ON THE TEMPERATURE IN DYNAMIC CRACK SURFACE LIGAMENTS

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Small ligaments connecting the crack surfaces just behind the moving crack front are assumed to exist during fast cleavage crack growth. The production and conduction of heat during the process of tearing these ligaments are studied. The straining is computed by assuming an elastic visco-plastic material model. The produced heat during plastic work is calculated. Heat conduction is considered. The results reveal that the temperature increases several hundred centigrades for a low yield stress. For a high yield stress the temperature increases to over one thousand centigrades. The appropriateness of the selected constitutive relationship is discussed in view of the result.

INTRODUCTION

At fast cleavage crack growth in structural steels, the rate-dependent material behavior during plastic straining often has to be considered (Freund and Hutchinson(1)). Consider a crack propagating by coalescence of trans-granular micro cracks. The mismatch of crystallography and mechanical properties at grain boundaries will leave unbroken parts connecting upper and lower crack surfaces. During increasing separation of the crack surfaces the unbroken parts will become ligaments, bridging the gap between the separating crack surfaces.

In a previous study by Nilsson et al. (2) a ligament model is studied as it deforms plastically to a state where very little remains of its initial cross-sectional area. The dissipated energy is calculated as a function of the ligament extension rate. At crack tip speeds of practical interest for structural steels the energy dissipation rate in the ligaments is found to be comparable to total energy release rates. However, the effects of temperature rise due to plastic work was never examined. It was assumed that the process was adiabatic which resulted in unrealistically high temperatures and temperature gradients.

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In this paper dissipated energy is assumed to be converted into heat. The heat conduction, affecting the temperature distribution, is computed. The deformed geometry is considered as the mechanical and the thermal state is calculated simultaneously.

**THE MODEL**

A large body described in the coordinate system \( x_1 \) and \( x_2 \) is considered. The body is separated along the plane \( x_2 = 0 \) except in the region \( |x_1| > a, x_2 = 0 \) are assumed to be traction free. Plane strain is assumed. Large strains are given by Green’s strain tensor, \( \varepsilon_{ij} \). The strain rate, \( \dot{\varepsilon}_{ij} \), is decomposed into an elastic part, \( \dot{\varepsilon}_{ij}^{e} \), and a visco-plastic part, \( \dot{\varepsilon}_{ij}^{p} \). The elastic strain rates are given by Hooke’s law using Young’s modulus \( E \) and Poisson’s ratio \( \nu = 0.3 \). The visco-plastic strain rates are given by the following constitutive relationship

\[
\dot{\varepsilon}_{ij}^{e} = \dot{\gamma}_{o} \left( \frac{\sigma_{e}}{\sigma_{o}} - 1 \right)^{n} \frac{s_{ij}}{\sigma_{e}},
\]

where \( \dot{\gamma}_{o} \) is the strain rate sensitivity, \( \sigma_{e} \) is von Mises effective stress, \( \sigma_{o} \) is the yield stress, \( n \) is the strain rate exponent and \( s_{ij} \) is the stress deviator (Perzyna (3)). The ratio between Young’s modulus and the yield stress \( E/\sigma_{o} = 251 \) and 502 is used which is believed to represent common structural steels. The strain rate exponent, \( n = 5 \) is chosen as proposed by Malvern (4).

A polar coordinate system, \( r \) and \( \theta \), is attached to the crack tip as Fig. 1 shows. An elastic solution is employed for a boundary layer analysis of a part bounded by \( r \leq R \). A prescribed displacement rate \( \dot{\nu}_{o} \) control the load. The following boundary conditions are applied

\[
u_{1} = 0 \quad \text{and} \quad \nu_{2} = \dot{\nu}_{o} t \left[ \ln \left( \frac{2R}{a} \right) - 1 \right] \left[ \frac{1}{2(1 - \nu)} \right], \quad \text{at} \quad r = R, \quad 0 \leq \theta \leq \pi/2.
\]

where \( t \) is the time (cf. Nilsson et al. (2)). The ratio \( R/a \) is chosen to be 40 which is assumed to be sufficiently large to make the boundary conditions reasonably well modeled by eq. (2).

The problem of transient heat conduction is computed in the deformed geometry. The temperature rate due to plastic deformation is given as

\[
\dot{T} = \frac{\sigma_{e} \dot{\varepsilon}_{ij}^{p}}{c_{p} \rho_{o}},
\]

where \( \rho_{o} \) is the density and \( c_{p} \) is the heat capacitivity. The crack surfaces at \( |x_1| > a, x_2 = 0 \) and the boundary at \( r \leq R \) are assumed to be thermally isolated.

All material parameters are assumed to be temperature independent.
NUMERICAL MODEL

The commercial finite element code ABAQUS (5) is used for the numerical calculations. An up-dated Lagrangian method for large deformation elastic visco-plastic materials is used. Full integration of element stiffnesses and conductivity's is used. The body is covered by a mesh containing 481 nodes and 432 isoparametric elements.

The suggested initially sharp crack tips at $\sqrt{x_1} = a, x_2 = 0$ are modeled as notches with a small finite root-radius, $\rho = 0.2a$. The center of the half circle forming the notch bottom, is at $x_1 = 1.2a$ and $x_2 = 0$, see Fig. 2. The finite radius improves the convergence rate. The modification has little influence on the result since the radius increases several times during the loading (cf. McMeeking (6)).

RESULTS

Temperature distribution in the ligaments as a function of the ligament extension rate is studied. The heat conductivity $\lambda = 84$ N/s°C is used. Calculations are performed for constant extension rates from \( \dot{\varepsilon}_0 = 0.08\dot{\gamma}_0 a \) to $80\dot{\gamma}_0 a$. The force in the ligament versus extension for five ligament extension rates can be studied in Fig. 3. The results are presented for $E/\sigma_0 = 502$ and 251, the latter are included for comparison reasons and for one extension rate only. The calculations are interrupted when the load has decreased to 30% of its maximum value.

The temperature distributions for the case $\dot{\varepsilon}_0 = 7.79\dot{\gamma}_0 a$ is shown at $\dot{\varepsilon}_0 = 0.03a, 0.06a$ and $0.12a$ in Fig. 4. Rupture is assumed to occur at the latter displacement. Early during the straining (Fig. 4a) the temperature gradient is observed to be very large at the notch bottom. The gradient has completely vanished later during the process (Fig. 4c).

Figure 5 shows temperatures at the center of the ligament, $x_1 = x_2 = 0$ as function of ligament extension. An observation is that the temperature is low during the initial essentially elastic phase. At onset of plastic deformation, i.e. when a substantial deviation from the linear elastic response occurs (see Fig. 3), temperature is increasing rapidly and is strongly dependent of the ligament extension rate. The heat flow from the ligament region is probably responsible for the decreasing temperature slopes in Fig. 5. For low extension rates this even leads to a decreasing temperature at the end of the calculation. Fig. 5 also compares temperatures at the notch bottom, i.e., at $x_1 = a, x_2 = 0$, with those at the center of the ligament. It is interesting that the temperature difference, at the end of the calculation, is very small even for the largest extension rates.

The normalized temperature at the time of rupture at the center of the ligament is shown in Fig. 6 as function of extension rate for two
values of the parameter $E/\sigma_0$. Only a small difference in normalized
temperature is observed for the different ratios between $\sigma_0$ and $E$. As
temperatures are normalized by $\sigma_0/\rho_0 C_v$ and assuming that different
steels have approximately the same $E$, $\rho_0$, $C_v$ and $\lambda$ this means that
temperature rise essentially scales with $\sigma_0$.

The difference between the temperatures for different ratios between
$E/\sigma_0$ is small and is believed to be due to the chosen criterion to
interrupt the calculations.

**DISCUSSION AND CONCLUSIONS**

It is shown by Campbell and Ferguson (7) that the strain rate
dependence decreases with increasing temperatures. Therefore the
dissipated energy in a ligament is lower than what is predicted by
Nilsson et al. (2) especially at high extension rates and yield stresses. It
should be noted that dependent on how the measurements of the
constitutive behavior are done heat develops during the
measurements. A constitutive relation taking strain, strain rate and
temperature into consideration explicitly would be of great interest for
modeling the material behavior.

The constitutive model used by Nilsson et al. (2) coincide well with
measurements done by Campbell and Ferguson (7) and Huang and
Clifton (8). The tests reported in (7) are dynamic shear tests done with a
drop weight testing technique. The temperature increase during those
tests is estimated to be approximately 1 °C in the medium strain rate
region (1 to $10^4$ 1/s ). The tests presented in (8) are pressure-shear
impact tests on high purity iron. There numerical simulations of the
experiments indicate a temperature increase of several hundred
centigrades due to large strains.

It is believed that the constitutive relation chosen by Nilsson et al.
(2) have to be refined in future research except maybe for the highest
ratios between $E/\sigma_0$ and the lowest extension rates.

**REFERENCES**

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(4) Malvern, L. E., Plastic wave propagation in a bar of material exhibiting a strain rate effect, Quarterly Journal of Applied Mathematics, Vol. 8 (1951) 405-411.


Figure 1 Geometry

Figure 2 Finite element mesh

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Figure 3 Ligament extension forces vs extension

Figure 4 Temperature distribution at a) $\nu = 0.03a$, b) $0.06a$ and c) $0.12a$

Figure 5 Temperature in the ligament for $E/\sigma_0 = 502$

Figure 6 Temperatures at the center of the ligament at the time of rupture