. In stress intensity terms this condition reads:

$$\frac{\partial K_{appl}}{\partial a_{eff}} \geq \frac{\partial K_R}{\partial a_{eff}} \tag{10}$$

where K_R is the crack growth resistance obtained through Eq. (9).

When the calibration function of the structural part under consideration is known the tangency construction can be done easily by varying the applied stress; a family of K_{app}-curves is generated thereby.

This procedure is facilitated significantly by generating one K_{app}-curve with $\sigma = 1$ for each width. Thus, the parameter σ disappears. If the K_{app}-curves and the R-curve are drawn on semilog paper a vertical displacement of the R-curve relative to the K_{app}-curves yields immediately the critical stress if the common tangency point of the R-curve and the K_{app}-curve of the appropriate width is determined (Creager (1973)).

BUCKLING

Small thickness, low modulus of elasticity, large width, large crack length, and high load can cause buckling of the specimen or structural part. Buckling of the specimen must be avoided by anti buckling guides which should consist of two pieces only, one for the front face and one for the rear face of the specimen. Nevertheless, it is advisable to check if the specimen is really unbuckled. This can be done by partially unloading the specimen. If buckling occurs the unloading or reloading curves may have nonlinear parts. In addition, a comparison of a_{phys} determined by unloading with the physical crack length measured by an independent method (potentialmethod, microscope) may give an information about possible buckling. Unfortunately, quantitative rules are not available at present.

Buckling causes a loss $\Delta\sigma_{cB}$ of the load carrying capacity the amount of which depends on the boundary conditions of the sheet under consideration. For sheets with free long edges Fig. 7 shows an empirical relation for the buckling induced reduction of critical stress (Allen (1970)).

NOMENCLATURE

a	crack length
a_o	fatigue precrack length
a_{eff}	effective crack length
a_{phys}	physical crack length
B	thickness
E	Young's modulus
F	load
K	stress intensity calculated with a_{phys}
K_{appl}	applied stress intensity
K_{eff}	effective stress intensity calculated with a_{eff}
K_{max}	maximum stress intensity in an R-curve test
K_R	crack growth resistance
r_y	plasticity correction
U	current potential drop
U_o	U for $a = a_o$
v	half crack mouth opening
W	specimen half width
y	half gage length for measurement of v or U
ν	Poisson's number

σ applied stress

σ_n net section stress

$\sigma_{0.2}$ 0.2% yield strength

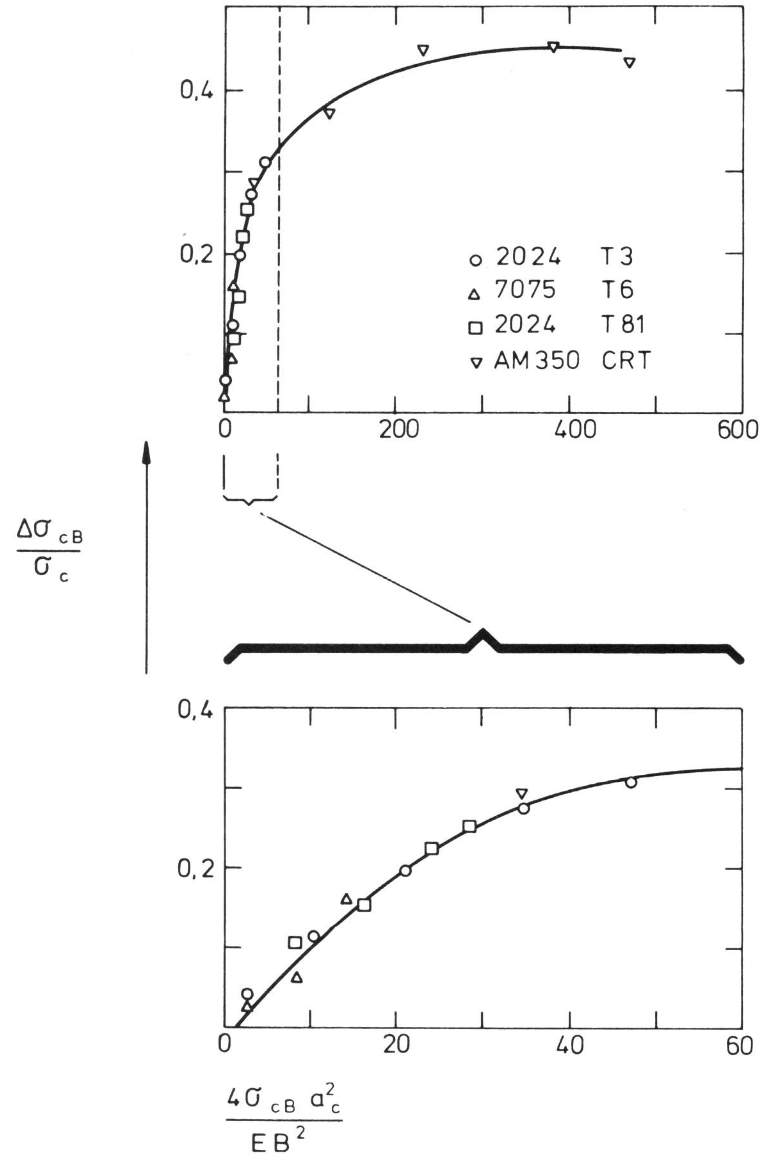

Fig. 7: Loss of load carrying capacity due to buckling [9].

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