# MODELING OF DEFORMATION AND FRACTURE PROCESSES UNDER THE THERMOMECHANICAL ACTION

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**ABSTRACT:** This paper is dedicated to modeling deformation and fracture processes whose localization and development are restricted to some part of the surface of a structural element and are caused either by pronounced local thermomechanical effects (problem A) or by a local corrosion damage of the part (problem B). Within the problem A the model for describing deformation and fracture of the material of a plate subjected, on a part of its surface, to thermomechanical action is considered. Within the problem B the model for describing deformation and fracture of the cylinder with corrosion zone subjected to thermomechanical action is considered. In both cases the thermal action applied to the surface is determined by a temperature jump on one of its sides, whereas the other side is heat insulated. The mechanical action is due to stresses applied to one side of the deformed part of the surface.

#### **INTRODUCTION**

Many technological processes as well as structure operation conditions are accompanied by the thermomechanical loading of the structural element surface (problem A). The local thermomechanical action induces a variation of local deformation and strength properties and damage accumulation. The thermal action is applied to a part of the surface and determined by a temperature jump on one of its sides, whereas the other side is heat insulated. The mechanical action is due to stresses applied to one side of the deformed part of the surface. Problems of this type occur when studying laser radiation effects in materials [1] or strength of structural elements in thinning areas that may appear, for instance, because of corrosive wear [2, 3], or technological treatment of surfaces of plates [4].

We performed modeling of the deformation and fracture of an elastic plate under the action of local nonstationary thermomechanical loading taking into account the effects of the geometric and physical nonlinearity. The geometric nonlinearity is associated with taking into account the bending moment caused by the longitudinal stresses. The physical nonlinearity is related to taking into account the elastic moduli dependence on the temperature. In this case the heated part of the plate becomes to be effectively nonhomogeneous through the plate thickness. Stress-deformation calculations for the nonhomogeneous plate are based on the method developed in [5]. The criterion of attaining limit tensile strength of the material (taking into account the temperature dependence of the strength) was used as the criterion of fracture initiation at the plate surface. It was shown that two characteritics fracture regimes are possible: fracture at the central part of the heated zone or fracture at the edges of this zone.

Another considered problem (problem B) is connected with safety and residual lifetime providing for the pressurized components (e.g., pressure vessels, apparatus, pipelines) with the zones of local corrosive wear. Structural integrity and service conditions for pressurized components containing defects are regulated by a series of national and international Codes (e.g., [7-9]). Note, that the recommendations of the Codes are related to the stationary service conditions (including the values of operating pressure and temperature). However, transitional technological regimes associated with the planned start and stop of the components are often realized. Recommendations given in the Codes for these cases are too general (see, e.g. [10], Appendix 17) and do not make provision for the defects presence. Hence, the problem on providing the structural integrity at the transitional regimes needs a special analysis taking into account the service conditions and possible influence of the defects. The appropriate practical recommendations need to be given in the start and stop regulations. The problem B is devoted to the modeling of the effects of the transient regimes on the strength and fracture of structural elements under thermomechanical loading taking into account the zones of local corrosive wear. The thermal action is related to a temperature jump at the inner surface of the structural element (cylinder). The mechanical action is caused by the inner pressure. The dependences of the critical pressure on temperature and heating time, geometric sizes of the zone of local corrosive wear and physal parameters of the material were computed.

## THE PROBLEM A

Consider the deformation of a clamped plate of length  $2\ell$  and thickness H=2h in the coordinates  $X_1X_2X_3$  with (the  $X_3$ -axis being normal to the plane of the plate and the  $X_1$ ,  $X_2$  axes are directed along the plate).

The boundary value problem corresponding to the one-dimensional nonstationary heat conduction and deformation of the plate has the form

$$T(x_{1},0)=T_{0}, \Delta T(+h,t)=\Delta T_{0}, q(-h,t)=0; \text{ or } (1)$$
  

$$T(x_{1},0)=T_{0}, \Delta T(-h,t)=\Delta T_{0}, q(+h,t)=0; \text{ and}$$
  

$$\sigma_{33}(x_{1},-h,t)=p, \sigma_{33}(x_{1},h,t)=0, \sigma_{31}(x_{1},\pm h,t)=0, u_{1}(\pm \ell, x_{3},t)=0, |x_{1}| \le \ell, |x_{3}| \le h$$

where  $\Delta T(x_3,t)$  is the temperature variation,  $q(x_3,t)$  is the heat flux along the X<sub>3</sub>-axis,  $\sigma_{ik}(x_1,x_3,t)$ ,  $u_i(x_1,x_3,t)$  are the stress and displacement components, *t* is time counted from the initial instant of heating, p is the mechanical load (pressure) applied to the lower surface of the plate.

The deformation of the plate is considered under the following assumptions: the relation  $\ell >>h$  holds, the material of the plate undergoes elastic deformation, and Young's modulus depends on temperature. The upper edge of the plate is unloaded. The boundary condition of the "clamped edge" ensures that during the deformation of the plate a longitudinal force *F* and a moment *M* appear at its edges  $x_1=\pm \ell$ .

Thus, the variation of the stress-deformation state of the plate will be influenced by the heat increment  $\Delta T = \Delta T(x_3, t)$  and by the dependence of Young's modulus E(T) on temperature.

Under loading, a critical state may occur in some part of the plate. The critical state of the material is characterized by the condition that the longitudinal tensile stress components attain the ultimate strength, whose temperature dependence  $\sigma_B(T)$  should be taken into account. The critical state is attained if there is at least one point at which the longitudinal tensile stresses become equal to the value of the ultimate strength

$$\max \sigma_{11}(x_1, x_3, t) = \sigma_B, |x_1| < \ell, \ 0 < x_3 < h$$
(2)

Thus, the deformation of the plate is caused by the pressure p, the longitudinal force F and moment M arising at its edges, and also by the induced nonhomogeneity with respect to its thickness.

To determine the parameters ensuring the attaining of the critical state we have to solve a self-consistent problem for the nonstationary equation of heat conduction and the nonlinear deformation of the plate, taking into account the dependence on the heating time and condition given by Eq. (2). As a result we find critical values of the temperature increment  $\Delta T$  and pressure p for given geometrical, mechanical and heating parameters. The

parameters in the calculations were adopted as follows:  $\alpha = 5 \cdot 10^{-6} K^{-1}$ ,  $E_0 = 95$  GPa  $\mu \sigma_B = 588$  MPa at T=20<sup>o</sup>C, h=0.01m, L varied from 0.05 to 0.15 m.

The first series of calculations was performed for the thermal action applied to the upper surface of the plate, with its opposite surface being thermally insulated, as specified by the first boundary condition in Eq.1. The calculation results are given in Figure 1 (the dashed lines correspond to the attaining of the critical state at the center of the plate, while the solid lines correspond to the critical state at the edge).

The relations obtained show that under heating up to  $550^{\circ}$  C, the critical state is attained only at the edge on the inner side of the plate. When heated beyond  $550^{\circ}$  C, the plate attains the critical state at its center at the initial instants of the nonstationary heating (Figure 1).

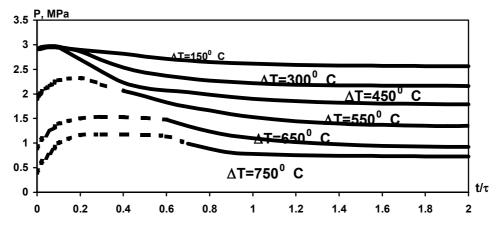


Figure 1 : Dimensionless time dependences of critical pressure P ( $\Delta T$ -temperature jump;  $\tau$  - characteristic heat conductivity time; L = 0.1 m; h=0.01 m)

The second series of calculations was performed for the thermal action on the bottom surface of the plate, with the opposite side being thermally insulated, as specified by the second boundary condition in Eq.1. The relations obtained show that the critical state is attained only at the edge of the plate on the inner surface, irrespective to the time of heating or the temperature jump. Here, all factors that affect the attaining of the critical state come from the inner surface, and consequently, the critical pressure is lower than in the previous case.

### THE PROBLEM B

Let us consider a cylinder of radius R and wall thickness H = 2h having a corrosive defect of rectangular cross-section of length  $2\ell$  and depth d at the inner surface as a model of the cylindrical part of the pressurized component. Multiple experiments show [3, 11] that the pipe strength is determined by the maximal depth of the defect and its size in the longitudinal direction.

Denote by  $T_o$  the temperature of cylinder after stopping the component operation. Next start is accompanied by the action of a heated medium of pressure p on the inner surface of the wall. Denote by  $\Delta T$  the temperature jump at this inner surface. The critical regimes and combinations of the parameters are searched for. To describe the process of deformation of the cylinder with the corrosive defect one needs to solve a coupled problem on the heat transfer and deformation. The condition (H/R) << 1 is assumed later on and the shell theory is used for modeling the stress-deformation state of the cylinder. Material deformation is assumed to be elastic up to attaining the limit state. To search for the critical parameters of mechanical (pressure) and thermal actions one needs to formulate criteria of the limit state of the zone containing a corrosive defect. Two such criteria will be used.

First criterion combines the following requirements: 1. Strength of the component with the corrosive defect under nonstationary thermomechanical actions needs to be provided; 2. Corrosive wear localization leading to leak formation or catastrophic failure of the component is not tolerated.

In this case the criterion of attaining the minimal value of the yield stress by the maximal equivalent stress can be used [12]

$$\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} \le \sigma_{\rm Y} \tag{3}$$

where  $\sigma_1$ ,  $\sigma_2$  are the main stresses,  $\sigma_Y(T)$  is yield stress. The characteristic temperature dependence of the yield stress  $\sigma_Y(T)$  was obtained by generalization of the experimental data for carbon low allow steels [5,12].

The first criterion is more conservative than the second used criterion which determines the fracture conditions at the zone with the corrosive defect at the cylinder surface. It is assumed that the fracture process for the steels of low and medium strength is determined by attaining the plastic state within the whole section of the component wall. A similar criterion of the limit state is used in the known ASME Code B31G [7] for searching for the critical sizes of the corrosive defects and is based on the experimental

results on fracture of pipes with zones of corrosive damage. We used the following criterion of the limit state related to the cylinder fracture

$$\frac{\sigma_{\theta}}{\sigma_{Y}} + \left(\frac{M_{Z}}{M_{Y}}\right)^{2} = 1$$
(4)

where  $\sigma_{\theta}$  is the medium circumferential stress in the cylinder,  $M_z$  is the bending moment along its element,  $M_Y(T)=(\sigma_Y\Delta^2/4)$  is the limit bending moment related to formation of a plastic hinge at bending,  $\Delta = H - d$  for the zone of the corrosive defect and  $\Delta = H$  outside of this zone.

The condition given by Eq.4 determines the critical parameters of the thermomechanical actions at the cylinder fracture in the zone of the corrosive defect. The critical values of the parameters calculated according to Eq. 3 are compared with ones calculated according to Eq. 4.

The nonstationary heat transfer problem will be considered in the 1D – approximation taking into account the nonuniform temperature distribution through the cylinder wall. The boundary conditions have the form of Eq. 1. The 2D – stress-deformation state occuring in the cylinder wall near the corrosive defect (concentrator) is determined by the pressure p and temperature jump  $\Delta T_o$  taking into account the nonstationary heating of the wall. Attaining the limit state in a point of the cylinder is determined by the circumferential stresses and the stresses of longitudinal bending according to criteria given by Eqs 3, 4 accounting for the temperature dependence of the yield stress.

We use the basic relations and equilibrium equations of the shell theory of thermoelasticity [5, 13]. The boundary conditions at the boundary of the corrosive defect zone are determined by continuity of the deflection w and its derivative  $(\partial w/\partial x_1)$  as well as the longitudinal stresses  $T_{11}$ , the longitudinal bending moment  $M_{11}$  and crosscutting force  $Q_3$ .

Hence, the boundary value problems were formulated for self-consistent computing the stress-deformation state at the given temperature jump,  $\Delta T_o$ , pressure, geometric parameters of the cylinder and corrosive defect and thermal properties of the material.

The stress-deformation state and critical parameters (pressure and temperature jump) for the cylinder with the corrosive defect were computed for the following material and initial parameters typical for carbon low-allow steels:  $\alpha = 5 \cdot 10^{-6} \text{K}^{-1}$ ,  $\lambda = 40$  wt/m grad (coefficient of thermal conductivity),  $C_p = 460$  Joule/kg grad (the heat capacity),  $\rho=7800$  kg/m<sup>3</sup>

(density), R = 0.01 m, E = 200 GPa,  $H_o = 0.008$  m, L = 0.08 m,  $T_o = 0^0$ C (initial temperature). The computations were performed for different values of the pressure, temperature and residual wall thickness, H, in the zone of the corrosive defect.

The interrelation between the critical pressure and temperature jump determined by criterion given by Eq.3 is shown in Figure 2 for the cylinder of the nominal wall thickness  $H_o = 0.008$  m and the residual wall thickness in the zone of the corrosive defect H = 0.003 m (the lower curve) and H = 0.005 (the upper curve). The parameters of the component save operation are associated with the regions located under the limit curves of Figure 2.

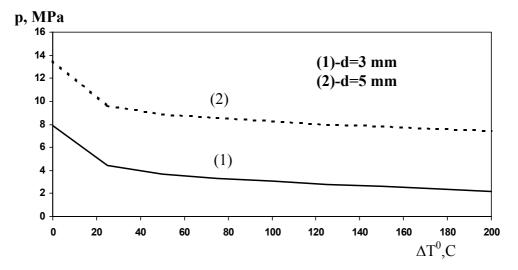


Figure 2: Limit curves of critical pressure dependence vs temperature jump according to criterion 1 (Eq. 3)

The other limit curves were obtained for the criterion given by Eq.4. However, the difference between the critical parameters determined by two criteria does not exceed 10-15%. Just this difference determines the margin of safety. Hence, the development of the transient regimes regulation requires introducing additional margins of safety accounting for possible variations of the loading parameters as well as geometric and material parameters such that the appropriate calculations of the safe parameters for the transient regimes need using the probabilistic models of the structural strength and the defect statistics.

The above described analysis of the limit states of an element of a pressurized component containing a corrosive defect showed that to evaluate the safe parameters of the transient regime one can recommend to use the criterion of the limit state given by Eq. 3 and to introduce additional margins of safety taking into account possible variations of the parameters of loading, geometry and materials as well as the statistical distribution of the corrosive defect sizes detected in operation.

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