Constraint phenomena on the pre-cracked specimens: numerical and experimental evaluation

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ABSTRACT: An extensive investigation has been carried out on the sensitivity parameters determination describing the fracture behaviour of body with crack with respect to the character change of true stress-strain curve with dominant region of Lueders deformation. The attention is paid on the influence of hardening exponent of deformation to the history of the idealised true stress-strain material curve described by the Ramberg-Osgood relation. Above mentioned tests are used for the correct Weibull stress determination, which is as a measure of the failure probability of cracked body. The Weibull stress model for cleavage fracture of cast steel requires calibration of two micromechanics parameters \((m, \sigma_u)\). Local material parameters have been calculated arising from Beremin approach and calibration is based on the Gao and Ruggieri approach. The aim of the paper can be seen in fracture toughness transfer and correction from pre-cracked specimens to small scale yielding (SSY) represented by 1T (SENB) specimens and their precise computation using FEM. The fracture resistance has been assessed using data from static tests of the three point bend specimens.

Introduction

To quantify the effects of constraint variation on the cleavage fracture toughness the form of the toughness-scaling model based on the Weibull stress \(\sigma_w\) is investigated. Method is based on weakest link assumption and incremental fracture probability, which depends not only on the maximum principal stress, but also on the equivalent plastic strain. It seems that for transferring of fracture-mechanical data from test specimens to exposed real constructions or to its monitored parts, it is necessary to use two-parameter fracture approach. Recent extensive investigations on crack tip constraint effects provide a necessity of testing various constraint configurations, such as shallow-cracked SEN(B) specimens.

Determining of static fracture toughness on SEN(B) specimens is one of the basic fracture mechanics test. It must be emphasised that the most important values are critical K-value, in case of using linear-elastic fracture
mechanics and critical value of J-integral, in case of using elastic-plastic fracture mechanics. Subsequently we confine our investigation to elastic-plastic material behaviour.

More realistic description of crack tip stress and deformations fields has been developed. Approaches are based on two-parameter characterization of crack tip fields, such T-stress and nondimensional Q-stress. These J-T and J-Q approaches retain contact with traditional fracture mechanics. Laboratory measurements on the specimens with varying crack length (changing the relation a/W) and with the same ligament showed increasing values of fracture toughness expressed using $J_c$ versus decreasing crack length. Following the idea of Sumpter [1], Kirk and Dodds [2] investigated several possibilities of J-integral and CTOD estimation for SEN(B) specimens with shallow crack. For fracture toughness valuation on the base of two-parameter fracture mechanics the evaluation of parameters, which express the constraint ahead the crack tip, in our case Q-parameter is critical. Several approaches exist: (i) On the base of experimentally determined dependence $J_c$ on a/W the Q calculation comes from numerically given stress fields received by FEM for every analysed body separately. (ii) Statistical approach using so called local approach [3]. We limit our focus to a stress controlled, cleavage mechanism for material and adopt the Weibull stress ($\sigma_w$) as the local parameter to describe crack-tip conditions. Unstable crack propagation occurs at a critical value of ($\sigma_w$) which may be attained prior to or following some amount of stable, ductile crack extension. The procedure focuses on an application of the micromechanical model to predict specimen geometry and crack effects on the macroscopic fracture toughness $J_c$ (Dodds [4] and Anderson [5]). The procedure requires attainment of equivalent stressed volumes ahead of a crack front for cleavage fracture in different specimens. This can be done e.g. on the base of Weibull stress, because the Weibull stress incorporates both the effects of stressed volume [6].

**Experiments and modeling**

As an experimental material C-Mn cast steel was used. This material was modelled as homogenous and isotropic with elastic constants $E=2.05 \times 10^5$ MPa and $\nu=0.3$. The average value of yield stress was 360 MPa. The testing temperature was -100 ºC. In case of using incremental theory of plasticity the curve $\sigma-\varepsilon$ was modelled by 23 points, which were connected to linear parts. These points belong to experimental measured stress-strain curve.
In case of using deformation theory of plasticity material was described by Ramberg-Osgood relation:

\[
\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left( \frac{\sigma}{\sigma_0} \right)^n,
\]

(1)

where \( n \) is hardening exponent, \( \alpha \) is hardening coefficient, \( \varepsilon_0 \) is yield strain and \( \sigma_0 \) is yield stress.

### TABLE 1: Test specimen geometry in mm

<table>
<thead>
<tr>
<th>a/W=0.1</th>
<th>a/W=0.2</th>
<th>a/W=0.5</th>
<th>Pre-cracked Charpy</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>120</td>
<td>140</td>
<td>250</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>W</td>
<td>26</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>l</td>
<td>104</td>
<td>120</td>
<td>200</td>
</tr>
</tbody>
</table>

**Figure 1: 3PB test**

All computations are based on 3D elastic-plastic analysis using FEM, concretely Abaqus version 6.1 [7]. Test and 3D model are shown in Fig. 1 and Fig. 2, where only one quarter of real body is shown because of two symmetry planes. Models were meshed with eight-node hybrid elements included in Abaqus. 15 680 of elements (C3D8H) were used (17884 nodes). Fig. 3 shows enlarged area around the crack. As can be seen a very fine mesh is required. Element size is increased when the radial distance is retreated from the crack front. Outer radius of the area (Fig. 3) was 0.1 mm and the crack tip radius was 0.01 mm. Twelve elements were used for dividing this radius. Thus, the characteristic element length was \( 8.3 \times 10^{-4} \) mm; at least ten layers of elements in the direction of thickness were used.

Four examples were solved for a range of values of \( n \) in case of using deformation theory of plasticity in order to choose the proper value \( n \). Selection of proper \( n \) value was based on comparing the relations between external force and load point displacement. The value of hardening exponent \( n=8 \) was selected as the best fit, along with the value of hardening coefficient \( \alpha=1 \) (Fig. 4).
Behaviour of the model is very similar to the real material with Luders strain region.

**Toughness scaling model based on Weibull stress**

The local approach for cleavage fracture is based on the weakest link concept that postulates that failure of the body of a material containing a large number of statistically independent volumes is triggered by the failure
of one of the reference volume [3]. In the local approach to cleavage fracture, the probability of failure is assumed to follow a two-parameter Weibull distribution [1,8] in the form:

\[ P_f(\sigma_w) = 1 - \exp\left[-\left(\frac{\sigma_w}{\sigma_u}\right)^m\right] \]  

(2)

The stress integral over the fracture process zone is denoted \(\sigma_w\) and is termed the Weibull stress. This stress is defined by

\[ \sigma_w = \left[\frac{1}{V_0} \int \sigma_1^m dV\right]^{1/m}, \]

(3)

where \(m\) is so-called Weibull slope, \(V_0\) is a reference volume, the integral is computed over the plastic zone, and \(\sigma_1\) is the first principal stress. The parameters \(\sigma_u\) and \(m\) of the Weibull stress \(\sigma_w\) at fracture are material parameters, i.e. independent of the stress state of materials, but may depend on the temperature.

![Figure 4. True stress strain curve and its approximation](image)

The first method of transferability of the fracture toughness was tested on the pre-cracked Charpy specimens and standard specimens (1T). Koppenhoefer and Dodds [9] proposed to quantify the relative effects of constraint variation on the cleavage fracture toughness in the form of toughness-scaling model (TSM). The first studies can be found in the same
works, where on principle two approaches can be regarded: (i) the Dodds and Anderson approach, (ii) the Koppenhofer approach and others. The method demonstrates the dependence of Weibull stress $\sigma_w$ on the crack-tip stress triaxility and the transfer diagram $\sigma_w$ versus computed value of $J$ is constructed. The idea of TSM is to use to same value of probability of failure for both specimen geometries.

The steps of calibration procedure used for TSM:
- Rank probability diagram ($P_f$ versus $J_c$) for two geometries is generated.
- FEM computation for tested specimens and for SSY conditions (BLM).
- Weibull stress determination for tested specimens and for SSY conditions.
- Constraint correction according to weakest link based thickness correction procedure of E-1921. Results of this transformation can be seen in [10].
- Determine $\beta$. Assume that constrain corrected toughness values obey Weibull distribution with fixed exponent of 2. Where $\beta$ defines toughness value at a 63.2 percent failure probability. Equating failure probabilities leads to

$$\frac{J}{\beta}^2 = \left(\frac{\sigma_w}{\sigma_u}\right)^m$$

(4)

The plane-strain, boundary layer model [10] simplifies the generation of numerical solution for stationary cracks under SSY conditions with varying levels of constraint in Fig. 5 (for both approximation), where the reference volume $V_o$ equals $(100 \ \mu m)^3$ for convenience in all calculations. To determine toughness-scaling diagram based upon the Weibull stress with varying Weibull moduli two types of diagrams were constructed.

Figure 5: Diagram of Weibull stress determined by boundary layer method for both approximation of true stress strain curve
Figure 6: Diagram of Weibull stress determined by experimental data for both approximation of true stress strain curve

Figure 7: Toughness scaling diagram based upon the Weibull stress with varying Weibull moduli for both approximation of true stress strain curve

Figure 8: Rank probability diagram

Figure 9: Calibration of Weibull stress parameter
The former for experimental data is presented in Fig. 6, the later for SSY conditions is not presented in this paper. Making these two diagrams in one we can receive the diagram presented in Fig. 7. For the material considered in this paper $\beta_{SSY}=0.064$ MPam was determined according data presented in Fig. 8. Calibrated $m$-values were found out for numerical FEM model based on the incremental theory of plasticity $m=24.1$, for numerical model based on the deformation theory $m=28.3$. The estimation of calibrated $m$-value is clear from plot given in Fig. 9.

Conclusions

The main results obtained can be summarized into the following points:
- The fracture toughness-scaling diagram based on the local approach was determined and used for the transformation of data received on small pre-cracked specimens. Other computations (see Tab.1) are currently being carried out to test this approach.
- The calibration procedure based on the work presented in [8] has been applied and calibrated $m$-value was found to be $m=24.1$ for first model and 28.3 for the second model.
- Calibrated $m$-values are differing to each other. It shows that a precise approximation of stress train curve is very important.

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REFERENCES
