

# Strength and Fracture Properties of Structural Graphite Materials

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**Abstract.** The results of experimental studies of elastic properties, strength and fracture toughness characteristics of isotropic structural graphite of different grain sizes are presented and discussed. The comparative analysis of experimentally determined values of strength and fracture toughness of different types of graphite is carried out. The method for the determination of crack length and fracture toughness of materials during the crack growth under the conditions of bending is considered. In the experiments, the unsteady or catastrophic crack growth from short initial cracks was observed. The condition of unsteady crack growth is determined. The processes of unsteady crack growth are analyzed for different loading conditions. It is shown that in the case of a short crack some part of the stored energy transforms into kinetic energy, which can be described by corresponding equation.

## Introduction

Graphite materials are widely used as structural members in different branches of techniques, which work under various conditions of thermal and mechanical loading. These heterogeneous materials have small nonlinearity of stress-strain curves and contain pores, micro-cracks and other defects of structure in considerable extent, which can't be eliminated by supplementary treatment, such as high temperature pressing. They are possessed of another type of physical nonlinearity, connected with the dependence of their mechanical properties on the type of loading [1]. For these materials the stress-strain curves under uni-axial tension, uni-axial compression, shear and other types of stress state differ significantly, and instead of a single curve of dependence between the effective von Mises stress and equivalent strain intensity, there is a fan of effective stress-strain curves for different types of loading. In these materials, the process of bulk deformation is related with the shear deformations. The corresponding constitutive relations to describe these effects were proposed [2]. Different crack problems for plane stress and plane strain conditions were considered and it was shown that in some cases the traditional approaches to the solution of crack problems can't be used [3,4]. In the present paper, the results of experimental studies of strength and fracture properties of graphite materials and corresponding theoretical analysis of crack growth are presented to substantiate the applicability of linear fracture mechanics approaches to the analysis of fracture characteristics of materials, which demonstrate rather unusual behavior.

## Elastic and strength properties of graphite

The polished samples of the fine grained graphite materials MPG-6 and MPG-8 of grain sizes about 0.09 mm and the coarse grained graphite VPP of the grain size about 2.3 mm were tested under the temperature 23°C. The Young modulus, the Poisson's ratio and the strength properties of graphite were determined on the base of the tests of smooth specimens under the conditions of tension. The specimens had the thickness of 5 mm, the width of 9 mm and the gage-length of 40 mm. The plates of wood were glued to the sample surface to prevent the destruction of samples in the pneumatic grips of testing machine. Instron extensometers were used for the determination of longitudinal and lateral deformation. The stress – strain diagrams were linear ones with the small deviation from the linearity near the breaking stress. The strength properties

were determined under the conditions of bending, too. The experimental values of elastic and strength properties of graphite materials are given in Table 1.

Table 1. Elastic and strength properties

Material	Strength [MPa]		Young modulus $E [10^4 \text{ MPa}]$	Poisson's ratio, $\nu$
	Bending	Tension		
MPG-6	65.5	31.6	0.884	0.204
MPG-8	87.7	44.5	2.173	0.306
VPP	24.4	17.0	0.934	0.231

### Fracture toughness characteristics

For the determination of the fracture toughness characteristics of graphite, the three-point bending tests of cracked specimens with different depth of cuts were carried out (Fig. 1).

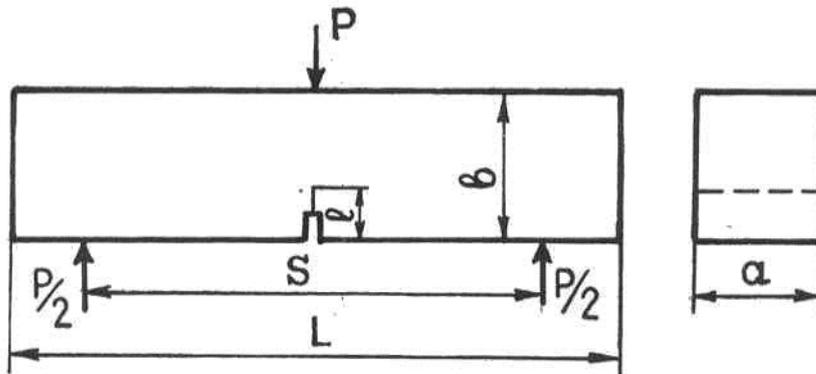


Fig. 1. Specimen for crack toughness testing.

The specimens had the following dimensions:  $b = 14.80 \div 15.00$  mm,  $a = 14.92 \div 15.00$  mm,  $L = 80$  mm,  $S = 60$  mm. The values of the intensity of strain energy release can be determined on the base of the compliance method and corresponding analytical formula for energy release [5]

$$G = \frac{P_c^2}{2ba} \frac{dC}{d\omega}, \quad (1)$$

where  $\omega = \ell/b$  is relative crack length,  $P_c$  – critical load corresponding to the initiation of crack growth and  $C$  – compliance of notched specimen. For this type of loading we have the following formula for stress intensity factor [6]:

$$K_I = \frac{6M\sqrt{\ell}}{b^2a} Y(\omega), \text{ where } Y(\omega) = A_0 + A_1\omega + A_2\omega^2 + A_3\omega^3 + A_4\omega^4. \quad (2)$$

For the conditions of three-point bending  $M = \frac{PS}{4}$ ,  $S/b = 4$ ,

$$A_0 = 1.93; A_1 = -3.07; A_2 = 14.53; A_3 = -25.11; A_4 = 25.80;$$

There is a simple relation between the intensity of stress energy release and the critical value of stress intensity factor  $K_{Ic}$ , which is a material characteristic

$$G = \frac{1-\nu^2}{E} K_{Ic}^2, K_{Ic} = \frac{3P_c S \sqrt{\ell}}{2b^2 a} Y(\omega). \quad (3)$$

The specimens were polished before tests. The tests were carried out at constant cross-beam velocity of 2 mm/min, and the diagrams were linear up to the maximum load corresponding to the crack growth initiation. Two of these diagrams obtained for graphite MPG-6 are shown in Fig. 2. The diagrams obtained for other graphite materials were similar.

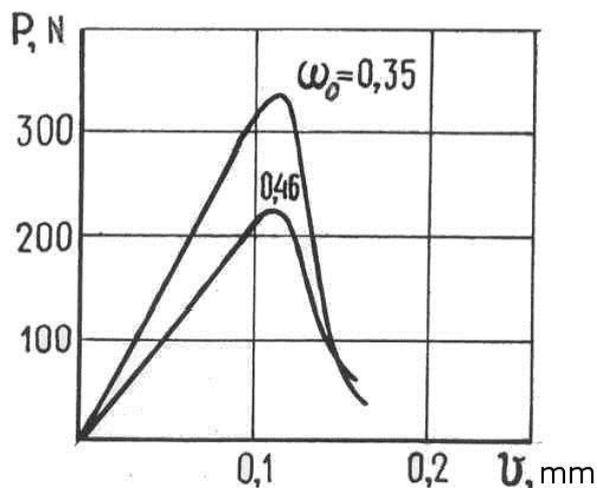


Fig. 2. Typical load-flexure curves for different initial crack length.

A series of five specimens were tested for each initial crack length. The experimental values of fracture toughness are given in Table 2.

Table 2. Mean values of fracture toughness for different initial crack length

Material	$\ell$ , [mm]	$\omega_0$	$P_c$ , [N]	$K_{Ic}$ , [N/mm <sup>3/2</sup> ]	$G$ , [N/mm]
MPG-6	2.33	0.158	583	41.2	1.84
	3.35	0.224	477	41.2	1.85
	5.01	0.349	352	42.0	1.92
	6.97	0.464	248	40.5	1.79
MPG-8	2.84	0.191	674	53.2	1.18
	4.93	0.332	455	52.1	1.13
VPP	2.29	0.152	552	38.4	1.55
	3.37	0.224	502	43.1	1.88

On the base of experimental data, the comparison between the strength and fracture toughness of various types of graphite can be carried out. The strength of graphite MPG-6 is about two times higher than the strength of VPP graphite but the fracture toughness or the resistance of the material to the crack growth is almost the same. The strength of MPG-8 is about 2.5 times higher than the strength of VPP but the fracture toughness is only in thirteen per cent higher than the value obtained for VPP. Thus these graphite materials have almost the same fracture toughness characteristics; despite the strength properties are quite different.

In the case when the length of initial crack is not very short, it is possible to determine a number of values of fracture toughness during the test of a single specimen. The bending is a type of hard loading where the displacement of the cross-beam is controlled. So, it is possible the repeated loading of a specimen and one can determine a number of values of critical loads for various crack lengths, which can be measured on the polished surface of a specimen. One of these diagrams obtained for graphite MPG-6 is shown in Fig. 3.

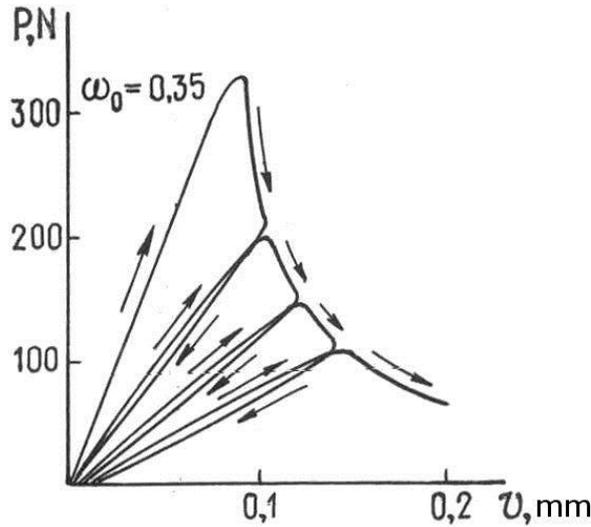


Fig. 3. Load-flexure diagrams for repeated loading.

This procedure was used for the analysis of the process of steady crack growth. According to this procedure, the fracture toughness can be determined during the crack growth. The values of load under repeated loadings were almost the same as the values of load before unloading. The values of fracture toughness determined during the test of a single specimen were in the range of experimental data scattering.

According to Eqs. 1 – 3, the differential relation between the compliance and relative crack length can be obtained

$$\frac{dC}{d\omega} = \frac{9S^2\ell(1-\nu^2)}{2b^3aE} [Y(\omega)]^2 \quad (4)$$

The load is not entered into this equation. The compliance depends only on two material properties – Young modulus and Poisson's ratio, and specimen dimensions. This equation can be integrated

$$C = C_0 + \frac{9(1-\nu^2)S^2}{2Eb^2a} [F(\omega) - F(\omega_0)] \quad (5)$$

where  $C_0$  is the compliance of the specimen with initial relative crack length  $\omega_0$ ,

$$\begin{aligned}
F(\omega) = & \omega^2 \left[ \frac{1}{2} A_0^2 + \frac{2}{3} A_0 A_1 \omega + \frac{1}{4} (A_1^2 + 2A_0 A_2) \omega^2 + \right. \\
& + \frac{2}{5} (A_1 A_2 + A_0 A_3) \omega^3 + \frac{1}{6} (A_2^2 + 2A_0 A_4 + 2A_1 A_3) \omega^4 + \\
& \left. + \frac{2}{7} (A_1 A_4 + A_2 A_3) \omega^5 + \frac{1}{8} (2A_2 A_4 + A_3^2) \omega^6 + \frac{2}{9} A_3 A_4 \omega^7 + \frac{1}{10} A_4^2 \omega^8 \right].
\end{aligned} \tag{6}$$

It is possible to accept  $C_0$  in Eq. 5 equal to the compliance of un-notched specimen ( $\omega_0 = 0$ ),  $C_0 = S^3 / 4Eb^3a$ . Then for the compliance of a notched specimen we can obtain the following equation:

$$C(\omega) = \frac{S^3}{2Eb^2a} \left[ \frac{S}{2b} + 9(1-\nu^2)F(\omega) \right]. \tag{7}$$

The described method can be used for the determination of the values of stress intensity factor during the crack growth and to study the influence of crack growth velocity on the fracture toughness of a material.

### Steady and dynamic crack growth

In the experiments, it was observed unsteady or catastrophic crack growth from short initial cracks. This effect can be analytically approved and the conditions for steady crack growth under bending can be determined. Eq. 3 establishes the relation between critical load and corresponding crack length. This dependence can be represented in non-dimensional form

$$\frac{3P_c S}{2K_{Ic} b^{3/2} a} = \left[ Y(\omega) \sqrt{\omega} \right]^{-1}. \tag{8}$$

To each value of critical load, we have corresponding value of critical flexure according to relation

$$v_c = C(\omega) P_c. \tag{9}$$

From Eqs. 7 – 9, the relation between non-dimensional critical flexure and relative crack length can be determined

$$\frac{v_c 3E\sqrt{b}}{SK_{Ic}} = \left[ \frac{S}{2b} + 9(1-\nu^2)F(\omega) \right] \left[ Y(\omega) \sqrt{\omega} \right]^{-1}, \quad S = 4b. \tag{10}$$

Eqn. 8 and Eqn. 10 can be regarded as parametric dependencies for critical load and critical flexure on the crack length, and corresponding graphs for the critical load (curve 1) and the critical flexure (curve 2) are shown in Fig. 4.

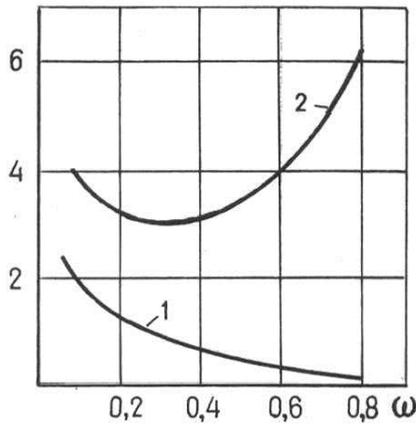


Fig. 4. The diagrams of dependence of critical load (curve 1) and critical flexure (curve 2) on the relative crack length.

On the base of Eqs. (7) and (9) or using this two graphs, the relation between critical load and critical flexure can be established, which is shown in Fig. 5. The dependence of critical load on the critical flexure is not even and it is not simple, too. From this graph, the condition of unsteady crack growth can be determined. Strait line corresponds to the dependence between load and flexure for un-notched specimen.

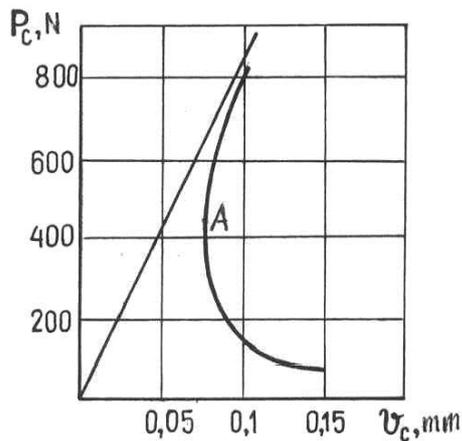


Fig. 5. The dependence of critical load on critical flexure.

The point A corresponds to the relative crack length equal to  $\omega = 0.27$ . If the test is carried out under the control of flexure alteration and initial relative crack length  $\omega_0 = \ell_0 / b$  is less than 0.27, unsteady or dynamic crack growth is to begin after the load reaches its critical value, as it is observed in experiments. Similar result was obtained in the case of tension of a plate with central crack under conditions of hard loading [7,8].

#### Analysis of kinetic energy

It is possible to analyze the energy expenditures during non-steady crack growth. The energy balance equation of a solid with moving crack without heat flux can be represented in the following form:

$$\Delta U + \Delta K + \Delta \Pi = \Delta A, \quad (11)$$

where  $\Delta U$  is the increment of elastic energy,  $\Delta K$  is the increment of kinetic energy,  $\Delta \Pi$  is the change of energy in consequence of the formation of new crack surface and  $\Delta A$  is the increment of the work of external forces.

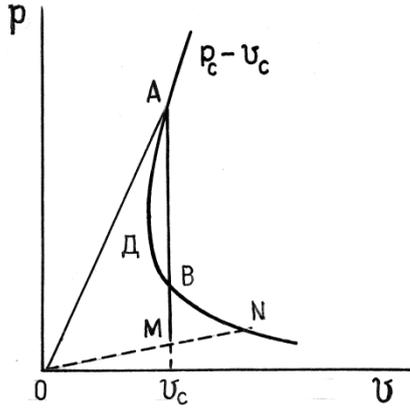


Fig. 6. Constant flexure after the critical state is reached.

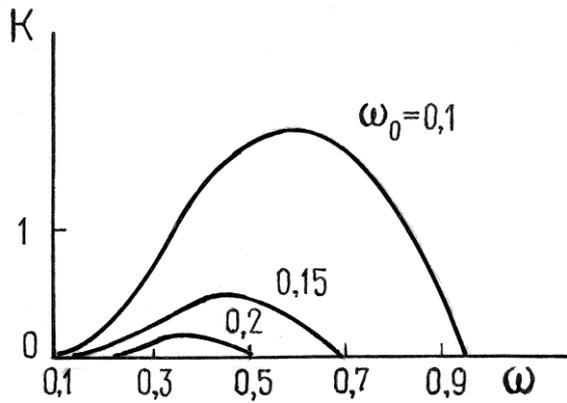


Fig. 7. Change of kinetic energy during the crack growth under constant flexure conditions

For linear elastic material, the relation between the flexure and the load is represented in the form  $v = CP$ , where  $C$  is the compliance, which is the function of relative crack length  $\omega$ . The elastic energy stored in a solid medium under the loading is

$$U = \frac{1}{2} P v . \quad (12)$$

The energy loss due to the formation of the new crack surface for the specimen shown on Fig. 1 can be represented in the form:

$$\Delta \Pi = G a \Delta \ell . \quad (13)$$

The increment of the work of external force is

$$\Delta A = P \Delta v . \quad (14)$$

Substituting Eqs. 12 – 14 into Eq. 11 and using the relation  $\Delta v = P \Delta C + C \Delta P$ , we can obtain the following energy relation:

$$\Delta K + G a \Delta \ell - \frac{P^2}{2} \Delta C = 0 .$$

Substituting increments by differentials, we can obtain differential equation for kinetic energy

$$\frac{dK}{d\omega} = \frac{P^2}{2} \frac{dC}{d\omega} - Gab. \quad (15)$$

Eq. 15 can be integrated for different loading conditions.

### Crack growth under condition of constant flexure

Let us consider the case, when flexure remains constant  $v = v_c$  after the load reaches the critical value  $P_c$  for initial crack length  $\omega_0 < 0.27$  (Fig. 6). Eq. 15 can be represented in a following form:

$$\frac{dK}{d\omega} = \frac{v_c^2}{2C^2} \frac{dC}{d\omega} - Gab. \quad (16)$$

The integral of Eq. 17 according to initial conditions  $\omega = \omega_0$ ,  $K = 0$  has a simple form

$$K = \frac{v_c^2}{2} \left[ \frac{1}{C(\omega_0)} - \frac{1}{C(\omega)} \right] - Gab(\omega - \omega_0). \quad (17)$$

The critical flexure and the compliance are characterized by Eq. 10 and Eq. 8, respectively. Substituting Eqs. 3, 7 and 10 into Eq. 17, we obtain the following equation for kinetic energy

$$\bar{K} = \frac{S/2b + 9(1-v^2)F(\omega_0)}{9(1-v^2)Y^2(\omega_0)\omega_0} \left[ 1 - \frac{S/2b + 9(1-v^2)F(\omega_0)}{S/2b + 9(1-v^2)F(\omega)} \right] - (\omega - \omega_0), \quad (18)$$

where  $\bar{K} = \frac{KE}{abK_{Ic}^2(1-v^2)}$  is non-dimensional kinetic energy.

The diagrams of variation of kinetic energy during the crack growth corresponding to different initial crack lengths  $\omega_0$  are shown on Fig. 7. The process of fracture can be represented in the following manner (Fig. 6). After the critical conditions are reached at point  $A$ , the accelerating crack growth begins. Kinetic energy increases from 0 and reaches its maximum value at point  $B$  and then decreases to 0 when the crack stops at point  $M$ . Kinetic energy is represented by the area between the graph  $P_c \sim v_c$  and straight line  $AB$ . At point  $M$  all the kinetic energy transforms into energy required for the formation of new crack surface. Thus the area of  $ADB$  is required to be equal to the area of  $MNB$ .

### The analysis of kinetic energy for arbitrary change of flexure

On the base of Eq. 15 it is possible to study kinetic energy in the case of arbitrary change of flexure  $v = v(t)$  using the load-time dependence  $P = P(t)$  obtained during test. Using Eq. 5 and the known variation of flexure and load we can obtain the dependence  $P = P(\omega)$  in the process of crack growth. The kinetic energy is determined by the following equation:

$$\bar{K} = \int_{\omega_0}^{\omega} \left( \frac{36P^2(\omega)}{a^2bK_{Ic}^2} \frac{dF}{d\omega} - 1 \right) d\omega. \quad (19)$$

Let us consider particular case of constant flexure rate equal to  $\alpha$ . This condition was realized in the performed experimental studies. If the load decreases with some overall rate  $\beta$ , the compliance in the process of crack growth can be represented by the following equation:  $C = (v_C + \alpha t) / (P_C - \beta t)$ . Introducing non-dimensional parameters  $\alpha_1 = \alpha / v_C$ ,  $\beta_1 = \beta / P_C$  and non-dimensional time  $t$  referred to unit time, the following expression for the compliance can be obtained:  $C = C_0 [(1 + \alpha_1 t) / (1 - \beta_1 t)]$ . Using Eq. 5, the dependence of the relative crack length  $\omega$  on the time  $t$  can be determined:

$$t = \frac{[9(1-\nu^2)S^2 / 2Eab^2C_0][F(\omega) - F(\omega_0)]}{\alpha_1 + \beta_1 \{1 + [9(1-\nu^2)S^2 / 2Eab^2C_0][F(\omega) - F(\omega_0)]\}}$$

As a result, we obtain the relation between the load  $P$  and the relative crack length  $\omega$ :

$$P = \frac{v_C}{C_0} \frac{\alpha_1 + \beta_1}{\alpha_1 + \beta_1 \{1 + [9(1-\nu^2)S^2 / 2Eab^2C_0][F(\omega) - F(\omega_0)]\}} \quad (20)$$

The diagrams of variation of load during the test obtained for specimens of graphite MPG-6 with initial crack lengths  $\omega_0 = 0.155$  and  $\omega_0 = 0.215$ , and the limit diagram of dependence  $P_C \sim v_C$  are shown in Fig. 8.

It can be seen that the dependence of load on time in the falling parts of the diagrams is approximately linear. The parameters are  $\alpha_1 = 15.4$ ,  $\beta_1 = 68.4$  for  $\omega_0 = 0.155$  and  $\alpha_1 = 16.5$ ,  $\beta_1 = 83.4$  for  $\omega_0 = 0.215$ . It is possible to obtain the graphs of the variation of kinetic energy in the process of crack growth (Fig. 9) using the values of these parameters and Eqs. 19, 20. Calculations were carried out up to  $\omega = 0.8$  because the Eq. 2 for stress intensity factor is valid in this range of relative crack length. In this case the kinetic energy increases with  $\omega$  and reaches maximum value at  $\omega \approx 0.8$ . Similar behavior was observed in the tests of specimens with short initial cracks under the conditions of constant flexure rate.

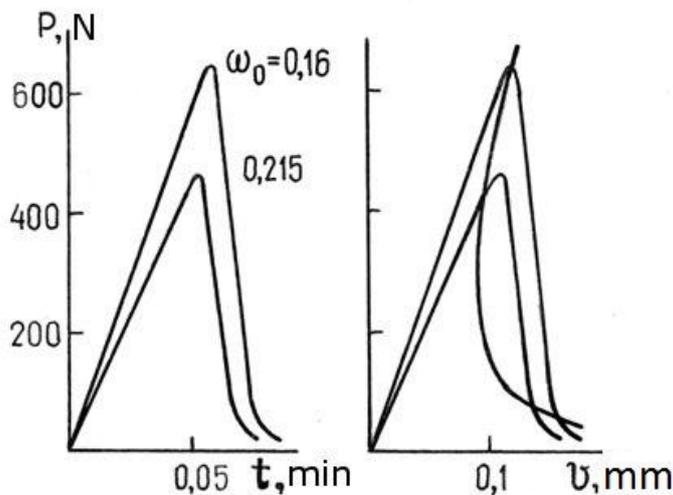


Fig. 8. The load-time diagrams for different initial crack length (left) and the load-flexure diagrams (right)

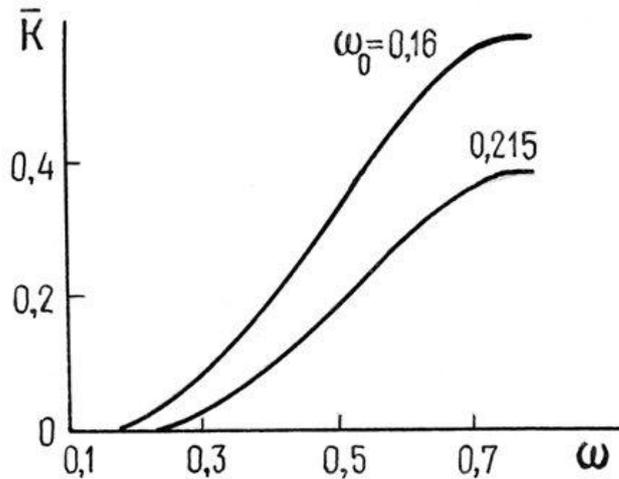


Fig. 9. The change of kinetic energy during the crack growth under conditions of constant flexure rate

### Summary

The elastic, strength and fracture toughness properties of structural graphite materials are investigated. The theoretical analysis and experimental study of crack growth was carried out for the conditions of bending. The steady crack growth is observed for the initial relative crack length  $\omega_0 \geq 0.27$  and unsteady (catastrophic) growth — for  $\omega_0 < 0.27$ . Under steady behavior of crack, the fracture toughness can be determined during the process of crack growth and the crack growth rate can be determined during the test, too. The cases of unsteady crack growth are analyzed for different loading conditions. It is shown that in the cases of short cracks some part of the stored energy transforms into kinetic energy, which can be described by corresponding equation. This equation is obtained in the general form, which can be used for the arbitrary conditions of hard loading. Some particular cases are analyzed and the kinetic energy is evaluated. This method can be applied to elastic-plastic materials in the cases when the requirements of linear fracture mechanics are satisfied.

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