MICROCRACK MODEL OF CREEP UNDER NON-PROPORTIONAL LOADING

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A microcrack-model of creep deformation of an initially isotropic materials under non-proportional loading is presented. This model describes simultaneously the difference between the creep processes under compressive and tensile loading types, damage induced anisotropy, creep compressibility and the Poynting-Swiil-effect.

INTRODUCTION

The creep theory of materials with different behaviour in tension and compression is being intensively developed now. The process of creep deformation and damage accumulation occur under creep conditions in parallel with each other, and they have a reciprocal effect.

One of the modern difficulties of continuum damage mechanics is connected with two specific features (Dragon (1), Chaboche (2), Chaboche (3), Altenbach et al (4), Voyiadjis and Pakr (5), Voyiadjis and Venson (6)).

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-the damage induced anisotropy under loading due to the directionality of the microcracks,
-the unilateral character of damage, when the damage is active or inactive, and the microcracks are respectively open or closed.

The most of existing models do not take into account both these specific features. The purpose of the work reported here, is to elaborate the microcrack-model for creep deformation of an initially isotropic materials and its application. This model describes simultaneously the difference between the creep processes under compressive and tensile loading types, damage induced anisotropy, creep compressibility and the Poynting-Swift-effect.

CONSTITUTIVE EQUATIONS

It is assumed that the damage is connected with the degradation of the creep properties due to parallel surface-like microcracks. The orientation of an array of parallel microcracks may be characterized by a unit vector \( \hat{n} \). Constitutive equation of creep is based on the assumption of the existence of a potential in the form

\[
F = \frac{1}{2} \sigma_{ij}^2
\]

where the equivalent stress has the following structure

\[
\sigma_i = A \sigma_i + B \sigma_{ii} n_i n_i
\]

Here \( \sigma_{ij} \) \((i,j=1,2,3)\) are components of the stress tensor; \( \sigma_i \) is the stress intensity, 
\( \sigma_{ii} = \frac{1}{3} \delta_{kk} \sigma_{kk} \). \( \sigma_{ij} \) \((i,j=1,2,3)\) are components of the stress deviator, 
\( \sigma_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{mm} \delta_{ij} \). \( \delta_{ij} \) is the Kronecker symbol; \( A, B \) are constant material parameters; \( n_k \) \((k = 1, 2, 3)\) are components of the vector \( \hat{n} \). The creep strain rate tensor is determined by the rule

\[
\dot{\epsilon}_{ij} = \lambda \frac{\partial F}{\partial \sigma_{ij}}
\]
where $\lambda$ is a certain scalar multiplier.

It is possible to show on the basis of (1)-(3) that constitutive equation has next form:

$$\dot{\varepsilon}_{kl} = \lambda \sigma_{ij} \left( \frac{3}{2} A \frac{S_{ij}}{\sigma} + B \nu_k \varepsilon_l \right)$$  \hspace{1cm} (4)

For the creep of non-strain-hardening materials, one has

$$\lambda \sigma_{ij} = \mathcal{S}(\sigma_{ij})$$  \hspace{1cm} (5)

The representations

$$\mathcal{S}(\sigma_{ij}) = \sigma_{ij}^\nu$$  \hspace{1cm} (6)

$$\mathcal{S}(\sigma_{ij}) = \sinh(\sigma_{ij}/\alpha), \text{ or } \mathcal{S}(\sigma_{ij}) = \exp(\sigma_{ij}/\beta)$$ are possible for the function $\mathcal{S}(\sigma_{ij})$.

To describe the strain-hardening of materials, we introduce the parameter $\varphi$, defined by the kinetic equation (it is a time):

$$\frac{d\varphi}{dt} = \mathcal{R}$$  \hspace{1cm} (7)

where

$$\mathcal{R} = 1$$  \hspace{1cm} (8)

or

$$\mathcal{R} = \sigma_{ij} \dot{\varepsilon}_{ij}$$  \hspace{1cm} (9)

Then we have

$$\lambda \sigma_{ij} = \mathcal{S}(\sigma_{ij})\mathcal{P}(\varphi)$$  \hspace{1cm} (10)

The function $\mathcal{P}(\varphi)$ can be taken as:

$$\mathcal{P}(\varphi) = \varphi^n$$  \hspace{1cm} (11)

In the case of tertiary creep of non-strain-hardening materials we have.
\[ \gamma(\omega) = \frac{\omega \gamma}{(\omega - \omega_c) \gamma} \]  

(12)

where \( \omega = \int \sigma_i \dot{e}_i dt, \omega \in [0, \omega_c], \) and \( \omega = \omega_c \) corresponds to the creep fracture. In this case

\[ \lambda \sigma_c = \dot{\varepsilon}(\sigma_c) \gamma(\omega) \]  

(13)

BASIC EXPERIMENTS

Let us consider a method of determining the material parameters in the constitutive equation (4). We assume brittle character of creep rupture and we have that \( \dot{\varepsilon} \) is a vector in the direction of the maximum principal stress. In analyzing the creep curves, let the following relationship be established between the strain rate \( \dot{\varepsilon} \), stress \( \sigma \), and time:

\[ \dot{\varepsilon} = K \sigma \dot{\varepsilon} \]  

(14)

in uniaxial tension and

\[ \dot{\varepsilon} = -K \sigma \dot{\varepsilon} \]  

(15)

in uniaxial compression (\( K > 0 \)).

Then we consider relations (4), (6)-(8), (10), (11) to be valid and we obtain following expressions for creep rates:

\[ \dot{\varepsilon} = (A + B) \sigma \dot{\varepsilon} \]  

(16)

and

\[ \dot{\varepsilon} = -A \sigma \dot{\varepsilon} \]  

(17)

Then making a pairwise comparison of relations (14) and (16), (15) and (17), we find the parameters of the material:

\[ \frac{1}{A} = K^{n+1}, \quad \frac{1}{B} = K^{n+1} - K^{n+1}, \]  

(18)
SYMBOLS USED

$A, B$ = material constants
$F$ = creep potential
$i$ = unit vector of the orientation of parallel microcracks
$\sigma_i$ = stress intensity
$\sigma_e$ = equivalent stress

REFERENCES