Cone Crack Development during Ball on the Three Balls Test

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Abstract. A standardized uniaxial four respective three point bend loading of beams is used for strength evaluation at present time. However, in many applications are ceramics components loaded in biaxial mode. Therefore appropriate results from biaxial tests are required. A number of testing procedures for biaxial strength testing of plates exists; the ring on ring test is being the most common and standardized. The disadvantage of ring on ring test setup is significant sensitivity on small deviation from ideal plan parallel surfaces of test-piece. Proposed ball on three balls test takes advantage of biaxial loading and very simple experimental setup allowing usage of various test-piece geometries. The loading configuration is insensitive to small inaccuracies of disk shape. The only relevant parameter playing significant role is the thickness of test-piece. The three supporting and one loading balls eliminate disadvantage of ring of ring test mentioned above. From the experiences of numerical analyses as well as experimental results is still unclear influence of contact stresses between test-piece and loading ball on resulting stress field respectively the influence of cone crack developed due to such stresses. The main aim of this paper is to describe possibility of cone crack development due to contact stresses.

Introduction

Development in advanced ceramics and brittle matrix composites enable use of these materials for industrial application irrespective of their brittleness. Obtaining relevant mechanical properties is extremely essential for components design. However, mechanical properties corresponding to the real loading conditions are unknown due to absence of testing procedure providing such results. There are present usually testing procedures giving data under uniaxial loading conditions. In case of strength measurement of inherently brittle materials is possible to use two standardize methods \cite{1, 2}. First is uniaxial tensile test and the second is bend test. The tensile test of brittle materials is very complicated and therefore is commonly used bend test. The flexural strength is determined using rectangular beams loaded in four point or three point bending. However, even results obtained from bend test face to a number challenges connected with statistical nature of fracture behaviour. Can be mentioned influence of loaded volume, surface quality, edge quality, loading rate etc. on resulting strength value \cite{3, 4}.

During the time a several other methods was proposed with the aim to eliminate disadvantages of presently used methods. Some of them based on multi-axially loaded plate are constantly of the interest \cite{5-8}. Recently the ball on three balls testing method was proposed as a candidate testing method for brittle materials for standardization \cite{9, 10}. The proposed method is been known for a long time in theory but difficulties around analytical solution of stress field (i.e. singularity under loading ball) was the factor limiting the successful application in the past. With the rapid development in computation technique together with shortening computational time it is possible to use a numerical simulation to obtain stress field for given testing geometry and corresponding loading conditions. However, there are still challenges to overcome and it is necessary to analyse different geometrical configuration to find the limits for smooth application of testing procedure.
The biggest advantage of involving the ball on three balls test into ceramics materials is relatively easy machining of the samples in some cases there can be used as fired components or discs. Important issue is the time necessary for testing of one specimen which is in case of ball on three balls test very shorten and can be easily tested statistically significant number of samples to obtain representative view about material behaviour.

Over all advantages of proposed testing method there are uncertain issues. Influence of Hertzian stresses and possible ring crack or further cone crack development on the loaded sample side being in contact with the loading ball is one of them. From the numerical analyses conducted on 3D model of test configuration there is indisputable presence of strong compressive stress field under the loading ball together with significant high tensile stress field around the contact area. The paper is focused in description of possibility to ring crack or cone crack formation and development during the test and influence of this process on results.

Experimental

An alumina ceramic was selected as a representative of widely used structural ceramics what the test were developed for. Experimental material was supplied in plate form of nominal dimensions 50x50x10 mm³. The plate was subsequently cut by diamond saw Struers Acutom50 into rectangular specimens of the dimensions 10x8.5mm² suitable for B3B testing having thickness of 2mm. All specimens were grinded and polished on either side (tensile and compressive) by standard ceramographics procedures to the final mirror like surface quality corresponding using a ¼ micron diamond paste. Especial attention was paid to quality of tensile surface of the specimen. For loading was used electromechanical testing machine Zwick equipped with TestXpert software for data acquisition and fixture for B3B test. The standard testing layout of the ball on the three balls test is displayed on Fig. 1.

![Fig. 1 Scheme of typical testing configuration of ball on three balls test.](image)

The experimental part was divided into three steps. First was to load plain samples up fracture in the ball on the three balls configuration to obtain maximum fracture forces. Loading of samples
containing artificial cracks in different distance from the centre of the loading ball up to predetermined load level below to the fracture force was next step. Artificial cracks were introduced into selected samples using a Vickers indenter by application of load 2kg which was sufficient to create cracks in indents corners having length of 150 μm. Scheme of the typical indents distribution along the compressive specimen side and an example of specimen containing indents after test with indicated contact area is displayed in Fig. 2. The red dashed line indicates boundary of the contact area. The last was to determine experimentally force necessary to initiate ring and/or cone crack. This force was determined as a force corresponding to final specimen fracture. For this reason were used both procedures i) sample loaded up to fracture ii) sample loaded on to certain load level with consequent microscopical observation checking presence of ring or cone cracks.

![Fig. 2 Scheme of indents distribution on compressive side of a specimen (left) and corresponding micrograph of tested specimen with contact area marked by dashed red line (right).](image)

All experiments were supported by optical microscopy observation of the loaded surface with the aim to observe possible crack development after load application as well as for fracture surfaces observation and cone crack detection. An image analyses were used to determine dimensions of the contact area and crack length measurement.

**Results and discussion**

Ball on three balls test was developed to simplify testing of structural ceramics and basics of this procedure are described elsewhere [9-11]. The fundamental approach is based on determination of the fracture force which is recalculated to fracture stress using following formula

$$\sigma_{\text{max}} = f \frac{F}{t^2},$$

(1)

where $f$ is the coefficient dependent on the loading conditions and usually is determined by FEM calculation and $F$ is the fracture force and $t$ is the thickness of sample. An example of the maximal principal stress distribution (from the sample centre to edges) determined for both sides of the sample (the tensile - red line and compressive surface - blue line) using three-dimensional numerical model is on Fig. 3. Details about numerical calculation and influence of selected geometrical and materials parameters can be found elsewhere [10].
Fig. 3 An example of maximum principal stress distribution from centre to edge of the sample for both tensile and compressive side.

Four critical points can be found with respect to possible ring and cone crack development in places where high compressive stresses appear. These places can be identified using combination of Fig. 1 and Fig. 3. First place can be found in the contact between a sample and loading ball. Other three are situated in contact areas between a sample and supporting balls. These three places are equivalent and therefore can be investigated as one case. The most severe loading conditions and corresponding compressive stress concentration is under loading ball. Supporting balls are loaded by one third of applied force however, there can be significant influence of specimen edges especially in cases when contact area of supporting balls is close to the edge (i.e. where specimen radius $R$ is similar to $R_a$ or specimen has rectangular shape).

Fig. 4 Force – displacement traces for specimen tested by B3B test.

The ball on the three balls was conducted on a series of alumina specimens and the fracture force was determined. Series of loading curves is in Fig. 4. The average fracture force on level of 1818N ± 139N was obtained from tests conducted on specimens without any artificial defects.

Typical example of broken specimen and its fragmentation is on the Fig. 5, where on the left hand side is top view on tensile side and on the right on compressive side. There was no significant
difference between fracture patterns within all tested specimens. Shape of fragments indicates that final fracture start in the middle of the tensile side and than propagates conically to the opposite compressive side. There was neither sign of ring or cone crack contribution on final fracture nor evidence of their presence. Identification of the possible presence of a ring or cone crack was not convictive due to an extensive fragmentation in the middle region of the specimen.

**Fig. 5** Top view on tensile side (left) and compressive side (right) of reassembled broken specimen having the fracture force of 2035N.

To proof absence of a ring or cone crack around contact area of the loading ball and the specimen before final fracture it was conducted series of test where load was applied only up to specific level. **Fig. 4** shows both selected levels 1500N and 1700N. A detailed microscopical observation of the compressive surface around the contact area was carried out. Neither ring nor cone crack formation was found in all specimens tested at mentioned loading levels.

To analyse possible ring or cone crack development under loading ball there was necessary to find conditions suitable for the crack formation. Similarity of loading conditions of the central (loading) ball in the B3B test with ball indentation test named also Hertzian indentation for brittle materials was used. A simplification in the way how is specimen support during both tests seems to be acceptable for specimens where the specimen thickness - ball radius ratio is relatively low.

**Fig. 6** Example of cone crack developed during Hertzian indentation test.

First step in this analysis was experimentally determined load level necessary for ring or cone crack formation. Basically the same experimental approach and specimen type was used with the difference in supporting part where flat support instead of three balls was used. Specimen was loaded up to fracture where ring and cone crack developed. The minimum applied force causing
ring or cone crack development and consequent final fracture of the specimen was observed on the level of 2500N. An example of developed cone crack is displayed in Fig. 6.

Proposed approach is taking into account fracture toughness of the specimen as a criterion to assess when fracture induced from the ring or cone crack is probable.

![Fig. 7 The geometry of Hertzian indentation test.](image)

There were many attempts to use Hertzian indentation where a hard sphere is pressed into the flat surface of brittle material for fracture toughness determination. The relatively simple and precise theory for fracture toughness determination brought Warren [12]. The theory is based on the presumption that ball and specimen is loaded in elastically. The geometry of used Hertzian contact is displayed on Fig. 7.

The peak pressure under the ball is given by following formula

\[ P_{\text{peak}} = \frac{3P}{2\pi r_c^3}, \tag{2} \]

where \( P \) is applied load and \( r_c \) is the contact radius. The contact radius is given by equation

\[ r_c = \left( \frac{3R_b P}{4E} \right)^{1/3}, \tag{3} \]

\[ \frac{1}{E^*} = \frac{1-v_2^2}{E_2} + \frac{1-v_1^2}{E_1}, \tag{4} \]

where \( R_b \) is the ball radius, \( E_1 \) and \( v_1 \) are elastic constants of the loading ball and \( E_2 \) and \( v_2 \) are those of the specimen. In the case when is possible to predetermine contact radius before fracture using an appropriate model or experimentally is possible to use presented theory for fracture toughness determination

\[ K_{lc} = \frac{E^* P_{\text{min}}^{FN}}{P_{\text{min}}^{FN}/R_b}, \tag{5} \]

where \( P_{\text{min}} \) is minimal force necessary for surface ring crack formation and \( P_{\text{FN}}^{\text{min}} \) is parameter dependent only on Poisson ratio and \( a/r_c \) ratio is determined by
where \( a \) is length of ring crack. Calculated values for selected materials can be found in tabulated form in [13]. The minimum force necessary for a ring crack formation can be determined experimentally as was proposed by Warren [12] where minimally 25 tests is recommended to obtain the minimal fracture force. Experimental approach for \( P_{\text{min}} \) determination is complicated for the purpose to estimate level of force where allows cone crack development in the ball on the three balls test. Therefore other approach can be selected for estimation of the minimal fracture force using the following formula suggested by Hertz [14, 15] for the crack initiation stage.

\[
P_{\text{min}} = \frac{3E^2r_c^3}{4kR_b},
\]

where

\[
k = \frac{9}{16}\left[(1-v_1^2)+(1-v_2^2)\frac{E_2}{E_1}\right].
\]

Table 1 Elastic and geometrical parameters for B3B test of alumina plates

<table>
<thead>
<tr>
<th>( E_1 ) [GPa]</th>
<th>( E_2 ) [GPa]</th>
<th>( v_1 ) [-]</th>
<th>( v_2 ) [-]</th>
<th>( R_b ) [mm]</th>
<th>( r_c^* ) [mm]</th>
<th>( r_c^{**} ) [mm]</th>
<th>( r_c^{***} ) [mm]</th>
<th>( t ) [mm]</th>
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* Estimated value by Thimosenko approximation \( r_c = t/6 \).
** Estimated using Eq. 3 when was selected to average measured force \( P = 1818N \).
*** Measured using image analyses from optical micrographs.

The elastic constants for both loading ball and specimen, measured and estimated contact radius, ball radius and specimen thickness is summarised in Table 1. For the calculation of the minimum force necessary to induce ring or cone crack can be done by two ways as was indicated earlier.

First straightforward way is to use Eq. 7 where contact area radius can be either measured or estimated. The value of \( P_{\text{min}} \) was calculated on the level of 750N when measured contact radius was used.

The second way is based on knowledge of fracture behaviour of the tested material where is possible to apply Eq. 5 proposed by Warren for fracture toughness determination. If the fracture toughness (for used alumina around 3.7 MPa.m\(^{0.5}\)) is known can be \( P_{\text{min}} \) expressed. Calculated value of \( P_{\text{min}} \) when \( P_{\text{FN}} = 2790 \) according to Warren [13] is 1845N.

From the comparison of obtained \( P_{\text{min}} \) and fracture forces is obvious that cone crack can nucleate for these testing conditions. Minimum force \( P_{\text{min}} \) calculated according to Eq. 5 seems to be closer to reality because there was no cone crack nucleation observed during B3B experiments. However, it was proved that favourable oriented pre-existing (artificially prepared by indentation) defect can grow due to presence of Hertzian stress even at level of 1700N.

By both approaches is possible to estimate force limit when ring or cone cracks can be developed. Comparing this value with the maximal force achieved by B3B test can by easily qualified the risk of ring or cone crack influenced results and when the \( P_{\text{min}} \) will be lower than maximum fracture force a detailed fractographical observation has to be conducted for each test to qualify if ring or cone crack were present and further validate or invalidate the received strength value.
Summary
Conce crack development during the ball on the three balls test was examined. Generally is can be claimed that the cone crack formation during the B3B test is possible and can have an influence on resulting strength values. However, it was demonstrated that using knowledge about stress conditions necessary for cone crack development is possible to significantly lower the risk. Two ways how estimate minimal force needed for ring or cone crack formation were described. Described approach based on knowledge of fracture toughness seems to be the effective tool for an optimal testing conditions assessment which can prevent ring or cone crack nucleation and further their development.

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References