USE OF THE INTERCEPT METHOD FOR J-R CURVE CONSTRUCTION OF C(T) SPECIMENS WITHOUT CRACK EXTENSION DATA

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ABSTRACT

The construction of a *J-R* curve requires consideration of the stable crack extension process, and the appropriate correction for the incremental crack growth. Following Standard ASTM E1820, the *J* resistance curve is obtained from a single specimen test, in which the actual crack length is measured concurrently with the fracture test, by the unloading compliance method, or other similar techniques. Recently, Donoso, Vasquez and Landes (DVL) developed a method for obtaining the crack size in a test in which there is stable crack extension, but has only the *P-v* data, and the initial and final crack sizes, a_o and a_f, respectively, as inputs. The DVL method was introduced as an alternative to the unloading compliance and normalization procedures included in E1820, and presents notable advantages when the full *a-v* data are not available. Inherent to this novel crack size evaluation methodology, is the notion of the "crack growth law" postulated earlier by Donoso, Zahr and Landes. The method presented here, designated as the "*intercept method*", is used with C(T) specimen data that have crack extension measured by the unloading compliance method. The results are quite encouraging, and the method is extended to one example in which a full *a-v* record is lacking, and there are only initial and final crack sizes available.

KEYWORDS

J-R Curve, Fracture toughness, Stable crack extension, Common Format, Crack growth law

INTRODUCTION

ASTM Standard E 1820 provides the guidelines for the evaluation of the fracture toughness of a ductile material [1]. The fracture toughness may be evaluated as a point value, J_{Ic}, provided certain size and constraint requirements are met in the fracture test. On the other hand, fracture toughness may be evaluated as a fracture toughness resistance curve, known as the *J-R* curve. The construction of *J-R* curves involves consideration of the stable crack extension process, and therefore, requires some sort of incremental crack growth correction. Following E 1820, the *J*-resistance curve is obtained from a single specimen test, in which the actual crack length is measured concurrently with the fracture test, either by means of elastic unloading compliance changes, or by other similar techniques. Thus, the fundamental data for *J-R* testing are force (*P*), displacement (*v*), and crack size (*a*). In other words, the actual crack extension value, $\Delta a = a_i - a_o$, where a_i and a_o, are current and initial crack size, respectively, is essential in *J-R* testing to perform the crack growth correction.

Recently, Donoso, Vasquez and Landes [2] developed a method for obtaining the crack size in a test in which there is stable crack extension, but has only the *P-v* data, and the initial and final crack sizes, a_o and a_f, respectively, as inputs. The method, which presents remarkable advantages, has been introduced as an alternative to the unloading compliance and normalization procedures included in E 1820 [1]. Inherent to this novel crack size evaluation methodology, is the notion of the "crack growth law" — or CG law — postulated earlier by Donoso, Zahr and Landes (DZL) [3]. The fundamentals of the CG law will be presented next, followed by some applications of the alternative method, designated hereinafter as the "intercept method".

FUNDAMENTALS OF THE DZL CRACK GROWTH MODEL

Figure 1 (a) shows the *P-v* curve — the end result of a fracture toughness test — for an ASTM A 508 1T-C(T) specimen tested at room temperature. The experimental data, shown here by a full black symbol (●), show that as the displacement *v* increases, so does the force *P*, until it reaches a maximum value, P_{max}, of approximately 52 kN, at a total displacement $v \approx$ 1.2 mm. A fracture toughness test for a ductile material — like the A 508 that shows a fair amount of stable crack extension before plastic collapse ensues — is usually carried to a displacement beyond that at maximum force. Thus, the force for such a specimen test first increases, reaches a maximum, then decreases with increasing total displacement. Then, the test is ended at a point labeled "P_f, v_f, a_f", meaning that the force and displacement at the test final point are (P_f, v_f), while the final crack size is a_f. The total stable crack extension at the end of the fracture test is $\Delta a_f = a_f - a_o$, where a_o is the initial crack size of the specimen (26.2 mm). For the example shown here, the test was terminated at a displacement $v_f \approx 3.2$ mm, giving out an amount of stable crack extension of $\Delta a_f \approx 6.8$ mm.

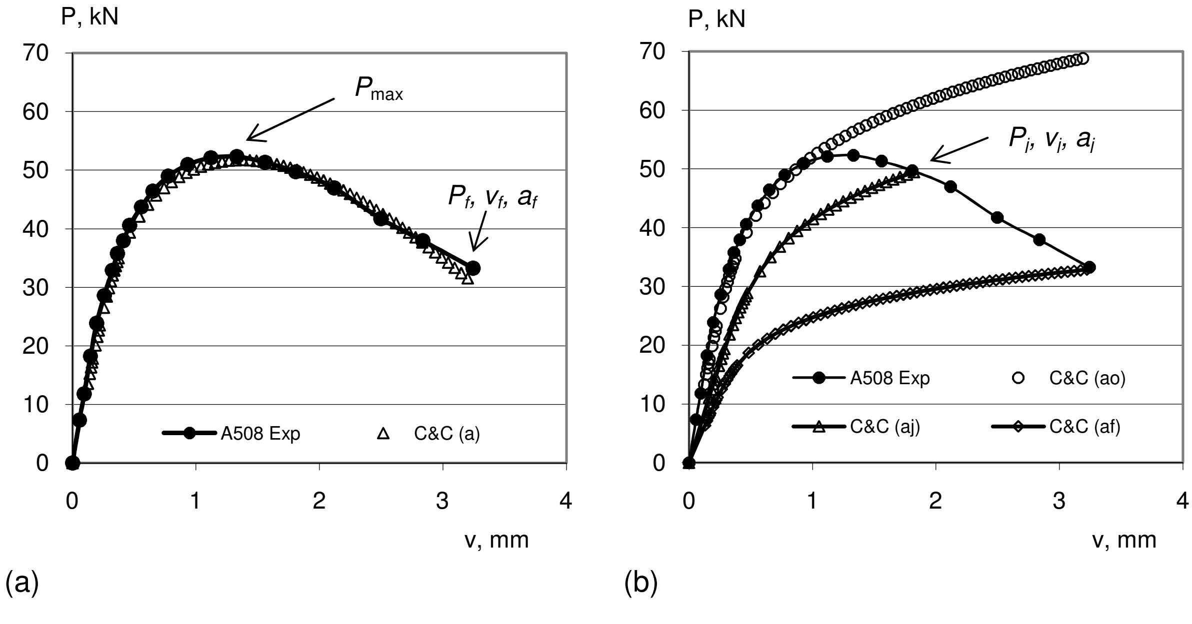


Figure 1.- (a) Force-displacement curves for the A 508 specimen. The curve C&C(*a*), was obtained with the DZL crack growth model, and follows closely the experimental curve. (b) The curves C&C(a_o), C&C(a_j) and C&C(a_f) were constructed with constant crack sizes a_o (initial crack size), a_j (such that $a_o < a_j < a_f$) and a_f, (final crack size), respectively.

Beyond the displacement at maximum force, the force decreases and the displacement increases, due to stable crack extension. As indicated above, the test is ended at a point chosen by the experimentalist; in this case, the point corresponds to the triad "P_f, v_f, a_f". At each one of the data points of this curve past maximum force, the crack size is known from unloading compliance measurements carried out concurrently with the fracture test. Figure 1 (a) includes one other curve related to the original "A508 Exp" curve. This second curve (Δ), follows the experimental one, and is labeled as C&C(*a*), being the result of the application of the DZL crack growth model [3]. Figure 1 (b), which includes three "constant crack size" curves in addition to the experimental one, will be dealt with momentarily.

A test specimen that undergoes stable crack extension shows a large amount of plastic displacement compared to the elastic component. Thus, it is more convenient to carry out the analysis using the Common Format. The Common Format Equation, CFE, proposed by Donoso and Landes [4], describes the force-plastic displacement relationship for a blunt-notch fracture specimen, and relates the force P to two variables representing the non-linear deformation of a fracture specimen with a stationary crack: v_{pl}/W, the plastic component of the force-line displacement, normalized by the specimen width W, and b/W, the normalized ligament size. The CFE also includes a term that denotes the out-of-plane constraint, Ω^*, and is usually written as:

$$P = \Omega^* BCW\,(b/W)^m\,\sigma^*\,(v_{pl}/W)^{1/n} \tag{1}$$

In Eq. (1) B is the specimen thickness; C and m are the geometry function parameters, and σ^* and n are material properties. On the other hand, the product, $\Omega^*\sigma^* = D$ is obtained directly from the specimen normalized force-normalized displacement curve. For a non-growing crack, the crack (or ligament) size is constant, and P and v become the variables of the calibration function, at constant crack size. The three "*a* constant" curves of Figure 1(b) were constructed on this basis.

When there is stable crack extension, however, the crack size a also becomes a variable, so that a separate relation between a and v_{pl} is needed. For such purpose, DZL proposed the "*crack growth law*" concept [3] to account for the relation between stable crack extension Δa, and plastic displacement v_{pl}. The DZL crack growth law is a two-parameter power law equation that relates the change in normalized crack size, $\Delta a/W$, with normalized plastic displacement, *vpl/W*, i.e.,

$$\frac{\Delta a}{W} = l_o \left[\frac{v_{pl}}{W}\right]^{l_1} \tag{2}$$

In Eq. (2), l_o is a coefficient to be determined and l_1 an exponent, which for C(T) specimens is of the order of 2.0 [3].The crack extension, Δa, may also be written in terms of the change in ligament size, that is, $\Delta a = b_o - b$, where b_o is the initial ligament size ($b_o = W - a_o$).Thus, Eq. (2) yields the following expression for the current ligament size, b:

$$\frac{b}{W} = \frac{b_o}{W} - l_o \left[\frac{v_{pl}}{W}\right]^{l_1} \tag{3}$$

Substitution of Eq. (3) into the geometry term of Eq (1) yields the following expression for the CFE in terms of the plastic displacement alone, when there is stable crack growth:

$$P = DCBW \left[\frac{b_o}{W} - l_o \left[\frac{v_{pl}}{W}\right]^{l_1}\right]^m \left[\frac{v_{pl}}{W}\right]^{1/n} \tag{4}$$

Equation (4) represents the relation between force and plastic displacement when there is stable crack growth, according to the DZL model. The curve C&C(a) of Figure 1 (a) — which follows very closely the experimental curve — has been constructed with Eq. (4), keeping in mind that the elastic component of the displacement has to be added to v_{pl} to account for the total displacement observed. The shape of Eq. (4) clearly indicates the existence of a maximum value for P in terms of plastic displacement. According to Standard E1820, the fracture toughness test data are usually given as force *vs.* total displacement. The data may also include the unloading-reloading lines from which the compliance values, changing with increasing crack size, are evaluated. The alternative look at the *J-R* curve construction proposed by DZL includes the crack growth law, Eq. (2), and the relation between force and plastic displacement for a situation in which the crack is growing, Eq. (4). Both equations (2) and (4) relate Δa and P to the plastic displacement; the elastic displacement, on the other hand, may be obtained with the use of the Concise Format [5].

THE INTERCEPT METHOD FOR A C(T) SPECIMEN

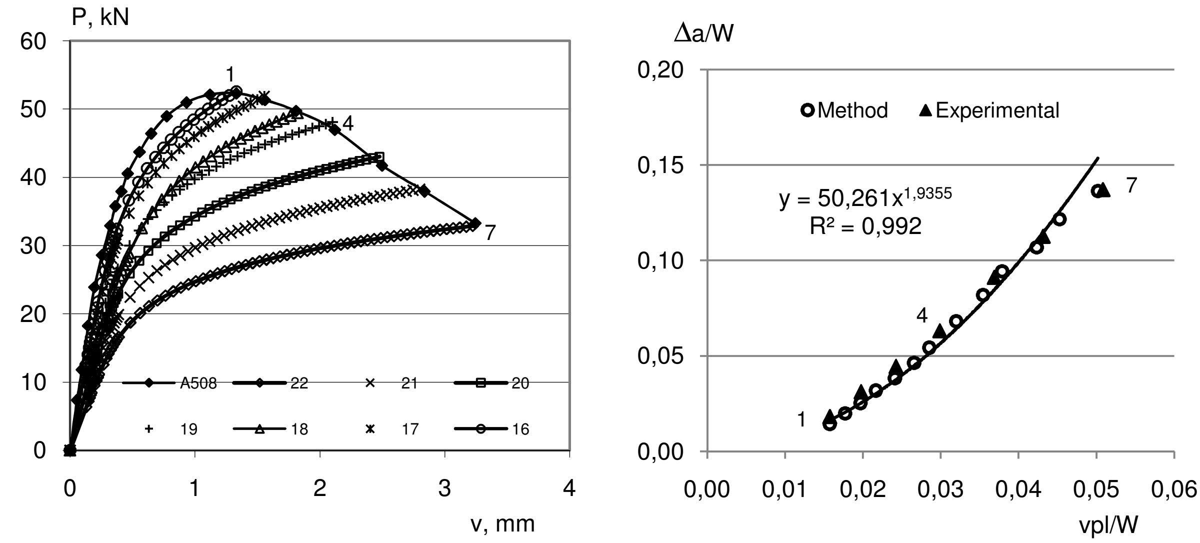
The original DZL model used a value of the exponent of the crack growth law, l_1, Eq. (2), of 2.0 for the C(T) data analyzed [3]. The value of the coefficient l_o, on the other hand, was obtained by calibration, either at the final point, or at the point of maximum force [6]. It should be noted that P_{max} is an extreme case of termination of the test; also, it should be clear that at this point, the crack has already grown from its original value a_o.

Figure 1 (b) shows graphically the basis of the "intercept method". It has been shown that if one constructs an *"a constant"* curve based on the C&C formats, with a known crack size a_j, obtained from any triad "P_j, v_j, a_j" belonging to the experimental curve [2], then the *C&C(a_j)* curve will intersect the experimental curve *at exactly* the point "P_j, v_j". Conversely then, if one constructs a series of *"a constant"* curves of crack sizes a_j such that $a_f > a_j > a_o$, thus repeating the process outlined in Figure 1 (b), the result would look like that shown in Figure 2(a). The intersection points of these *"a constant"* curves with the experimental curve will produce two out of the three data of the triad: force and total displacement. The third datum is the value of the crack size with which the *C&C "a constant"* curve was constructed.

Figure 2(a) shows seven *C&C "a constant"* curves with known crack sizes – a_1 to a_7 – with the original A 508 *P-v* curve. Each one of the crack sizes a_1 to a_7 is equal to that measured at each point on the curve, 1 to 7, by the unloading compliance method (in this case, point 7 is the final point, i.e., $a_7 = a_f$). The analysis of these intercepts, plus six other intercepts in between these seven points [2], is illustrated in Figure 2(b).

Figure 2(b) shows the amount of stable crack growth Δa normalized by W, as a function of normalized plastic displacement, v_{pl}/W, for the A 508 specimen. Two sets of data are shown: one for the actual seven experimental data points of Figure 1(a) (▲) and thirteen intercepts (o) obtained with the method outlined earlier [2, 3], of which seven correspond to the intercepts of Figure 1(a), and the other six, to points in between these seven.

The dependence of v_{pl}/W with $\Delta a/W$ — *i.e.*, the crack growth law of Eq. (2) — obtained with the intercept method, is shown in Figure 2(b) for the A 508 specimen by means of a power-law fit. The fit yields a value of l_o of 50.26; it is important to notice that l_1 = 1.936 for the method, a value which is close to that used earlier in the DZL model, i.e., l_1 = 2.0 [2, 3].

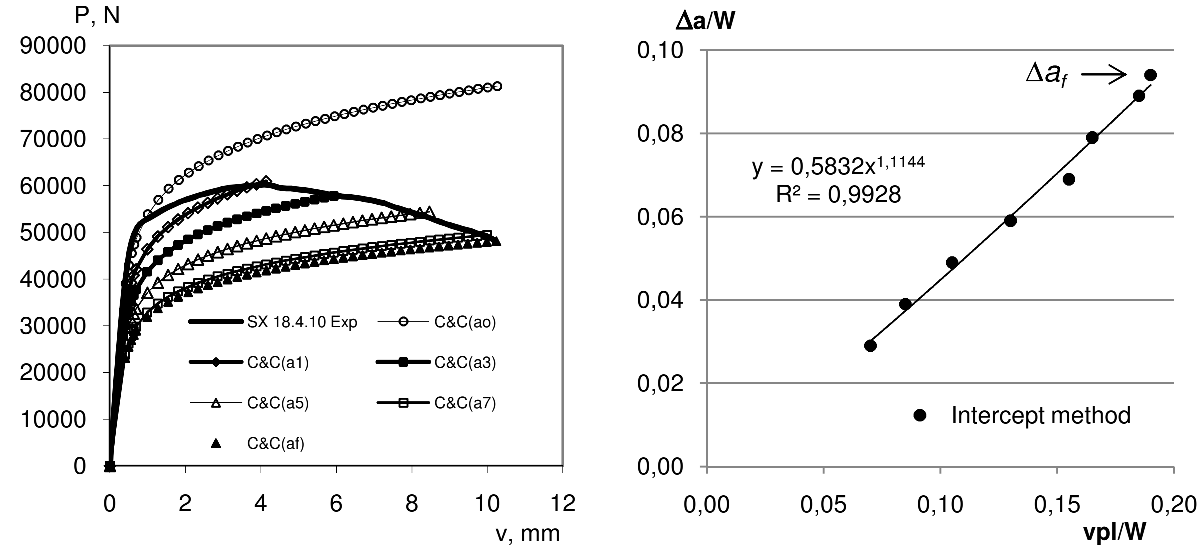


(a) (b)

Figure 2.- (a) The intercept method for A 508: seven "*a constant*" curves constructed with the actual data points 1 through 7. (b) The crack growth law for actual data points 1 through 7, plus the result of the intercept method for points 1 through 7, plus six points in between.

APPLICATIONS OF THE INTERCEPT METHOD TO *J-R* CURVE CONSTRUCTION

The intercept method presented earlier, is used now to evaluate crack sizes for a specimen of which only initial and final crack sizes are known. Figure 3 shows the *P-v* data and the crack growth values inferred from the method, for GKSS specimen SX 18.4.10 [7], tested at 0℃, in much the same way the values for A 508 of Figures 1 and 2 were obtained.

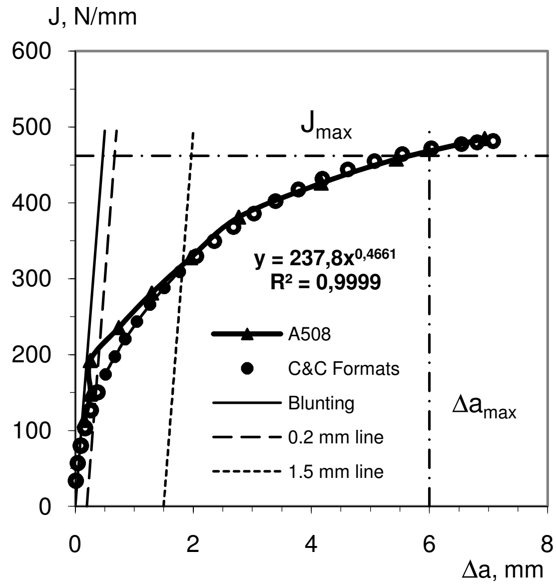


(a) (b)

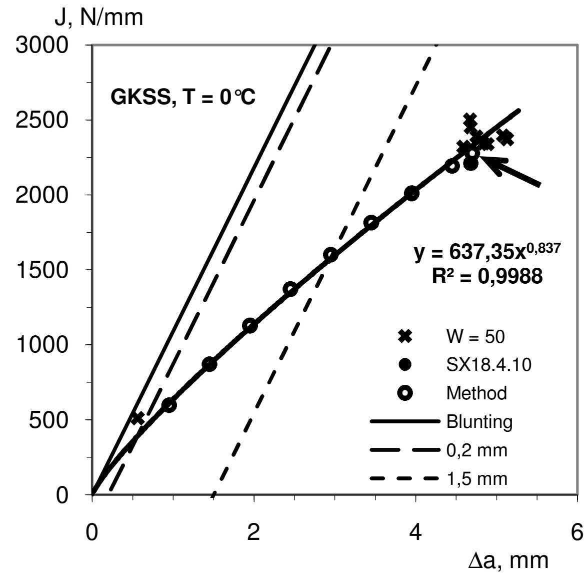
Figure 3.- (a) The intercept method applied to the *P-v* data of GKSS specimen SX 18.4.10. (b) The crack growth law generated with 8 points from the data of Figure 3(b).

The GKSS specimen, designated as SX 18.4.10, was part of a European round robin [7, 8], and has only *P-v* data, plus initial and final crack sizes measured on the fracture surface. Thus, there are no concurrent measurements of crack extension for this specimen. Figure 3(a) shows the experimental curve (thick continuous line) and six *C&C* "*a constant*" curves. Included in this figure are *C&C* curves for the initial crack size, a_o (o), the final crack size, a_f (▲) and four "*a constant*" curves for crack sizes in between a_o and a_f (out of seven used in total). Figure 3(b), on the other hand, shows the crack growth law derived from such data, with exponent l_1 = 1.114. This means that the rate of change of plastic displacement with crack extension is almost linear, a fact that somehow should be reflected both on the *P-v* curve and the *J-R* curve.

The *J-R* curves constructed with the method for both the A 508 and the GKSS specimen SX 18.4.10, are shown in Figure 4. A power-law fit for the A 508 data between the appropriate limits, gives $J = 237(\Delta a)^{0.47}$, implying a behavior which is almost parabolic. For the GKSS specimen, the fit, applied to all the range, gives $J = 637(\Delta a)^{0.84}$, implying a behavior which is short of being linear. No *J-R* data are available for this material for comparison, but there is plenty of information from GKSS regarding the "final" *J* value for several C(T) specimens with *W* = 50 mm. This "final" *J* value is calculated as the total area under the *P-v* curve of each specimen, at the termination of the test, multiplied by η/Bb_o. Thus, each final *J* value (shown by an "x") is plotted against the corresponding Δa_f value. The final GKSS *J* value for SX 18.4.10 is identified by the big black dot (●) in Figure 4(b). On the other hand, the value of *J* corresponding to the final crack extension Δa_f, calculated with the method presented here, is shown by the arrow, and it appears to be slightly higher than its GKSS counterpart. Thus, one may suggest that the *J-R* curve obtained with the intercept method represents fairly well the behavior of specimen SX 18.4.10. Considering the rest of the *W* = 50 mm specimens, one can venture that the intercept method *J-R* curve satisfies conservatively – on the lower side – the behavior of the *W* = 50 mm specimens.



(a)



(b)

Figure 4.- (a) *J-R* curve for A 508, constructed with the actual data points (▲), and with points obtained with the crack growth law inferred from the intercepts (o; ●). (b) *J-R* curve for GKSS specimen SX 18.4.10, constructed as per E1820 with crack sizes obtained with the intercept method; the only datum available for this specimen is that shown by the black symbol (●).

DISCUSSION AND CONCLUSIONS

A new method of construction of the *J-R* curve was proposed by Donoso, Zahr and Landes (DZL) [2, 3], using a crack growth law concept. The crack growth law relates normalized crack extension, $\Delta a/W$, to normalized plastic displacement, v_{pl}/W, and is of the power-law type relation with two parameters: a coefficient, l_o, and an exponent, l_1. In the original DZL proposal, the coefficient l_o was substituted on the basis of a known calibration point: the final point of the test, of which both the total displacement, v_f, and the force, P_f, are known. Thus, the crack growth law model had only one adjustable parameter, the exponent l_1.

The value of l_1 that proved to give the best fit to force and displacement data of 1T-C(T) specimens of A 508 and A 533B steels with known values of crack extension along the test (*i.e.*, to best reproduce the experimental *P-v* curve) was 2.0. This value of l_1, used in earlier work [2, 3], contributed to a very good fit between the experimental *P-v* curves and the C&C "*a variable*" curves for these specimens.

Therefore, an analytical treatment of both the *P-v* curve and the resulting J-Δa curve would be possible even when the inputs are the full *P-v* curve, but only the initial and final crack size values, without measurements of stable crack extension concurrently with the test. In order to make it useful, the concept of the crack growth law was in need of a solid analytical foundation and empirical proof of its significance. This was achieved elsewhere [2].

Thus, based upon the crack growth law concept, and on the Common and the Concise Format Equations of Donoso and Landes [4, 5], an alternative way of generating the amount of stable crack growth has been developed. The method, called the "*intercept method*", has shown to yield reasonable crack extension values when applied to *P-v-a* data in which crack sizes were experimentally measured by the unloading compliance method, as is the case of the A 508 specimen [3].

The "*intercept method*" applied in this work is based upon the fact that the Common and the Concise (*C&C*) Format Equations are able to generate "*a constant*" *P-v* curves, of given, known crack sizes whose values are in between the physically measured initial crack size, a_o, and final crack size, a_f. The intercept these "*a constant*" *P-v* curves make with the experimental curve will give a value of displacement and force at which the experimental curve will have a crack size equal to that of the corresponding *C&C* curve. From this intercept, the value of the plastic displacement at each point may be calculated, and thus, by using the *C&C* approach, a more complete set of crack extension values may be generated.

The method, verified on one example of known *P-v-a* data — the A 508 specimen — was applied to one example of 1T-C(T) fracture test specimen, GKSS specimen SX 18.4.10, for which there are no crack extension measurements available, except for the initial and final crack sizes. The results obtained with the intercept method, when compared to the GKSS final *J* value, are quite encouraging, and clearly suggest further work along these lines.

The crack growth law — a very convenient and useful tool — may not always have the exponent l_1 = 2.0. Whereas the A 508 data yield a value of l_1 close to two, the data for the GKSS specimen produce a value of the exponent close to unity, *i.e.*, the rate of generation of plastic displacement with crack extension is almost linear. Nonetheless, and regardless of the value of l_1, the most important use of the crack growth law is the generation of crack size values where there are none available. Thus, *J-R* curve construction may now be done from *P-v* data with only initial and final crack sizes, as measured on the fracture surface after termination of the test.

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