SCALING CHARACTERISTICS OF STOCHASTIC DAMAGE ACCUMULATION IN BRITTLE MATERIALS

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Stochastic damage evolution in brittle materials is analyzed with application of various scaling characteristics. Both temporal and spatial randomness of the process are studied. Characterization of a complicated morphology of failures is carried out in terms of a fractal approach. Effect of stochasticity extent on fractal dimension of failures and on load distribution near them is analyzed. Temporal scaling is investigated by means of transition to stochastic kinetic equations for damage accumulation. Three various type of temporal stochasticity (different variants of noise action) are studied. Introduction of a local failure criterion leads to estimation of time-to-fracture distributions under various types of stochastic action. Specific features of evolution of two different damage modes are discussed.

INTRODUCTION

Damage accumulation in real brittle solids is a result of a complicated, multi-scale process of evolution of various defects. Description of mechanisms, governing the development of all types of micro- and mesoscopic defects in terms of one general approach is hardly possible. Continuum damage mechanics (CDM) provides one with an opportunity to study the macroscopic effect of these phenomena. Vivid heterogeneity of real brittle materials - rocks, composites, etc. - results in a complicated scenario of damage evolution for arbitrary load schemes and thus caused the introduction of rather complicated damage parameters - tensors of high ranks - and respective constitutive equations. Still, there exists an alternative approach - to introduce various parameters, describing different modes of damage (failure). This is also stipulated by the difference in micromechanical mechanisms, responsible for respective damage modes. This paper is based on the utilization of two damage parameters, describing the macroscopic effect of the evolution of microcracks of normal separation (related to the mode I in terms of fracture mechanics) and of microshifts (mode II) (Silberschmidt and Chaboche

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(1), Silberschmidt and Silberschmidt (2)).

CHARACTERIZATION OF STOCHASTIC DAMAGE

Damage accumulation in brittle materials demonstrates stochastic behaviour both in its temporal and spatial evolution. This character is the result of following factors:

- spatial stochastic distribution of properties in real brittle and quasi-brittle materials.;
- presence of a stochastic component in acting load (additive noise);
- random evolution of defects, causing local stochastic load redistributions (inner noise).

Spatial scaling. Still, the known self-organized character of defects' evolution results in the universality of damage accumulation and failure development, which is reflected in the scaling behaviour of both processes. Transition to generation of macroscopic fracture is linked with the fulfillment of the local failure criterion. The latter is introduced as a critical value for the current damage in a physical point. Such a variant of criterion has a clear background in fracture physics (Betekhtin et al. (3)). Macroscopic failures - clusters of locally failed elements (representative volumes) - have a complicated morphology. Quantitative characterization of them necessitates an utilization of spatial scaling parameters, such as the fractal dimension of failures and multifractal spectra of load distributions near them (2). The type of materials stochasticity (for instance, a half-width of property's distribution) influences both characteristics: with increase in material's uniformity the fractal dimension of failures diminishes, as well as the width of the multifractal spectra for load distribution. In other words, the failures in more uniform solids have less indented edges and the stress distribution near them are more ordered.

Temporal scaling. Damage accumulation starts at the very early stages of loading process and dynamics of it is influenced both by material's structure and the intensity of external load (for a given stress-state). Localization of this process as a result of interaction and coalescence of defects can result in a crack generation and total rupture of a specimen/construction. Thus, analysis of damage dynamics can serve as a basement for reliability estimates.

Damage accumulation process (in terms of the time-to-fracture - stress dependence) was studied for damage under a constant load and for the regime of active loading. The former is characterized by the scaling exponent of value close to unity. In active loading there is a change of the scaling exponent with the increase of a threshold value of damage. The difference of the dynamics for two modes of damage is demonstrated.

The temporal stochasticity is accounted for by means of introduction of a stochastic term into kinetic equations of damage accumulation, thus transforming them into stochastic differential ones (Silberschmidt and Chaboche (1), Silberschmidt (4)). There are two main sources of temporal stochasticity in the process of damage accumulation:

1) presence of stochastic component in external load, linked with the loading conditions. It can be described in approximation of additional white-noise action;
2) local load fluctuations caused by a non-uniform evolution of the ensemble of defects. This is a result of multiple random processes of load redistribution under formation of new defects, which occur even under conditions of a pure deterministic load as a sequence of structural heterogeneity. Such type of stochasticity can be analyzed in approximation of an inner noise, with intensity proportional to the current level of damage (in the absence of defects the noise level is equal zero).

Recent developments in the numerical solution of stochastic differential equations (Kloeden and Platen (5), Milstein (6)) make possible the elaboration of algorithms for treatment of the respective kinetic equations of damage accumulation for above mentioned two mechanisms and also for a case of their mutual action.

Solutions of these equations (and comparison of them with ones for their deterministic analogues) is a base for the qualitative study of the effect of the noise intensity not only on the statistical characteristics of damage accumulation, but also on the change of the averaged behaviour. Figure 2 shows the development of fluctuations for I-mode damage under the action of inner noise of various intensity (for the same statistical realization). The multiplicative (inner) noise, in contrast to the additive one, causes also the shift of the averaged curve in direction of higher values of damage. For a moderate level of noise intensity this shift (after a short initial region) stays nearly constant (or rises rather slowly) (Figure 3 presents the data, averaged for 10000 statistical realizations of the process). But increase in a noise intensity results in the change of temporal asymptote for such a shift: it can reach the levels, comparable with the value of accumulated damage.

Such an acceleration of damage accumulation leads naturally to a more rapid failure of a specimen/construction. This is reflected in the graphs of time-to-fracture distribution (Figure 4; 10000 realizations for each curve). The extent of asymmetry of distributions increases with the growth of noise intensity, a larger part of realizations having time-to-fracture less as the respective one for a deterministic case. The averaged decrease in reliability reaches 20 per cent under such conditions. Analogous results were obtained for the II-mode damage (evolution of shear damage).

CONCLUSION

Thus, the specific features of the kinetics of damage accumulation for two various modes were studied in terms of modified continuum damage mechanics. Both temporal and spatial stochasticity sufficiently effects the behaviour of specimen or construction under the load. Such an approach allows quantitatively characterize rather non-uniform spatio-temporal evolution of failure. Spatial scaling parameters account for structural heterogeneity of material and its influence on damage accumulation. Study of effect of temporal randomness on damage kinetics provides with estimates of both averaged characteristics (reliability including) and the level of scatter for arbitrary loading conditions.

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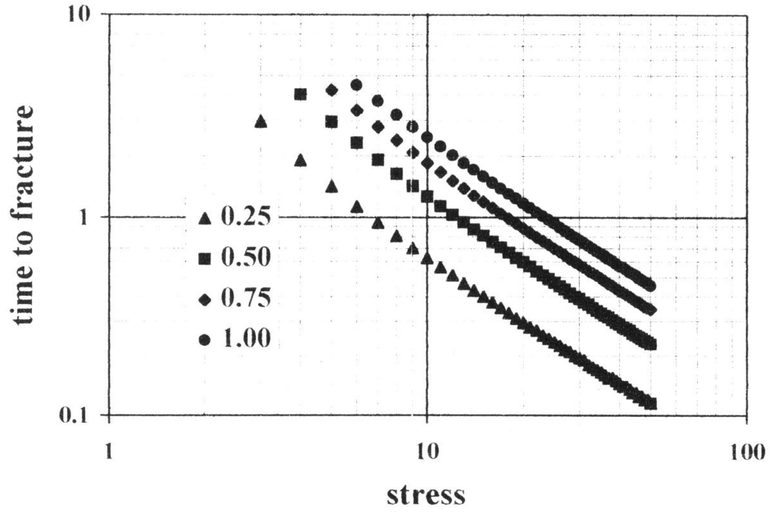

Figure 1 Effect of stress rate on time to fracture for I-mode damage for various levels of threshold damage

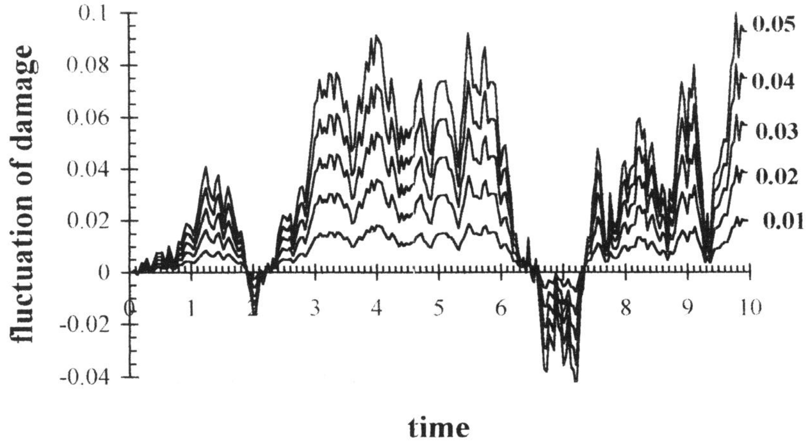

Figure 2 Pure effect of inner noise of various intensity on damage accumulation (stress $\sigma = 4.0$)

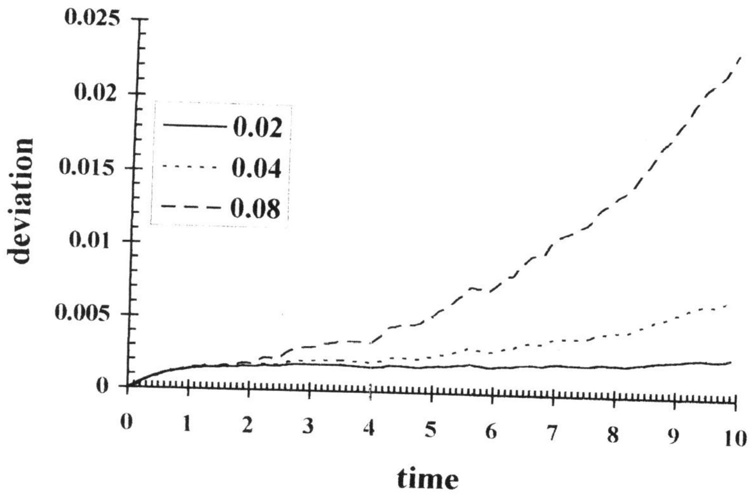

Figure 3 Deviation from deterministic curve of I-mode damage accumulation for various levels of inner noise intensity ($\sigma = 2.0$)

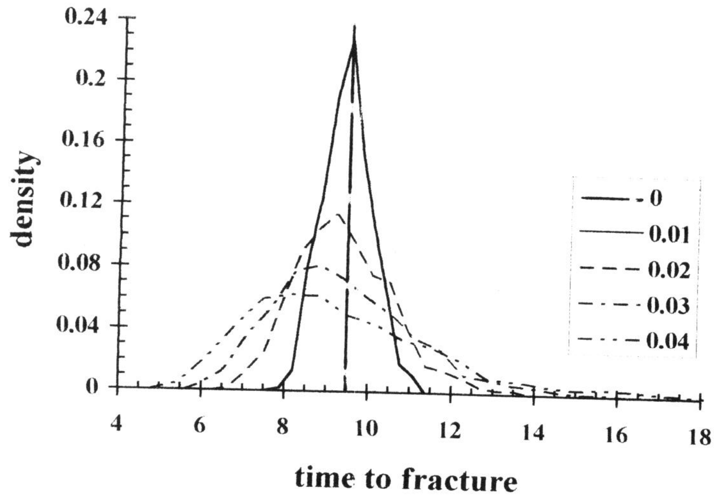

Figure 4 Distribution of time-to-fracture under action of inner noise of various intensity for I-mode damage ($\sigma = 2.0$)