

ADVANCES IN THE APPLICATION OF NON-LOCAL DAMAGE MODELS IN THE SIMULATION OF DUCTILE CRACK-EXTENSION

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Abstract

Local damage models usually have the disadvantage that results are strongly mesh dependent. The reason is that the type of the underlying partial differential equations changes under quasi-static conditions from elliptic to hyperbolic. Non-local damage models do not exhibit such behaviour under certain conditions. The usage of such non-local damage models in finite element analyses opens the possibility for preserving the ellipticity of the partial differential equations and thus avoiding mesh dependence of numerically obtained results. The loss of ellipticity for local models and its preservation for non-local models are demonstrated for a wide variety of examples enclosing ductile damage. In the present investigation, the non-local damage model is applied to the simulation of ductile crack extension in fracture mechanics specimens. The type of the underlying differential equations is permanently analysed and controlled.

Non-local extension of Gurson's model

The basic equations of the local Gurson model in the formulation of Tvergaard and Needleman include the yield condition

$$\Phi(\boldsymbol{\sigma}, \sigma_M, f) = \frac{\sigma_v}{\sigma_M^2} + 2 q_1 f^* \cosh\left(q_2 \frac{\text{tr}(\boldsymbol{\sigma})}{2 \sigma_M}\right) - (q_1 f^*)^2 - 1 = 0, \quad (1)$$

the evolution equation for the modified void volume fraction

$$f^*(f) = \begin{cases} f & \text{for } f \leq f_c \\ f_c + k_f (f - f_c) & \text{for } f > f_c \end{cases}, \quad k_f = \frac{(f_u^* - f_c)}{(f_f - f_c)}, \quad f_u^* = \frac{1}{q_1} \quad (2)$$

and the evolution equation for the void volume fraction

$$\dot{f} = \dot{f}_{growth} + \dot{f}_{nucleation} = (1-f) \text{tr}(\mathbf{D}_p) + A(\varepsilon_M) \dot{\varepsilon}_M. \quad (3)$$

The meaning of those parameters is explained several times elsewhere, e.g. [1]. It is typical for local models that only the equilibrium equations for the forces are fulfilled within the context of a finite element analysis. State dependent variables, such as the void volume fraction f , dependent on the evolution equations and the achieved equilibrium of the whole structure. This means that adjacent state dependent variables dependent

indirectly from each other by means of the achieved equilibrium. It is typical for the results obtained by local damage models that strain softening yields strongly mesh dependent results because the underlying differential equations do not remain elliptic [2].

An excellent method to overcome such problems are non-local damage models. The basic idea is to introduce an additional degree of freedom d as a non-local damage variable for which the balance equation

$$\dot{d} - c\nabla^2 \dot{d} = \dot{f} \quad \text{with } c \geq 0 \quad (4)$$

holds for the time derivative \dot{d} . The coupling between the spatial damage is done by the Laplacian ∇^2 and the equation is driven by its right hand side \dot{f} which is identical to the time derivative of the still existing local damage variable f . The newly introduced material parameter c describes the intensity of coupling the damage in spatial direction and is sometimes called critical length. If c equals zero, the gradient part in eq. 4 vanishes and the local model results. In the present investigation, c is assumed to be a constant.

Additionally, we introduce a modified yield condition, where the local modified void volume fraction f^* is replaced by the modified non-local void volume fraction d^*

$$\Phi_d(\boldsymbol{\sigma}, \sigma_m, d^*) = \frac{\sigma_v^2}{\sigma_m^2} + 2 q_1 d^*(d) \cosh\left(q_2 \frac{\text{tr}(\boldsymbol{\sigma})}{2 \sigma_m}\right) - (q_1 d^*(d))^2 - 1 = 0. \quad (5)$$

The definition of the non-local void volume fraction is almost identical to the local case:

$$d^*(d) = \begin{cases} d & \text{for } d \leq d_c \\ d_c + k_d (d - d_c) & \text{for } d > d_c \end{cases}; \quad k_d = \frac{(f_u^* - d_c)}{(f_f - d_c)}. \quad (6)$$

The boundary conditions for the non-local damage variable \dot{d} are

$$\dot{d} = 0 \quad \text{and} \quad \frac{\partial \dot{d}}{\partial \mathbf{n}} = \nabla \dot{d} \cdot \mathbf{n} = 0 \quad (7)$$

and hold for the cases that the rate of non-local damage production vanishes and that a “flow” of voids normal to the boundary is not possible, respectively. The constitutive approach is justified in the framework of rational thermodynamics as shown in [3, 4].

The implementation of the model into a finite element code requires a weak formulation of eq. (4)

$$\int_V \{(\dot{d} - \dot{f})\eta + c \nabla \dot{d} \cdot \nabla \eta\} dV - c \int_{\Gamma} \nabla \dot{d} \cdot \mathbf{n} \eta \, d\Gamma = 0, \quad (8)$$

where η is a test or variational function fulfilling the boundary conditions of \dot{d} . The model is implemented into a commercial finite element code allowing for an additional degree of freedom beyond the three degrees of freedom for the conventional static equilibrium.

Loss of ellipticity

The loss of ellipticity of partial differential equations is typically associated by the loss of uniqueness of the solution of such equations. The underlying system of differential equations for the local case is given in terms of the displacements \mathbf{u} . The rate form for these equations is written as

$$\operatorname{div}(\dot{\mathbf{T}}) = \operatorname{div}(\underline{\underline{\mathbf{C}}} : \mathbf{L}) \quad (9)$$

following from the equilibrium equations with the rate of Cauchy's stress $\dot{\mathbf{T}}$, the spatial velocity gradient \mathbf{L} and the constitutive law with its operator $\underline{\underline{\mathbf{C}}}$. The ansatz for the velocity field is

$$\dot{\mathbf{u}}(\mathbf{x}) = \dot{\mathbf{u}}_0 \exp(ik \mathbf{n} \cdot \mathbf{x}) \quad (10)$$

where \mathbf{x} denotes a material point, \mathbf{n} the direction of a wave front and k the wave number. The unknown velocity amplitude is $\dot{\mathbf{u}}_0$ and i the imaginary unit. Introducing the ansatz (10) into the equilibrium yields a system of linear equations, which has only solution, if

$$\det(\mathbf{n} \cdot \underline{\underline{\mathbf{C}}} \cdot \mathbf{n}) = \det \mathbf{H} = 0, \quad (11)$$

which is the condition for the loss of ellipticity for local models, wherein \mathbf{H} is introduced as the acoustic tensor. At this point, a strain rate jump with an unknown amplitude $\dot{\mathbf{u}}_0$ may occur in the direction of \mathbf{n} .

For non-local models not only the displacements \mathbf{u} are primary unknowns, but the non-local damage d as well. For that reason, the ansatz of eq. (9) is extended to the non-local case by

$$\dot{d} = \dot{d}_0 \exp(ik \mathbf{n} \cdot \mathbf{x}) \quad (12)$$

which is introduced into the balance equation (4) resulting together with eqs. (9, 10) in a 3x3 system of linear equations for every material point \mathbf{x} and every direction \mathbf{n} . This system of equations has only solutions, if

$$\det(\mathbf{n} \cdot \underline{\underline{\mathbf{C}}}^{\text{NL}} \cdot \mathbf{n}) = \det(\mathbf{H}^{\text{NL}}) = 0, \quad (13)$$

where the non-local constitutive operator is denoted as $\underline{\underline{\mathbf{C}}}^{\text{NL}}$ and the respective non-local acoustic tensor as \mathbf{H}^{NL} . Typically both, the constitutive operator and the acoustic tensor depend on the wave number k and the material parameter c . A schematic draw in Fig. 1 shows that $\det \mathbf{H}^{\text{NL}}$ depending on c and k may take values smaller or larger than zero meaning that even non-local equations may become hyperbolic. The condition (13) for preserving ellipticity, i.e. $\det \mathbf{H}^{\text{NL}} > 0$, is only fulfilled, if sufficiently high values of c and k are taken, which typically is ensured by the problem formulation and an adequate choice of c . The meaning from a more practical point of view is that the spatial coupling of damage expressed by the material parameter c has to be strong enough and that the wave number k should be chosen sufficiently high. The last point is easily achieved as only high wave number k are important because only they may lead to strain rate jumps.

Results

Fig. 2 shows a rectangular plate under plane strain conditions loaded by a prescribed displacement on its upper boundary. The initial imperfection consists of a slightly larger initial void volume fraction f_0 somewhere in the inner domain of the plate. The calculations are performed by using regular meshes with different mesh sizes. The load vs. displacement curves reveal the strong dependence on the mesh size for the local case, i.e. smaller meshes lead to earlier failure. In contrast to that, the results of the non-local model are almost identical for the investigated mesh sizes.

Fig. 3 provides a more detailed information on the damage in the rectangular domain. Especially for the local case, the thickness of the damage band decreases with the element size. Even the orientation of the damage band changes depending on the element size. In contrast to that, the orientation and the thickness remain constant for the non-local calculations.

The results regarding the loss of ellipticity are given in the lower part of Fig. 3. On the ordinate, $\det \mathbf{H}$ for the local and $\det \mathbf{H}^{\text{NL}}$ for the non-local case are given vs. the upper displacement. Both acoustic tensors are calculated at every integration point and the normal vector \mathbf{n} is taken for every position from $-\pi$ to $+\pi$. The minimum of all of these calculations is taken for Fig. 2 at one upper displacement. Loss of ellipticity occurs for the whole structure, if only in one point condition (11) or (13) is achieved during the whole process. For the local case, condition (11) is obviously violated and ellipticity is lost. In contrast to that, condition (13) is not fulfilled meaning that ellipticity is ensured and numerically obtained results are reliable.

The aim is the application of non-local models to simulate ductile crack growth in fracture mechanics specimens. As a first example, the influence of mesh sizes on load vs. displacement curves is shown in Fig. 4. In total, we investigate the influence of three element sizes with edge lengths from 0.05 mm to 0.2 mm in the crack tip region. The load vs. displacement curves in the upper part of Fig. 4 exhibits the strong influence of the mesh sizes on the results. The smaller the elements are in the crack tip region the earlier failure occurs thus leading to decreasing load vs. displacements curves. A convergence of load vs. displacement curves against a limiting curve with decreasing mesh sizes as it is known from elliptical problems does not exist for the local formulation of such damage models. From other investigations it is known that ellipticity is lost for local models in a very early loading stage in the application to fracture mechanics specimens [2, 4].

Additionally, the maximum of the damage does not appear on the symmetry line, but the second and third row of elements, which is unrealistic as ductile crack growth typically appears on the symmetry line of the specimen.

In the lower diagram of Fig. 4, the results for the non-local case are shown for the identical mesh sizes as used in the local case. The three load vs. displacement curves are much closer together than for the local equations. That these curves are not identical is related to the fact that the problem is elliptical and that the numerical solutions converge against a limit with decreasing mesh size. It was shown in earlier investigations that the equations remain elliptic for that example [2, 4]. For the non-local case, the evolution of damage occurs on the symmetry line, as shown in the lower part of Fig. 4. This result is absolutely realistic and shows the correctness of the proposed method.

Conclusions

Local damage models have the deciding disadvantage that the obtained results in any numerical procedure are mesh dependent resulting from the loss of ellipticity of the underlying system of differential equations. Thermodynamically founded non-local models preserve ellipticity over a wide range and they are a promising step to overcome such problems. The mesh dependence only occurs in a conventional way as it is typical for elliptic problems. The progress for the application of damage models is that such non-local models are applied to simulating ductile crack growth in fracture mechanics specimens. It is shown that the results are widely independent of the underlying FE-mesh.

References

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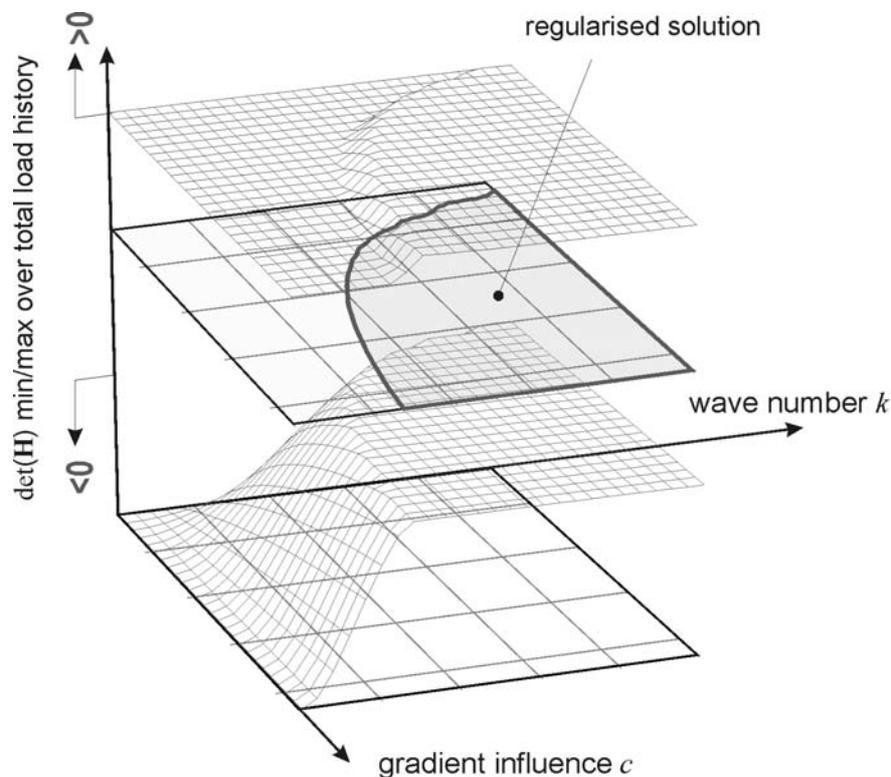


FIGURE 1. Influence of the gradient strength c and wave number k on the regularization

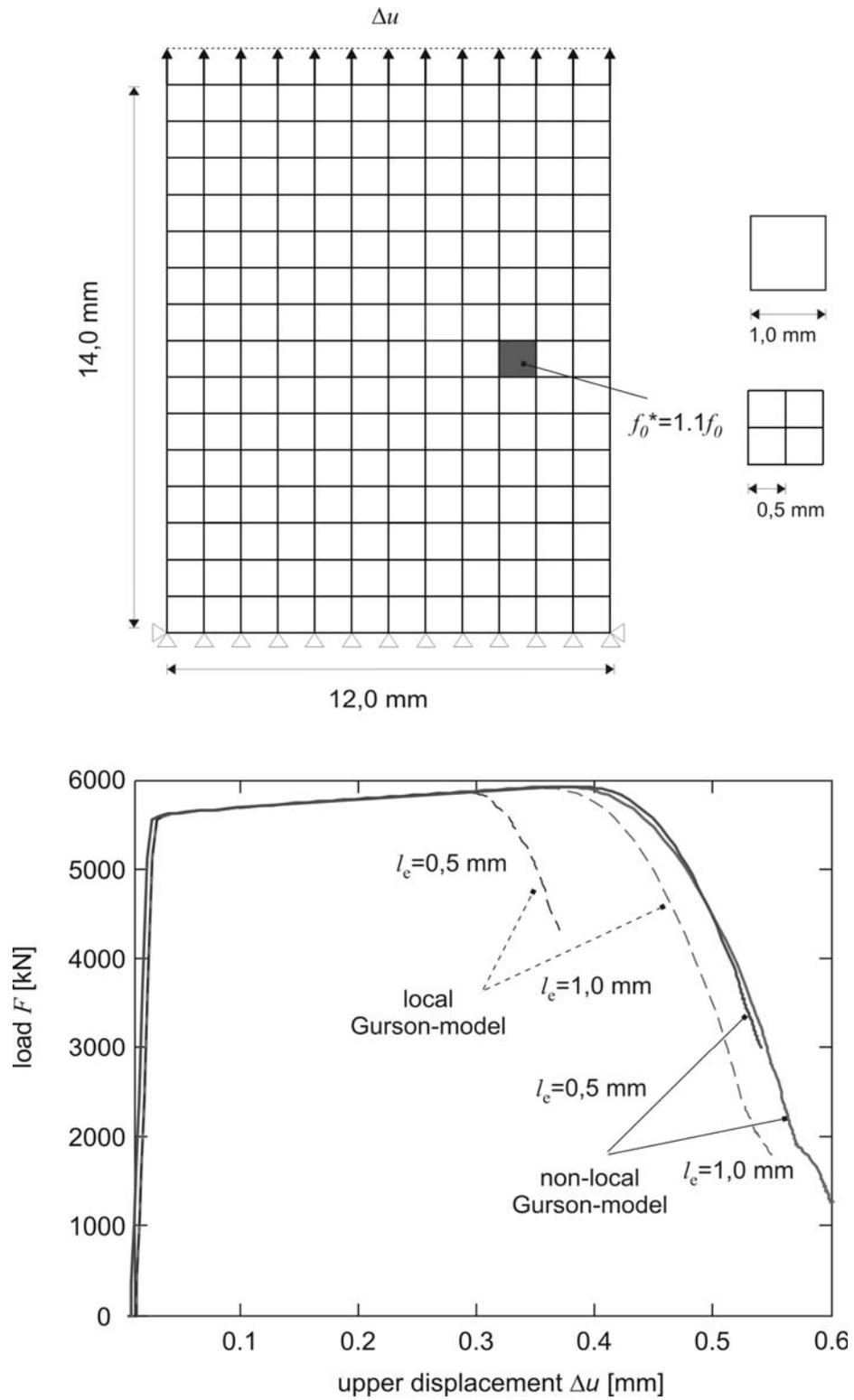


FIGURE 2. Load-displacement response for a rectangular plate with plane strain conditions under tensile load (local and non-local model)

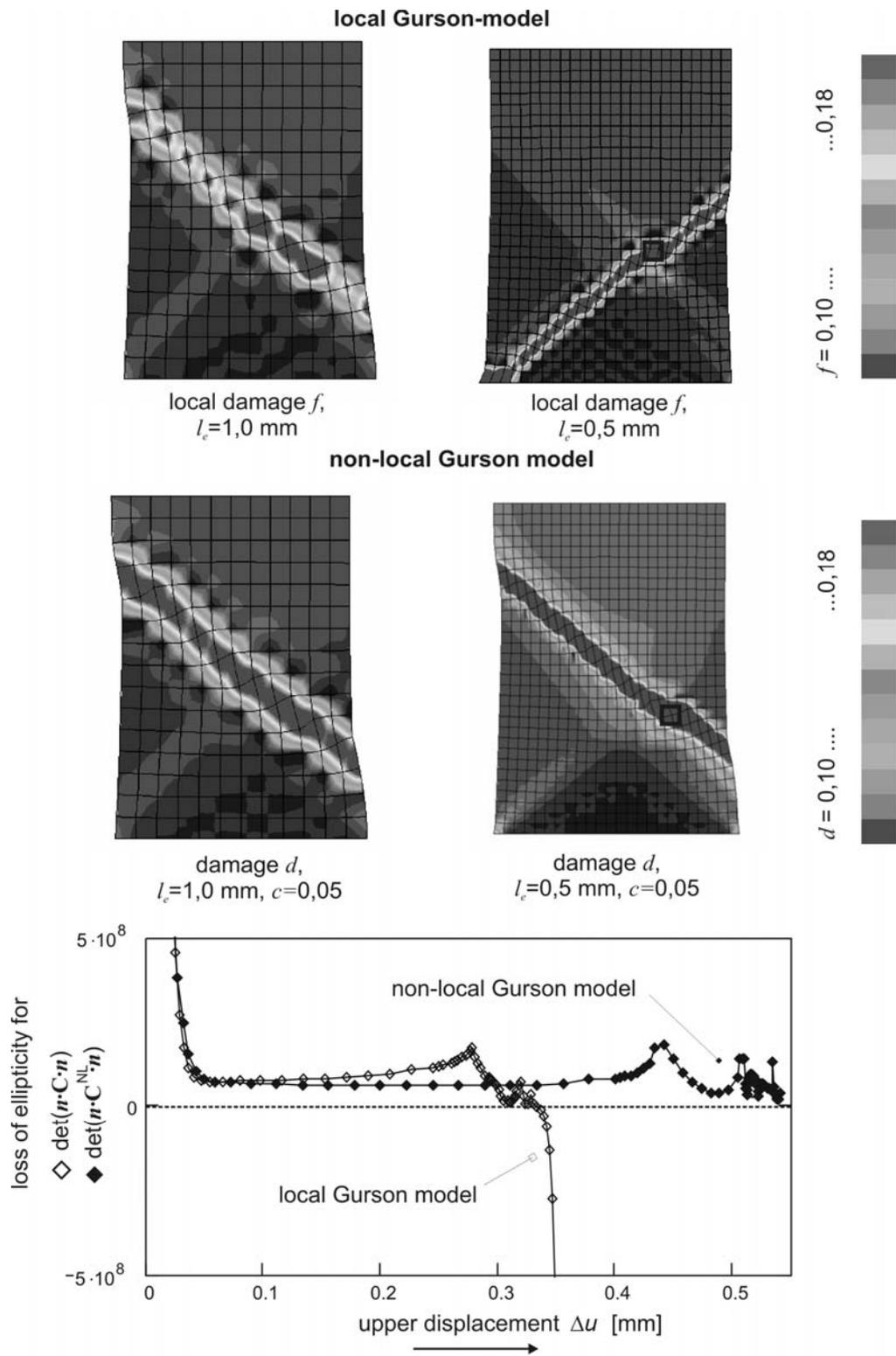


FIGURE 3. Damage distribution for a rectangular specimen under tensile load

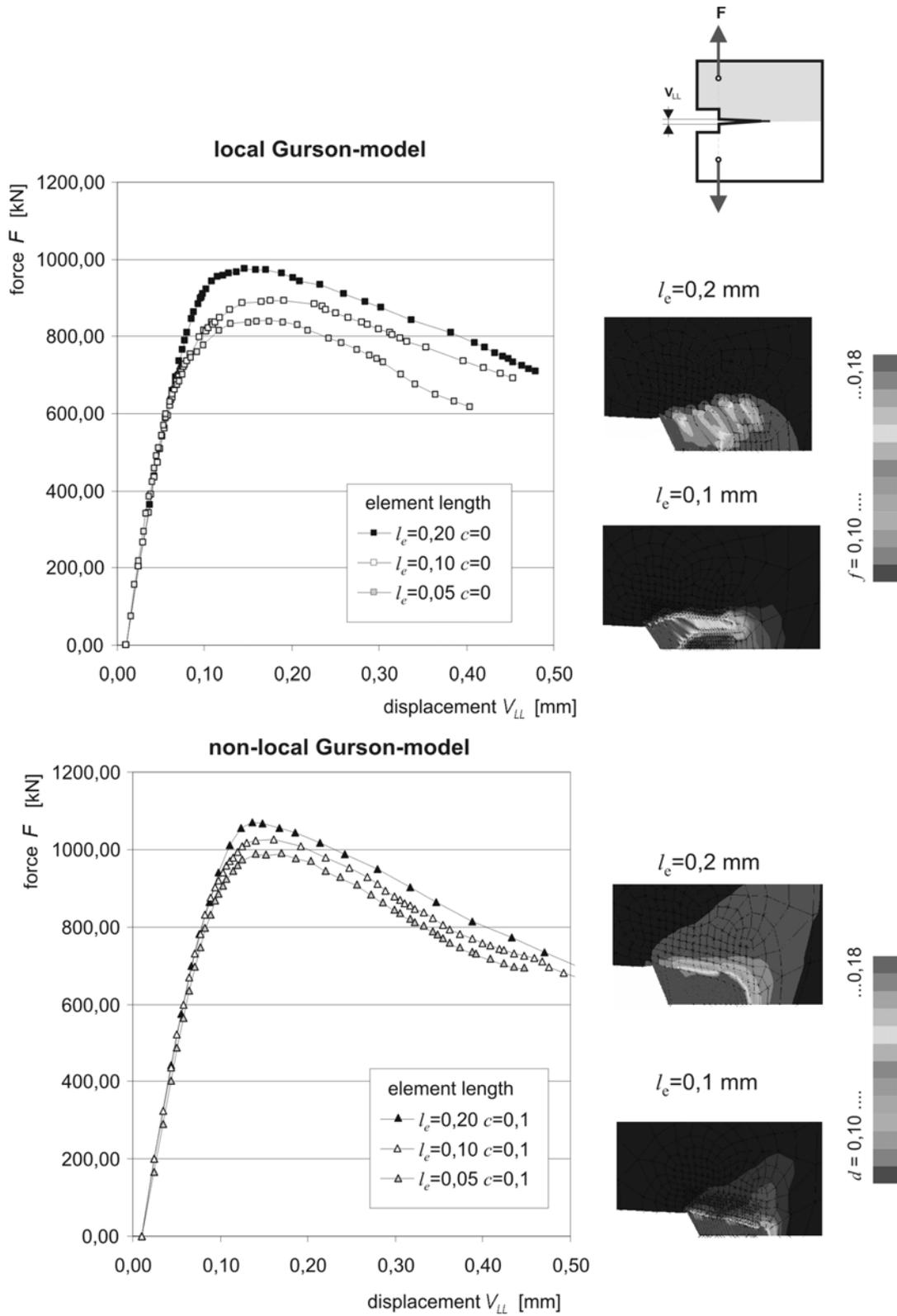


FIGURE 4. Local vs. non-local crack progress simulation for C(T) specimen with various mesh discretizations ($l_e=0,2$ mm and $0,1$ mm; material: 10MnMoNi5-5)