Single-Parameter Prediction of Stable Crack Growth in Large-Scale Panels

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**ABSTRACT:** Predictive capabilities of the so-called Unified Methodology (UM) are examined by the use of a Transferring Law (TL), to say, a common function of the test data on moving center cracks of different length in rather small specimens made from a ductile material. A combined analysis of these data and closely corresponding evidence from the literature indicate that the TL may be used, to a first approximation, as a simple quantitative tool to predict residual strength of proportionally scaled plates at least under uniaxial tension. The various effects of in-plane constraint, among them load biaxiality, are covered by the UM analysis.

**INTRODUCTION**

The mode of loading in thin-wall structures tends to be uniaxial or biaxial tension. It is highly improbable to predict stable crack growth which develops unconstrained flow fields using the R-curve concept in isolation from analyses of the global deformation pattern and necking [1, 2]. Nowadays it is generally agreed that the constant level $\psi_{ss}$ of the Crack-Tip Opening Angle is a more fundamental fracture criterion value than $K_R$ or $J_R$ resistance curves. However, experimentally measured and analytically derived $\psi_{ss}$ angles can differ significantly. From [3] it follows that for the large center-cracked panels respective values of $\psi_{ss}$ were 5.5 and 3.4 degrees. Such type of inconsistencies between computational and experimental data demonstrates convincingly that [4]: “a universal fracture law governing slow crack growth has not been found yet”.

This paper is presented from the viewpoint of “moving crack tip” embedded into the fully-developed “moving neck”. They both spread straight across the ligament under quasistatic loading. The aim is to check the potential of the UM [2,5] against predicting an upper limit to the critical load that a large-scale panel is able to sustain under uniaxial tension.
TEORETICAL BACKGROUND IN BRIEF TERMS

A center crack in an unconstraint Problem Domain (PD) of uniform thickness $B$ is modeled by an elliptic hole (Figure 1) having length $2c >> B$. In a stress-free PD, the hole has the fixed radii $\rho_n = b^2 / c$ and $\rho_m = c^2 / b$ of an extreme curvature at the points “n” and “m” on ideal crack profiles of different length. The procedure used for determination of $\rho_n$ and $\rho_m$ values, treated as characteristics of an actual crack in a given material, is outlined in [6]. An imaginary state of the stress-free plate relates to a virgin material, that is, to the zeroth level of a structural damage. Other important test events are the state “u” of the completely unloaded PD and the state “s” when both crack-tips are advancing under steady-state conditions.

Geometry-independent resistance to ductile tearing is only recovered under restrictive conditions of self-similar crack growth [2,5]. In an attempt to develop a simple TL, a new notion of the Steady-State Tearing (SST) has been incorporated in the analysis. The SST means that: (i) a crack is driven forward under a constant level $\sigma_{N_s}$ of the net-section stress resulting from proportional increments in applied loads or displacements, and, (ii)

![Figure 1: An actual crack in a rectangular plate (a) and the related ideal crack (b) together with postulated dependencies of the half-spacing $b_m$ (c) and displacement $v_m$ (d) upon the increase in the half-spacing $c_n = c$.](image-url)
during omnidirectional extension of crack borders the reversible, $2v_{st}$, and irreversible, $2v_{su}$, increments in the extreme spacing $2b_m$ are in direct proportion to nonreversible increments $2\Delta c_s$ in the extreme spacing $2c_n=2c$.

A new fracture parameter Crack Volume Ratio (CVR) is the ratio $V_g$ of the increment $\Delta M_g$ of the volume $M_g = A_g B_g$ enclosed by the surfaces of a growing crack at the moment of interest to the volume $M = A B$ of the same crack at the same moment but for the imaginary state of the PD without structural damage and internal stresses. Here $A_g$ and $A$ are in-plane areas of the ideal crack, $B_g$ and $B$ are the crack-tip thickness. When applied to the SST, the factors of thickness reduction $\beta_g = B_g/B$, $\beta_s = B_{su}/B$ and $\beta_u = B_{su}/B$ are equal in magnitude. The $V_g$ value consists of elastic

$$V^{el}_g = \left(1 + C_{mc} \frac{\sigma_g^{eff}}{E}\right) \left[1 + C_{nc} \frac{\sigma_g^{eff}}{E} + \frac{2\delta_g}{\pi(\rho_n c)^{0.5}}\right] \beta_g - 1 \quad (1)$$

and plastic

$$V^{pl}_g = \frac{2h_g \beta_g}{\pi(\rho_n c)^{0.5}} \left(1 + C_{mc} \frac{\sigma_g^{eff}}{E}\right) \quad (2)$$

components. Stress concentration factors $C_{nc}$ and $C_{mc}$ take the form

$$C_{nc} = F_{vc} \left[1 + 2 \left(\frac{c}{\rho_n}\right)^{0.5} - k\right], \quad C_{mc} = F_{uc} \left[k + 2k \left(\frac{\rho_n}{c}\right)^{0.5} - 1\right],$$

where $k = q/\sigma$ is the load biaxiality ratio, $E$ is the Young’s modulus. Compliances $F_{vc}$ and $F_{uc}$ are related to the points “m” and “n” on the actual crack profile, $\delta_g$ is the crack-tip opening displacement and $h_g$ is the characteristic size of an Active Damage Zone (ADZ). The effective tensile stress $\sigma_g^{eff}$ is taken as the sum of the internal stress $\sigma_u$ and the applied stress $\sigma$. The level $\sigma_u$ of fictitious loading is treated as a uniform tensile stress field that is internally generated during accumulation of structural damage represented in the analysis by the irreversible displacement $v_{su}$.

The SST is viewed as a process of continuous re-initiation with invariant values of the crack-tip driving forces $h_g$, $r_g$, and $\delta_g$. The ADZ length, $r_g = l_g - c_g$ and its height $h_g = 2 \left(h_g + v_g\right) \left[1 - (c_g/l_g)^2\right]^{0.5}$ are characterizing the generation of structural damage and $\delta_g$ governs crack
extension within the Fracture Process Zone (FPZ). Crack extensions $\Delta c_s$ are inversely proportional to the applied stress $\sigma_s$ and values $\delta_{su} = 0$, $h_{su}$, $\delta_{ss}$, $h_{ss}$, and $r_{ss}$ are kept constant during crack growth. To link displacement $v_{su}$ with the related value of stress $\sigma_u$, we use the following elastic solution $\sigma_u = E v_{su} / F_v \left(2c_s + b_s\right)$ for the PD in question.

**INSTABILITY AS A PRECURSOR OF STEADY-STATE TEARING**

In practical terms, a focal issue is prediction of extreme load levels for a large-scale component from data collected on relatively small specimens. Such load levels are usually related to a single-point critical event “c” which separates slow (controllable) crack extension from fast (uncontrollable) one. In the UM analysis, instability is treated as a continuing transition from the final point “p” of the pseudo-steady blunting via the pseudo-steady tearing stage “ps” to the SST stage “sa” in Figure 2.

Point’s “p” and “s” have the meaning of the lower and upper limits of the instability event “c”. The point “p” reflects some apparent state of the PD since it does not lie along the actual test record. However, the imaginary onset “p” of the pseudo-steady tearing is a distinct event derived directly from raw experimental data by using the diagram net-section stress $\sigma_N = \sigma c_f / (c_f - c)$ versus crack extension $\Delta c$ [7]. Here $c_f$ are the intersection points of the crack plane ($v = 0$) with the outer boundaries of the PD (see Figure 2), where $x = \pm (W-N)$. An appropriate example is the crack-

![Figure 2: Scheme of test records, test events and SST diagrams for two Middle-Cracked Double-Edge-Notched Tension specimens.](image-url)
extension data obtained in [8] which are presented here (Figure 3) in terms of the pseudo-steady diagrams. It can be seen that the critical points c1, c2 and c3 predicted with the R-curve concept are placed between the initial (p1, p2, p3) and the final (s1, s2, s3) points on tear diagrams.

The state “c” is difficult to define as the appearance of a distinct discontinuity in the mechanisms of ductile tearing. It is a continuous process giving an indication of equal fracture resistance either in advance of the point “c” or far after it proceeds. This is supported by a close agreement between the angles $T_{ac} = (0.23 \pm 0.002)$ MPa/mm over wide ranges of crack growth initiated from different saw cuts (see Figure 3). Thus, it is practical to concentrate attention on a softening branch of the test records representing the highest load carrying capability of a center-cracked plate.

**THE GLOBAL CONSTRAINT AND INSTABILITY PREDICTIONS**

SST diagrams have variable characteristics depending on the interaction between the global in-plane constraint and the accumulation of structural damage within the fully-developed ADZ and FPZ. Displacements $v_{su}$ and $v_{st}$ and ADZ sizes $h_{su}$, $h_{ss}$, and $r_{ss}$ serve as the easily obtainable damage parameters characterizing a transition from the stress-free state to the states “u” and “s”. This transition is affected by changes in each constraint-related test parameter. Our attention is directed to SST diagrams for the

![Figure 3. Test records (points) and related pseudo-steady diagrams (lines) for the M(T) specimens loaded to fracture under displacement control.](image-url)
simplest PD usually referred to as the M(T) specimen. The latter is similar to the M-DEN(T) configuration in every way, with one exception, the depth $N$ of both sharp notches (see Figure 2) is zero. One can ensure the conditions of almost zero straining along the crack plane by cutting the edge notches singly or together with a decrease in spacing $2H$ between the rigidly clamped boundaries. These variations in the PD geometry concentrate all the thinning, damage and cracking inside two localized necks spreading under the highest in-plane constraint to be expected.

The small specimens (see Figure 4) were made from 1.05-mm thick aluminium 1163AT having the composition similar to AL2024-T3 mentioned above. Its tensile properties: $E = 73$ GPa, 0.2% offset yield stress $\sigma_Y = 334$ MPa and ultimate strength $\sigma_{ult} = 446$ MPa are close to the characteristics $E = 71$ GPa and $\sigma_Y = 345$ MPa presented in [8]. We intend to predict the instability events $s1$, $s2$ and $s3$ (Figure 3) for the largest flat panels that have ever been tested by the use of data presented in Figure 4. The M(T) specimens and the panels are close to geometrical similarity. An important point is that the global constraint is treated here as an elevation of the tensile stress $\sigma_{ult}$ averaged over the net-section of the PD. When normalized by the ultimate strength, this stress is denoted as a tear constraint factor, $\alpha_s = \sigma_{Ns}/\sigma_{ult}$. The values of $\alpha_s$ for the small and large M(T) specimens are, respectively, 0.834 and 0.485. It means that high-constraint data ($\alpha_s > \alpha_{s,Y}$) must be put into correspondence with low-constraint one ($\alpha_s < \alpha_{s,Y}$). Here $\alpha_{s,Y} = \sigma_Y/\sigma_{ult} = 0.749$ is the constraint factor corresponding to the lower limit of plastic collapse. For reference, the peak value of $\alpha_s = 1.112$ has been obtained in tests of the M-DEN(T) specimens with $2W = 240$ mm, $2H = 480$ mm, $N = 0.2$ $W$, $c_u = 0.875c_f$.

In our case of proportional scaling, a simplified (two-dimensional) version of the TL incorporates the parameters characterizing: (i) the panel geometry ($B = 1.016$ mm, $2W = 2286$ mm, $2H = 3810$ mm, $2c_s = 458$, 712 and 932 mm); (ii) the boundary restraints (horizontal boundaries are rigidly clamped and vertical one are free-to-move); (iii) loading, loading history, and initial damage ($k = 0$, $\sigma_u = 0$, $\nu_u = 0$); (iii) the material ($E = 71$ GPa, $\sigma_{ult} = 345$ MPa) and finally the SST behaviour ($\rho_n = 0.262$ mm, $F_{vc} \approx [\sec(\pi c_s/2W)]^{0.5}$, $F_{uc} \approx F_{vc}$, $h_{su} = 3.948$ mm, $h_{ss} = 4.508$ mm, and $r_{ss} = 4.248$ mm). Most of the SST parameters have been
Figure 4: Basic test records, related SST diagrams and test events of practical importance for the M(T) specimens of small size.

derived from data presented in Figure 4 for the small M(T) specimens. Predicted and measured levels of the $\sigma_s$ stress are as follows: 214.9 and 185 MPa for $2c_s = 458$ mm, 171.5 and 148 MPa for $2c_s = 712$ mm, 139.2 and 120 MPa for $2c_s = 932$ mm. There is reason to think that the bias of theoretical and experimental data arise mainly from the influence of the loading systems in tests of the small and the large specimens. Machine compliance, coupled with specimen compliance, can have an effect on the crack driving force, $C$, on its derivative, $dC/dc$, [9], as well as on a transition from stable to unstable ductile crack growth. The ineffectiveness of the antibuckling guides used in [8] and mentioned in [3] is thought to be another main contributor of the above bias.
GENERAL REMARKS

The UM approach has the meaning of the simple mechanistic approximation based on the purely elastic single-parameter characterization of the overall response of crack borders to loading. It links together analyses of elasticity, plasticity, necking, damage and cracking and thereby offers an alternative to a popular concept “competition of fracture with plastic collapse”.

At the same time the UM is not an alternative to the conventional methodology of fracture mechanics as such. The well-defined sizes of ADZ and FPZ can be coupled with mechanism-based analyses of the crack-tip stress and strain employing a traction-separation law or void-containing cell elements.

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