Improvement of fatigue performance by residual stresses: a fracture mechanics approach

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Keywords: fatigue limit, residual stresses, closure effects, crack growth model

Abstract. The fatigue crack growth behaviour of the quenched and tempered steel 25CrMo4 is investigated at various load ratios and notch depths. The growth behaviour in the notch stress field is monitored in detail, whereby also information about the buildup of crack closure effects and the transition from short to long crack behaviour is gained. A crack growth model which is able to describe the growth behaviour for arbitrary crack length and stress ratio, is developed and adapted to the experimental results. Finally, the application of the model to real components is discussed, taking account of locally variable loads and residual stresses.

Introduction
The high number of cycles and the resulting low allowable flaw size pose a challenge in assessing the damage tolerance of drivetrain components. To decide whether a flaw of a certain length is still safe (at least until the next inspection of the component), it is important to be able to model the crack growth behaviour as precisely as possible. So far, the growth behaviour of short cracks has been described only inadequately or not at all in analytical models. In this paper a simple analytical model for describing the crack growth behaviour for any crack length and stress ratios will be introduced.

Analytical description of the crack growth behaviour
The following considerations on crack growth behaviour are based on the simplified model in Fig. 1a. Starting from a sharp notch with parallel flanks of length $a_0$, a crack of length $\Delta a$ grows. The sharp notch can be regarded as a crack of length $a_0$ which is not subject to any crack closure. Therefore the stress intensity factor is calculated via $K = \sigma(\pi a)^{1/2}$ using the whole crack length $a = a_0 + \Delta a$, whereas for the build-up of crack closure only the crack extension $\Delta a$ takes effect. The analytical description of crack growth under cyclic loading, with the maximum stress $\sigma_{\text{max}}$, the minimum stress $\sigma_{\text{min}}$ and the stress range $\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}}$ as well as the respective stress intensity factors $K_{\text{max}}$, $K_{\text{min}}$, $\Delta K$ for any stress ratio $R = \sigma_{\text{min}} / \sigma_{\text{max}} = K_{\text{min}} / K_{\text{max}}$ and crack extension $\Delta a$, is based on the crack growth equations according to Erdogan/Ratwani [1] and Forman/Mettu [2] including the Newman’s crack closure function [3]. These equations were modified to take account of the short crack behaviour (i.e. for small $\Delta a$). A summary and short discussion of these equations can be found in [4]. Already the crack growth equation according to Erdogan/Ratwani [1]

$$\frac{da}{dN} = C' \left( \frac{(\Delta K - \Delta K_{\text{th}})^m}{(1 - R)K_c - \Delta K} \right)^{2}$$

(1)
represent the three regions of the crack growth curve (Fig. 1b). Here the stress ratio $R$ is only used to determine the asymptote for unstable crack growth, $\Delta K = (1-R)K_c$ (region III), from the fracture toughness $K_c$. The position of the asymptote for the crack growth threshold (region I) is determined by $\Delta K_{th}$, and the position of the curve for stable crack growth (region II) is determined by $C'$ and $m$. Therefore it is necessary to find an analytical description for the dependence of the parameters $\Delta K_{th}$ and $C'$ on the stress ratio $R$ and the crack extension $\Delta a$.

\[ \frac{da}{dN} = C \left[ \frac{1-f}{1-R} \right]^m \left( 1 - \frac{\Delta K_{th}}{\Delta K} \right)^p \left( 1 - \frac{K_{max}}{K_c} \right)^q \Delta K^{m-p} K_c^{q} \left( \frac{\Delta K - \Delta K_{th}}{K_c - K_{max}} \right)^q \]  

(2)

Fig.1. (a) Sharp notch with growing crack; (b) fatigue crack growth curve

The Forman/Mettu equation [2]

\[ \frac{da}{dN} = C \cdot \left[ \frac{1-f}{1-R} \right]^m \left( 1 - \frac{\Delta a}{a} \right)^{\frac{\Delta K - \Delta K_{th}}{K_c - K_{max}}} \]  

(3, 4)

$F_{lc}$ describes the position of region II depending on the stress ratio $R$ for long cracks (large crack extensions $\Delta a$, index “lc”). The cause of this dependence is seen in crack closure effects, which are described by the crack opening function

\[ f = \frac{K_{op}}{K_{max}} \]  

(5)
Newman [3] achieved, based on finite element simulations of plasticity-induced crack closure for long cracks, the following analytical approximation for the crack opening function:

\[
f = \begin{cases} 
\max(R; A_0 + A_1 R + A_2 R^2 + A_3 R^3), & R \geq 0 \\
A_0 + A_1 R, & -2 \leq R < 0 \\
A_0 - 2A_1, & R < -2 
\end{cases}
\]  

(6)

\[
A_0 = \left(0.825 - 0.34\alpha + 0.05\alpha^2\right) \cos\left(\frac{\pi \sigma_{\text{max}}}{2\sigma_F}\right)^{1/\alpha}
\]

\[
A_1 = \left(0.415 - 0.071\alpha\right) \frac{\sigma_{\text{max}}}{\sigma_F}
\]

(7)

In what follows, the values \(\sigma_{\text{max}}/\sigma_F=0.3\) and \(\alpha=3\) are assigned, as it applies for approximate plane strain condition and largely elastic crack behaviour; for a more detailed discussion of these parameters cf. [3,4]. The curves in Fig. 5 (left) for the crack opening function \(f\) and Fig. 5 (right) for the crack velocity factor \(F\) for \(\Delta a = 10\) mm (\(\approx F_{\text{lc}}\)) show the behaviour valid for long cracks in graphical form. For cracks with arbitrary crack extension \(\Delta a\) an empirical approach for the crack velocity factor \(F\) is developed further below.

The \(R\)-dependence of the threshold for long crack growth propagation is approximated by [2,4]

\[
\Delta K_{\text{th,lc}} = \Delta K_0 \left(\frac{a}{a + a_{0,\text{fikt}}} \frac{1 - f}{(1 - A_0)(1 - R)}\right)^{-\left(1 + C_{\text{th}} R\right)}
\]

(8)

where \(\Delta K_0\) is the threshold for long crack growth at \(R=0\), and \(C_{\text{th}}\) is an adjustment parameter. \(a_{0,\text{fikt}}\) is a fictional intrinsic length scale based on the concept of El Haddad [5] for the approximate consideration of short crack effects. It is assumed that a crack of the total length \(a=0\) shows a threshold value of 0 and crack closure builds up until about the total length \(a=a_{0,\text{fikt}}\). At least in the presence of an initial notch \(a_0\), see Fig. 1a, this concept proves to be unsustainable because only the crack extension \(\Delta a\) and not the total crack length \(a\) is relevant to the build-up of crack closure effects. Furthermore, also in the absence of crack closure the threshold value is not 0 but equal to the intrinsic threshold for crack propagation \(\Delta K_{\text{th,eff}}\). For these reasons the application of the El Haddad correction is not considered (i.e., \(a_{0,\text{fikt}}=0\)) and Eq. 8 is used exclusively to describe the \(R\)-dependence of the long crack threshold.

Instead, for the description of the threshold build-up starting from the intrinsic value of \(\Delta K_{\text{th,eff}}\) at a crack extension of \(\Delta a=0\) to the long crack growth threshold \(\Delta K_{\text{th,lc}}\) (for large \(\Delta a\)) the empirical approach

\[
\Delta K_{\text{th}} = \Delta K_{\text{th,eff}} + (\Delta K_{\text{th,lc}} - \Delta K_{\text{th,eff}}) \cdot \left[1 - \sum_{i=1}^{n} v_i \cdot \exp\left(-\frac{\Delta a}{l_i}\right)\right] \text{ with } \sum_{i=1}^{n} v_i = 1
\]

(9, 10)

is proposed. The \(l_i\) can be interpreted as fictitious length scales for the formation of crack closure effects (similar to \(a_{0,\text{fikt}}\)) and determined in conjunction with the \(v_i\) by fitting of the experimentally obtained crack growth resistance curve (\(\Delta K_{\text{th}}\) plotted against \(\Delta a\)), see Fig. 4.

Since the crack velocity factor \(F\) also describes the effects of the same crack closure mechanisms, in analogy to Eq. 9 the approximation
\[
F = 1 - (1 - F_{lc}) \cdot \left[ 1 - \sum_{i=1}^{n} v_i \cdot \exp \left( -\frac{\Delta a}{l_i} \right) \right]
\]  

(11)

with the same values for the \( l_i \) and \( v_i \) is chosen. \( F_{lc} \) is calculated according to equations (4) and (6). In the crack growth equation (3) then \( F_{lc} \) is to be replaced by \( F \). Fig. 5 shows this behaviour graphically.

**Experimental investigations**

As material for the experimental investigations the QT steel 25CrMo4 widely used for drivetrain components was chosen. The material has a bainitic microstructure and a hardness of \( \sim 245 \) HV10. In the tensile test, a 0.2% yield strength of 512 MPa and a tensile strength of 674 MPa with an elongation at fracture of 18.9% is obtained. For determining the cyclic crack resistance curves, SENB (Single Edge Notched Bending) specimens measuring 100x6x20 mm with different notch depths \( a_0 \) (0.35mm, 1 mm, 5.3 mm) were machined. The notches were sharpened by means of a razor blade coated with diamond paste (1 µm). The samples were then compression pre-cracked at a stress ratio of \( R=10 \) to obtain an incipient fatigue crack, which is fully open due to the residual tensile stresses from compression pre-cracking so that crack closure effects can be excluded. The samples were then subjected to cyclic loading under eight-point bending in a resonance testing machine at a test frequency of 108 Hz. The crack growth was measured by DC potential drop method.

The experiments are conducted with step-wise increasing loads, see also [6]. The crack grows initially, but after a certain crack extension \( \Delta a \) crack arrest occurs due to the build-up of crack closure. Subsequently, the load is increased so that the crack can grow further. In this way, the crack...
resistance curve is obtained point by point (Fig. 2). As soon as the crack starts to grow through, one gets the crack growth curve. This crack growth curve is the one of long cracks, because at that time the crack closure has already built up completely.

**Parameter determination**

For the investigated material, in place of Eq. 3 the slightly modified crack growth equation

\[
\frac{da}{dN} = C \cdot F \cdot \frac{\Delta K^m - \Delta K_{th}^m}{1 - \frac{K_{max}}{K_C}}
\]

is used, which gives a significantly better fit of the transitions between the three regions of the crack growth curve.

\( F \) is calculated according to Eq. 11 with \( F_{lc} \) according to Eq. 4, and \( \Delta K_{th} \) is calculated according to Eq. 9 with \( \Delta K_{th,lc} \) according to Eq. 8. The model parameters determined from the experiment are summarized in Table 1.

The parameters were obtained as follows: first the parameters for the growth of long cracks were determined from the crack growth curves, Fig. 3. Subsequently, the crack resistance curves were adapted, Fig. 4.

Fig. 5 right shows the curves of the crack velocity factor \( F \) according to Eq. 11 for different crack extensions \( \Delta a \) as a function of the stress ratio \( R \). At all stress ratios \( R \), a clear difference between short and long crack behaviour is recognized. In addition, the threshold \( \Delta K_{th} \) is much lower for short cracks, as is evident from the crack resistance curves in Fig. 4. All this leads to a marked deviation of the growth behaviour of short cracks compared to that of long cracks, see Fig. 3 right.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>8</td>
<td>[nm / (MPa m(^{1/2}))]</td>
</tr>
<tr>
<td>( K_c )</td>
<td>90</td>
<td>[MPa m(^{1/2})]</td>
</tr>
<tr>
<td>( m )</td>
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<td>-</td>
</tr>
<tr>
<td>( \alpha )</td>
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<td>( \sigma_{max}/\sigma_F )</td>
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</tr>
<tr>
<td>( \Delta K_{th,eff} )</td>
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<td>[MPa m(^{1/2})]</td>
</tr>
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<td>( \Delta K_0 )</td>
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<td>[MPa m(^{1/2})]</td>
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<tr>
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<tr>
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<td>[mm]</td>
</tr>
<tr>
<td>( v_1 )</td>
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<td>-</td>
</tr>
<tr>
<td>( v_2 )</td>
<td>0,55</td>
<td>-</td>
</tr>
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</table>

Table 1. Parameters of the fatigue crack growth model for 25CrMo4
cracks, as is evident from the crack resistance curves in Fig. 4. All this leads to a marked deviation of the growth behaviour of short cracks compared to that of long cracks, see Fig. 3 right.

![Fatigue crack growth curves at different crack extensions Δa and stress ratios R – experiment and analytical model](image1)

**Fig. 3.** Fatigue crack growth curves at different crack extensions Δa and stress ratios R – experiment and analytical model

![Crack resistance curves: crack growth threshold ΔKth vs. crack extension Δa from Eq. 9 for different stress ratios R – experiment and analytical model](image2)

**Fig. 4.** Crack resistance curves: crack growth threshold ΔKth vs. crack extension Δa from Eq. 9 for different stress ratios R – experiment and analytical model

**Model verification**

In order to verify the model, the growth of a short crack starting from a notch of depth \( a_0 = 0.812 \) mm (\( R=-1 \)) is calculated and compared with the measured data. Good agreement between measurement and calculation is observed, see Fig. 6. The left limiting curve corresponds to a crack extension \( \Delta a=0 \) (short crack behaviour), the right to a very large \( \Delta a \) (long crack behaviour). In the first two load steps the crack slows down and stops (curves 1, 2). Only after a further increase of the
load the crack grows, after an initial slight deceleration, finally through, in the course of which it approaches the behaviour of long cracks (curve 3).

Fig.5. Crack opening stress function $f$ from Eq. (6) and crack velocity factor $F$ from Eq. (11) for different crack extensions $\Delta a$ depending on the stress ratio $R$

Fig.6. Growth of a short crack – experiment and computation

**Damage tolerance assessment procedure**

For assessing the damage tolerance of structural component, especially the influence of the locally varying stress fields due to the external loading as well as due to residual stresses must be considered. This is most easily done by using a Green’s function [7]
\[ \Delta K = \frac{2 \cdot 1.12}{\sqrt{\pi a}} \int_{0}^{a} \frac{\Delta \sigma(x)}{\sqrt{1 - (x/a)^2}} \, dx \]  

(13)

where at each point of the crack flank \( x \) the cyclic load range \( \Delta \sigma(x) \) is given by the difference between maximum and minimum load stress \( \sigma_{\text{max},L}(x) \) and \( \sigma_{\text{min},L}(x) \), respectively. Both for determining the threshold value (for endurance evaluation) from Eq. 9 and for determining the crack growth rate (for lifetime assessment) from Eq. 12 the stress ratio \( R \) must be known. Since \( \sigma_{\text{max},L}(x) \), \( \sigma_{\text{min},L}(x) \) and \( \sigma_{\text{res}}(x) \) vary locally, the local stress ratio \( R(x) \) is obtained from

\[ R(x) = \frac{\sigma_{\text{min},L}(x) + \sigma_{\text{res}}(x)}{\sigma_{\text{max},L}(x) + \sigma_{\text{res}}(x)} \]  

(14)

A conservative approach is to determine the maximum of \( R \) along the crack flanks; in a somewhat less conservative manner, also be the mean value of \( R \) can be used.

Acknowledgement

Financial support by the Austrian Federal Government (in particular from the Bundesministerium für Verkehr, Innovation und Technologie and the Bundesministerium für Wirtschaft, Familie und Jugend) and the Styrian Provincial Government, represented by Österreichische Forschungsförderungsgesellschaft mbH and by Steirische Wirtschaftsförderungsgesellschaft mbH, within the research activities of the K2 Competence Centre on “Integrated Research in Materials, Processing and Product Engineering”, operated by the Materials Center Leoben Forschung GmbH in the framework of the Austrian COMET Competence Centre Programme, is gratefully acknowledged.

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