Comparative study of different methods to estimate crack resistance curves from fracture tests using numerical crack growth simulation techniques

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Abstract

Different methods to estimate the instantaneous crack length and the crack resistance curve, respectively, from a single specimen fracture test are discussed. The Node-Release-Technique and a ductile damage model were used to simulate ductile crack growth in C(T) specimens under plane strain conditions. Three hypothetical materials have been considered, where the parameters related to material model and crack growth simulation technique were chosen in order to obtain three similar load versus displacement responses. It is shown that similar load displacement results may correspond to different crack growth behaviour and the capability of different methods to estimate the instantaneous crack length to distinguish between these different cases is discussed. The resistance curves predicted by the different estimation schemes are compared with the corresponding results of the numerical simulations.

Introduction

The ASTM standard E 1820 [1] proposes a method based on the work of Rice [2] and Ernst et al. [3] to estimate crack resistance curves from the load displacement response of a fracture test. To apply this method, the instantaneous crack length as a function of the load line displacement has to be known. A methodology to deduce the instantaneous crack length directly from the load displacement record of a single specimen test was first proposed by Joyce et al. [4] and has been applied recently to dynamic fracture tests by Joyce et al. [5]. On the other hand, a so-called normalization method was proposed by Landes and Herrera [6] and further extended and applied for example by Landes and Herrera [7] and many others. Based on the Common Format Equation (CFE) developed by Donoso and Landes [8], a method to estimate crack resistance curves from the load displacement response was proposed by Donoso et al. [9], where an approximation for the crack advance as function of the load line displacement was incorporated into the CFE. This finally lead to an analytic estimation for the crack resistance curve.

The aim of the work presented here is to discuss different methods to estimate the instantaneous crack length and the resistance curve, respectively, by means of numerical experiments. All of the considered methods can be written by the use of a load separation approach, where the Common Format Equation is used here.

Brief review of the Common Format Equation (CFE)

The total displacement of the load - displacement record of the fracture test is splitted into its elastic and its plastic part

$$v = v^{el} + v^{pl} \tag{1}$$

and the relation between load and plastic displacement is written as a product of three terms

$$P = \Omega^* G H \, . \tag{2}$$

with

$$G = CBW \left[\frac{b}{W} \right]^m , \tag{3}$$

where b is the ligament size, given approximately by the difference between the in-plane geometry parameter W and the crack length a ($b = W - a$) and B is the specimen thickness. The constants C and m have to be determined for the considered specimen geometry. In Donoso and Landes [10], $C = 1.553$ and $m = 2.236$ were obtained for the C(T) specimen. The function H in (2) is related to the hardening behaviour of the material and v^{pl}. Here, it is assumed that the stress-plastic strain relationship ($\sigma_Y - \bar{\varepsilon}$) can be approximated by a power law and therefore H is set to

$$H = \sigma_{Y_0} \left(\frac{E}{\sigma_{Y_0} \alpha} \right)^{1/n} v_N^{\frac{1}{n}} \quad \text{with} \quad v_N = v^{pl}/W \tag{4}$$

where E is the elasticity modulus, σ_{Y_0} the yield stress and n and α are constants used to fit the experimental data. Finally, the factor Ω^* in equation (2) is interpreted as a constraint factor. For plane strain, $\Omega^* = 0.38$ was found elsewhere. The counterpart of (2) with respect to the relationship between load and elastic diplacement provides an estimation for the elastic compliance of a blunt notched specimen in plane strain

$$c = \frac{A(1 - \nu^2)}{BE} \left(\frac{b}{W} \right)^{-\mu} \tag{5}$$

where $A = 7.60$ and $\mu = 2.28$ were found for the C(T) specimen in [10].

Brief review of the considered estimation methods

Compliance ratio methodology (CRM)

The approach proposed by Joyce et al. [4] takes advantage of the fact that the elastic compliance of a fracture specimen decreases with increasing crack length. The actual compliance
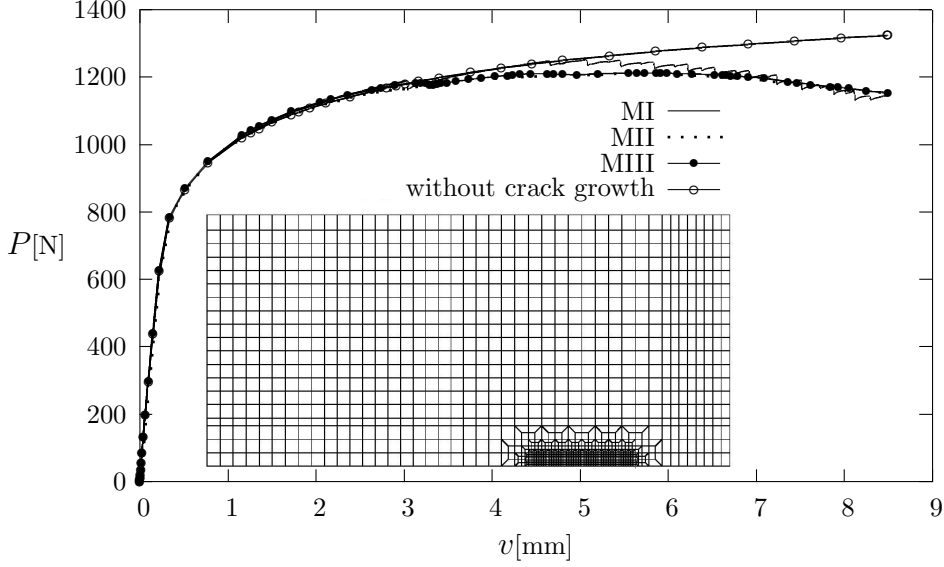
FIGURE 1. P-v response for the different materials and finite element mesh ($l_e = 0.2$mm).

is deduced by comparison with the load displacement record which corresponds to a blunt notched specimen without crack growth but with identical inicial ligament length b_0. The concept makes use of (1) and the instantaneous crack length is estimated from the compliance ratio assuming that after the onset of crack growth, the change of v^{el} can be neglected. Using (1), the actual load line displacement v can be written as

$$v = v_{bn}^{pl} + P_{bn}c_{bn} = v_{cg}^{pl} + P_{cg}c_{cg} \tag{6}$$

where the subscripts 'bn' and 'cg' are used to distinguish between the quantities which correspond to the blunt notched specimen and the fracture specimen, respectively. The compliance of the blunt notched specimen (c_{bn}) can be approximated by means of (5) as follows

$$c_{bn} = \frac{A(1 - \nu^2)}{BE} \left(\frac{b_0}{W} \right)^{-\mu} \tag{7}$$

and the compliance of the specimen with growing crack is written similarly

$$c_{cg} = \frac{A(1 - \nu^2)}{BE} \left(\frac{b}{W} \right)^{-\mu} \tag{8}$$

using the instantaneous ligament length b instead of the initial ligament length b_0. According to the assumptions mentioned above, v_{bn}^{pl} and v_{cg}^{pl} are approximately equal and it follows from (6) that the instantaneous ligament length is given by

$$\frac{b}{W} = \frac{b_0}{W} \left(\frac{P_{bn}}{P_{cg}} \right)^{-1/\mu}. \tag{9}$$

Once b/W is known, the ASTM scheme can be applied to calculate the resistance curve.
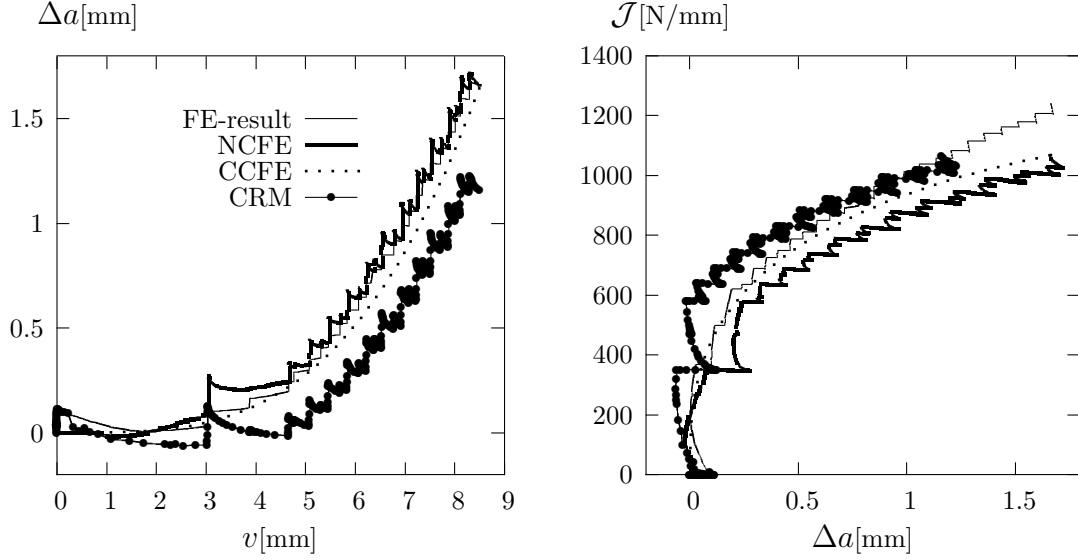
FIGURE 2. Comparision of the results for the material MI

Normalization using the CFE (NCFE)

Similar to (2), the load is written as a product of a deformation function $\tilde{H}$ which depends on the normalized plastic displacement and the terms which are related exclusively to the geometry of the specimen

$$P = \Omega^* G \tilde{H}(v_N) \,. \tag{10}$$

For a blunt notched specimen, the calibration function G depends on the initial ligament length b_0 whereas in the case of crack growth G depends on the instantaneous ligament length. The normalized load P_N is defined as

$$P_N = P/G \,. \tag{11}$$

It follows from (10) and (11) that

$$P_N = \tilde{H}(v_N) \tag{12}$$

and a functional form has to be chosen for $\tilde{H}(v_N)$. Here, the power law approximation

$$\tilde{H}(v_N) \approx c_0 v_N^{c_1} \tag{13}$$

is used, where c_0 and c_1 are related by the fact that one point of the normalized load versus displacement curve can be determined using the final crack length and the functional form has to coincide with this point. Furthermore, assuming that initiation of crack growth starts at $v_N > 0$, P_N has to coincide at least for very small v_N with the stationary crack response. It follows from (13), (10) and (3) that once the exponent c_1 is known, the instantaneous crack length is given by

$$\frac{b}{W} = \left[\frac{P}{P_f} \left(\frac{b_f}{W} \right)^m \left(\frac{v_N}{v_{N_f}} \right)^{-c_1} \right]^{1/m} \,. \tag{14}$$

where b_f is the final ligament length and P_f and v_{N_f} are the corresponding load and normalized plastic displacement, respectively.
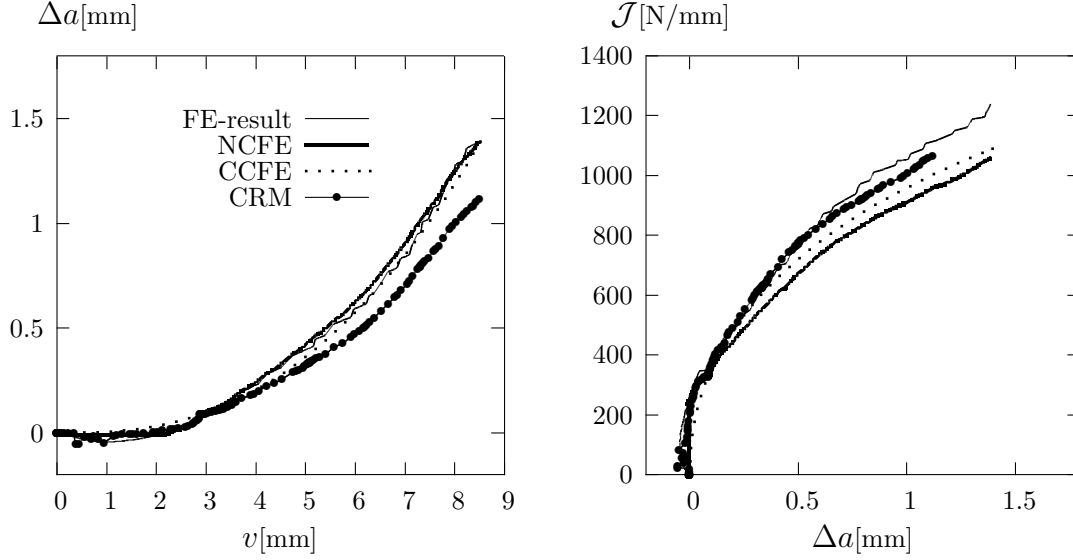
FIGURE 3. Comparision of the results for the material MII

Closed form of the resistance curve based on the CFE (CCFE)

The methodology proposed by Donoso et al. [9] applies a power law fit to approximate the instantaneous ligament length as a function of the load line displacement as follows

$$\frac{b}{W} = \frac{b_0}{W} - \frac{\Delta a_f}{W} \left(\frac{v_N}{v_{N_f}} \right)^{l_1}$$ (15)

where Δa_f denotes the total crack advance. The approximation (15) is incorporated into the CFE (2). The parameter l_1 is determined in order to obtain coincidence between the load versus displacement record predicted by the CFE and the corresponding fracture test data. Finally, the $\mathcal{J}$ integral was obtained in Donoso et al. [9] for plane strain as follows

$$\mathcal{J} = \frac{K_I^2}{E}(1 - \nu^2) + m\Omega^*\sigma^*CW\frac{n}{n+1} \left(\frac{b_0}{W} \right)^{m-1} \left(1 - \frac{\Delta a_f}{b_0} \left(\frac{v_N}{v_{N_f}} \right)^{l_1} \right)^{m-1} v_N^{1+\frac{1}{n}}$$ (16)

where the elastic component of $\mathcal{J}$ is expressed by means of the mode-I stress intensity factor K_I which can be determined using estimations given in [1] or other methods.

Numerical simulation of ductile crack growth

Here, the node-release technique and the continuum damage model proposed by Gurson [11] and further modified and extended by Tvergaard [12], Tvergaard and Needleman [13] were employed. The ABAQUS node-release facility Hibbit et al. [14] requieres that the instantaneous crack length has to be given in a discrete form as a function the load line displacement and according to these data, the progam releases the corresponding nodes during the calculation. In the following, the index notation in conjunction with the summation convention is

TABLE 1. Different hyphotetical materials considered here

Material	Simulation technique	parameters related to crack growth									
MI	node-release	$\Delta a = c_1 v^{c_2}$									
		f_0	f_c	f_N	s_N	ϵ_N	q_1	q_2	q_3	κ	l_e [mm]
MII	GTN	10^{-6}	0.3	10^{-6}	0.1	0.5	1.75	1.0	3.0625	4	0.1
MIII	GTN	$2\ 10^{-5}$	0.2	0.01	0.1	0.5	1.5	1.0	2.25	4	0.2

used to outline the essentials of the GTN model which consists of the yield condition

$$\Phi = \left(\frac{3}{2}\frac{\Sigma'_{ij}\Sigma'_{ij}}{\sigma_Y^2(\bar{\varepsilon})}\right)^2 + 2q_1 f^* Cosh\left(\frac{1}{2}q_2\frac{\Sigma_{kk}}{\sigma_Y(\bar{\varepsilon})}\right) - 1 - q_3(f^*)^2 \qquad (17)$$

and evolution equations for the internal variables. The Σ_{ij} and Σ'_{ij} in (17) are the components of the Cauchy stress and its deviatoric part, respectively, and $\sigma_Y(\bar{\varepsilon})$ stands for an averaged stress versus plastic strain curve of the matrix material. The fit parameters q_1, q_2, q_3 in (17) have been proposed in [12] in order to get a better agreement between the predictions of the original model and the results obtained by cell model calculations. To take into account the loss of stress carrying capacity associated with void coalescence, the modified damage parameter f^* was proposed in [13] as a piecewise linear function of the void volume fraction f

$$f^* = \begin{cases} f & f \le f_c \\ f_c + \kappa(f - f_c) & f > f_c \end{cases} \quad \text{with} \quad \kappa = \frac{f_U^* - f_c}{f_F - f_c}.$$

The parameter f_U^* is related to q_1 by $f_U^* = 1/q_1$ if $q_3 = q_1^2$ is used. The void volume fraction where void coalescence starts is indicated by f_c and the void volume fraction at final fracture is denoted by f_F. The change in void volume fraction is given by

$$\dot{f} = (1-f)\dot{E}_{kk}^{pl} + \frac{f_N}{s_N\sqrt{2\pi}}Exp\left(-\frac{1}{2}\left[\frac{\bar{\varepsilon} - \epsilon_N}{s_N}\right]^2\right)\dot{\bar{\varepsilon}} \quad , \qquad (18)$$

where the the plastic strain rates are termed by $\dot{E}_{ij}^{pl}$ and ϵ_N, s_N and f_N are material parameters. Finally, the evolution equation for the equivalent plastic strain $\bar{\varepsilon}$ is given by

$$\dot{\bar{\varepsilon}} = \Sigma_{ij}\dot{E}_{ij}^{pl}/(1-f)/\sigma_Y(\bar{\varepsilon}). \qquad (19)$$

The GTN model suffers from a spurious mesh dependence of the numerical results after the onset of localization of deformation. An engineering approach to handle this problem is based on the idea to interpret the dimension of the finite elements within the localization zone l_e as an additional parameter.

Three hypothetical materials (MI, MII, MII) are considered. The elasticity modulus $E = 200$GPa, the Poisson ratio $\nu = 0.3$ and the yield stress $\sigma_{Y_0} = 235$MPa have been chosen. It is assumed that the hardening behaviour can be characterized by a power law with $\alpha = 2$ and $n = 6$. All three materials have identical characteristics in terms of elasticity constants and stress versus plastic strain curve but different micromechanical properties as shown in Table
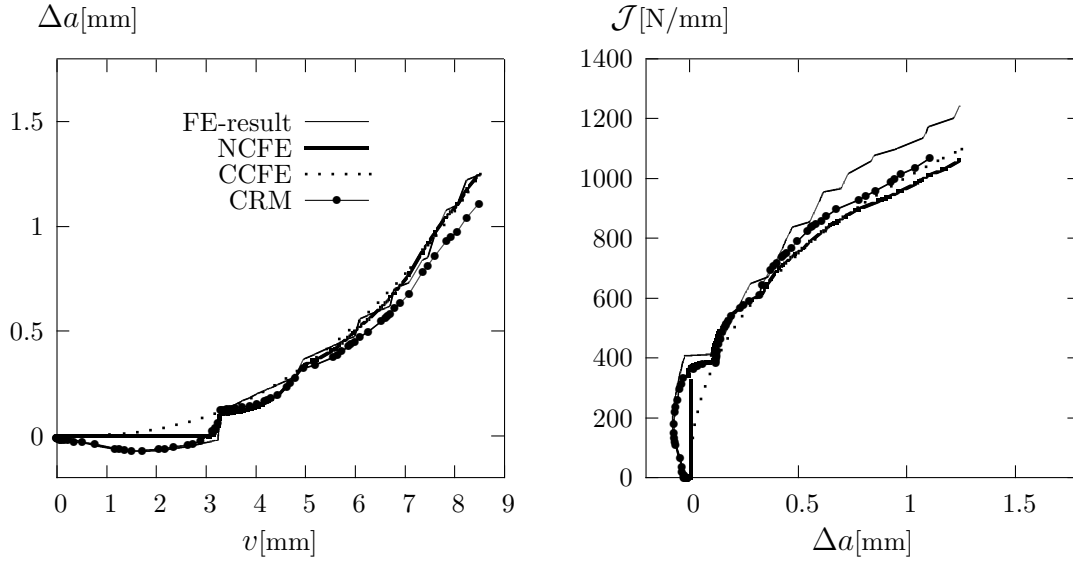
FIGURE 4. Comparision of the results for the material MIII

1.First the parameters for the material MI were chosen and the load displacement response was determined numerically by means of the node-release technique. Two different values l_e have been used for the materials MII and MIII and the other parameters given in Table 1 were determined in order to get approximately coincidence with the load displacement curve obtained for material MI as shown in Figure 1. All numerical experiments were carried out considering C(T) specimens in plane strain with $W = 50$mm and $a_0/W = 0.622$. An example for the finite element meshes can be found in Figure 1. Eight-node and four-node plane strain elements were used for material MI and for the materials MII, MIII, respectively.

Results and discussion

The results in terms of $\Delta a(v)$ and $\mathcal{J}(\Delta a)$ obtained by the different estimation methods are shown together with the corresponding numerical results in the Figures 2, 3 and 4, respectively. The crack resistance curve for the CCFE-method has been calculated by the use of (16). The ASTM scheme was employed for the other estimation methods. Due to the nature of the power law approximation (13), spurious numerical values Δa were obtained for very small v in the case of the NCFE method. Not only these spurious results but also the negative values obtained for v approximately less then 2.5 - 3mm were set to zero because in this case it cannot be decided objectively which of these negative values are valid or not. However, the finite element results show that due to the measurement of Δa by means of $W - b$, negative values for Δa before iniciation of crack growth are not unreasonable. The numerical results obtained for $\Delta a(v)$ are estimated quite well by the NCFE method and the CCFE method for all three materiales. However, both methods underestimate the $\mathcal{J}(\Delta a)$ response. Due the tendency to underestimate the crack resistance, conservative results were obtained for all three materials by these methods (NCFE, CCFE), where the use of the CRM method results in a non-conservative estimation for the material MI. As it was forced by

the choice of the material parameters given in Table I, almost no difference can be observed with respect to the final loads but the values obtained for final crack length differ significantly. Because the CRM deduces the instantaneous crack length exclusively from the load displacement response, it can not distinguish between these three cases considered here and provides almost identical values a_f for all three materials. On the other hand, because of the difference between $\tilde{H}$ (13) and H (4) the NCFE shows inconsistencies in the argumentation. Because the CCFE method can be interpreted as an inverse normalization it suffers as well from this inconsistencies.

Acknowledgements

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