MODELLING OF SURFACE DAMAGE AND FRACTURE OF POLYCRYSTALLINE COPPER UNDER PLASTIC FATIGUE

A. Baltov*, J. Mendez**, P. Violan** and A. Dragon**

On the basis of experimental data for OHFC copper the modelling of surface fatigue microcrack initiation and growth, as well as of the fatigue macrocrack propagation is attempted. Statistical and energetic approaches are applied. Special attention is given to the influence of the precycling on the fatigue durability of the material.

INTRODUCTION

There is an extending interest in the fatigue damage processes of metallic materials in connection with many contemporary industrial applications. In the elastoplastic regime, cyclic damage is frequently characterized by the initiation and growth of a population of surface microcracks. These microcracks grow and interact until the formation of a predominant macrocrack. Further propagation of this macrocrack into the bulk material produces its failure (Suresh (1), Anderson (2), Miller (3), Chaboche (4)). Modelling of such damage processes is required for estimating residual fatigue resistance accounting for previous loading history and physical damage mechanisms related to the material characteristics. The present work constitutes an attempt of modelling surface fatigue damage and fatigue failure of polycrystalline copper on the basis of experimental data. Experiments were previously carried out by Ghannoury (5) with cylindrical specimens of polycrystalline copper subjected to low cycle fatigue under global plastic strain amplitude $E_p = \text{const}$, resulting in symmetric tension-compression global stress amplitude $\Sigma M$. For the material

* Institute of Mechanics, Bulgaria Acad. of Sci., Sofia, Bulgaria.
**L.M.P.M. (URA CNRS 863) - ENSMA, Poitiers/Futuroscope, France.
considered (OFHC copper, 99.95% pure, mean grain diameter 45 μm) stochastic initiation and growth of surface intergranular fatigue microcracks is observed. At a certain cycle level \( N = N_p \), the predominant fatigue crack is formed. This crack initiates and enters into the specimen bulk with an intergranular mesomechanism up to a level \( N = N_v \). Above this level, up to the failure of the specimen \( N = N_f \) the leading mesomechanism is transgranular. A special attention is paid here to characterize the influence of a precycling on the subsequent fatigue durability of the material.

**SURFACE FATIGUE MICROCRACK GROWTH - STATISTICAL APPROACH**

Under the experimental conditions, summed up in the introduction and for a given initial number of cycles \( N_{(0)} \), surface fatigue cracks with depth \( a_{(0)} \) and width \( L_{(0)} \) are formed (see Figure 1).

For all cycles \( N \in [N_p, N_f] \), \( N_0 = \inf \{N_{(0)}\} \), the ratio \( a_i / L_i \) is assumed to be constant. The stochastic distributions of the initial depth \( a_{(0)} \) and of the corresponding rate of microcrack propagation \( V_{(0)} \), for \( N = N_{(0)} \) as well as of the depth \( a_i \) and of the rate \( V_i \) for given number of cycles \( N \) are observed (5).

On the basis of attentive microstructural analysis we assume that the resistance of the material against the microcrack penetration \( r_i \) follows a Boltzmann-like statistical law. The process of microcrack propagation is activated when the actual energy \( e_i \) exceeds the initial energetic barrier \( e_{(0)} \), i.e.

\[
r_i = r_{(0)} \exp \left( -\frac{\Delta e_i}{K_B T_0} \right) = r_{(0)} \exp \left( -\lambda_i (N - N_0) \right)
\]

(1)

where:

\[\Delta e_i = e_i - e_{(0)}, \quad e_i = \sigma_i^p \varepsilon_i^{cum} (N),\]

\[e_{(0)} = \sigma_i^p \varepsilon_i^{cum} (N_{(0)}), \quad \lambda_i = \frac{4K_\alpha}{K_B T_0}, \quad \sigma_i^p = K_i \sigma_p^{n-1},\]

(2)

where \( K_\alpha \) is the Boltzmann constant, \( T_0 \) is the initial temperature, \( K_i \) and \( \alpha \) are the material constants; in the Manson-Coffin law \( \alpha = 2 \), Suresh (1); \( \sigma_i^p \) is the local stress, corresponding to \( r_i \); \( \varepsilon_p \) is the mean local plastic amplitude over the surface of the specimen while \( \varepsilon_i^{cum} (N) \) is the mean...
accumulated plastic strain after N cycles. The rate of the microcrack propagation $V_f$ is inversely proportional to the material resistance $r$, and:

$$V_f = V_{f(0)} \exp \left( \frac{\Delta e_f}{K_BT_0} \right), \ a_f = a_{f(0)} \left[ \exp \left( \frac{\Delta e_f}{K_BT_0} \right) - 1 \right], \ V_{f(0)} = a_{f(0)} \lambda_f$$  \hspace{1cm} (3)

where $\frac{\Delta e_f}{K_BT_0} = \lambda_f (N - N_0)$, $V_{f(0)}$ is the initial value for $N = N_0$. According to our previous assumption $a_{f(0)}$, $V_{f(0)}$, $a_f$, $V_f$, $\lambda_f$ have statistical distribution depending on density $d$. The experimental evidence (5) allows the following form:

$$0 \leq x \leq x_i \quad x_i \leq x \leq x_{\max}$$

$$d = d_{\max} - a_0 x^2 \quad d = b_0 (x_{\max} - x)^2$$

$$a_0 = (d_{\max} - d_i)/x_i^2 \quad b_0 = a_0 x_i/(x_{\max} - x_i)$$ \hspace{1cm} (4)

In Fig. 2 an example for the distributions of $\lambda_f(d)$, $V_{f(0)}(d)$ is given for $E_p = 2.10^3$.

**PROPAGATION OF THE PREDOMINANT FATIGUE CRACK – ENERGETIC APPROACH**

The preponderant fatigue crack initiates for $N = N_p$ with depth $a = a_0$. For the copper under consideration in the cycle interval $N \in [N_p, N_q]$, resp. $a \in [a_0, a_0]$ the mesoscale crack propagation mechanism is still intergranular. For the second stage $N \in [N_q, N_r]$, resp. $a \in [a_r, a_q]$ the mesomechanism becomes transgranular. $a_q$ is the depth corresponding practically to the fatigue failure $N_r$. We assume, by the reasoning analogous to the one in the foregoing that the mesomechanism of the predominant fatigue crack propagation obeys also the statistical law of Boltzmann exhibiting activation character with activation energy $(e - e_s)$, resp. $(e - e_p)$, $e_p$ is the energetic barrier for the intergranular mechanism and $e_t$ the corresponding barrier for the transgranular mechanism.

According to this assumption we consider a macroscale level with the corresponding energy $e = \sigma^p e_{\text{cum}}^p (N)$ where $\sigma^p = KE_p^{-1}$, $K$ being a macroscopic material constant, $e_{\text{cum}}^p = 4N E_p$, $e_s$ and $e_t$ are the macroscopic energetic barriers. The rates of the predominant fatigue crack propagation during the two stages are $V_f^{(m)} = da^{(m)}/dN$ with $m = I, II$. $a^{(m)}$ is the macroscopic depth of the crack after $N$ cycles, while $a^{(m)}/L^{(m)} = \text{const}$. Then:
Stage I: \( N \in [N_s, N_T] \), resp. \( a \in [a_s, a_T] \)

\[
V^{(i)}_a = V_0 \exp \left( \frac{e - e_s}{K_B T_0} \right), \quad a^{(i)} = a_s \exp \left( \frac{e - e_s}{K_B T_0} \right),
\]

\[
a^*_s = \frac{V_0}{\lambda^{(i)}_p}, \quad \lambda^{(i)}_p = \lambda^{(i)}_p \left( N - N_s \right)
\]

Stage II: \( N \in [N_T, N_R] \), resp. \( a \in [a_T, a_R] \)

\[
V^{(ii)}_a = V_T \exp \left( \frac{e - e_T}{K_B T_0} \right), \quad a^{(ii)} = a_T \exp \left( \frac{e - e_T}{K_B T_0} \right),
\]

\[
a^*_T = \frac{V_T}{\lambda^{(ii)}_p}, \quad \lambda^{(ii)}_p = \lambda^{(ii)}_p \left( N - N_T \right)
\]

Taking into account that \( L^{(ii)} = a^{(ii)}/C \), \( C = \text{const.} \), we express:

\[
L^{(ii)} = L = L_T \exp \left\{ \lambda^{(ii)}_p \left( N - N_T \right) \right\}
\]

\[
\lambda^{(ii)}_p = \frac{\lambda^{(ii)}_p}{\eta_T}, \quad L_T = a_T/C
\]

For \( \lambda_p = \beta/N_p, \beta = \text{const.} \), we obtain the expression of the unified relation \( L = N/N_p \) for every \( E_p = \text{const.} \) (Suresh (1), Gammoumi (5), Alain (6)). In a sense, this is a proof that the assumed activation mechanism for the predominant crack propagation is physically reasonable.

**INFLUENCE OF PRECYCLING ON THE MATERIAL FATIGUE DURABILITY**

A number of experiments with OFHC copper (5) show that the precycling could increase or decrease the fatigue durability \( N_p \) in comparison with the reference value \( N_{R(e)} \).

The degree of this influence depends on: (a) the relation between the constant amplitudes in the two processes \( \psi = E_{ph}/E_p \) (the index \( h \) is reserved for the precycling); (b) the surface treatment after the precycling without surface polishing \( (N_p) \) and with polishing \( (N_R) \). We attempt to describe both processes, introducing the following parameters: \( \eta_p \) – characterizing the influence of the precycling on the energetic barrier \( e_T \); \( \eta_h \) – characterizing the material structural changes in the whole specimen after \( N_p \) preliminary cycles.
For the first process:

\[ a_h = a_{th} \exp \left\{ \lambda_{ph} (N_h - N_{th}) \right\}, \quad N_h > N_{th} \]  

For the second process we have:

\[ a = a_h \exp \left\{ \lambda_p N + \lambda_{ph} (N_{th} - N_h) \right\}, \quad \bar{N} = N - N_h \]  

Fatigue durability for the case without surface polishing:

\[ \bar{N}_R = \frac{1}{\lambda_p} \ln \frac{a_R}{a_h} + \lambda_h (\eta_h N_h - \eta_{th} N_{th}), \quad \lambda_h = \frac{\lambda_{ph}}{\lambda_p} = \frac{K_h}{K} \]  

Fatigue durability for the case with surface polishing:

\[ N_R = N_{R(0)} + \lambda_h (\eta_h N_h - \eta_{th} N_{th}), \quad N_{R(0)} = N_f + \frac{1}{\lambda_p} \ln \frac{a_R}{a_f} \]  

The analysis of the relations (9) and (10) allows the following estimations:

\[ \begin{cases} 
(\text{a}) \quad \text{If } \eta_h N_h > \eta_{th} N_{th}, \quad \bar{N}_R > \bar{N}_{R(0)} = N_{R(0)} - N_h \quad \text{and} \quad \bar{N}_R > N_{R(0)}; \\
(\text{b}) \quad \text{If } \eta_h N_h < \eta_{th} N_{th}, \quad \bar{N}_R < \bar{N}_{R(0)} \quad \text{and} \quad \bar{N}_R < N_{R(0)} \end{cases} \]

The above relations reflect well the experimental results concerning the increase or the decrease of the fatigue strength. The model presented enables to interpret many other effects, as e.g. the deviation from the unified relation \(L-N/N_h\) in the case of two successive processes, Ghannouri (5). The parameters \(\eta_f\) and \(\eta_h\) can be determined by the following procedure: (i) preliminary cyclic process with \(E_p=\text{const.}\), until \(N=N_h>N_{th}\) then (ii) two cyclic processes with different \(E_p(1)=\text{const.}\), \(E_p(2)=\text{const.}\), until \(N_{R(1)}\) and \(N_{R(2)}\):

\[ \phi^{(1)}(\eta_h N_h - \eta_{th} N_{th}) = \Delta N_{R(1)}, \quad \Delta N_{R(1)} = \bar{N}_{R(1)} - N_{R(1)} \]

\[ \phi^{(2)}(\eta_h N_h - \eta_{th} N_{th}) = \Delta N_{R(2)}, \quad \Delta N_{R(2)} = \bar{N}_{R(2)} - N_{R(2)} \]  

On the basis of the results obtained the authors are performing further work concerning the mesostructural analysis of the phenomena studied in preceding sections.
REFERENCES


Figure 1 Schematic view of damage processes.

Figure 2 Distribution of initial $V_{\lambda_0}$ and $\lambda_4$ values.