A MODEL OF DAMAGE AND FRACTURE BASED ON FUZZY SETS THEORY

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Damage evolution and fracture in nonhomogeneous materials is modeled with the use of the methods of the theory of fuzzy sets. The concept of damage as a membership function of the material into a fuzzy set of failed material is formulated. This concept is applied to model the damage evolution in a nonhomogeneous material. The damage in a loaded material versus time relation is obtained numerically. The influence of damage on the intensity of the destruction of material is studied as well.

INTRODUCTION

The heterogeneity of material and stochasticity of microfracture and damage formation have a pronounced effect on the strength and fracture of materials, but can be hardly modeled with the use of the traditional methods, like the fracture mechanics or damage mechanics. It is supposed usually that the stochasticity of fracture is determined by the statistical effects in materials at microlevel. Yet, the stochasticity of some processes of damage evolution and fracture is caused physically by the uncertainty of material behaviour at micro- (down to atomistic) level. Statistical models present only mathematical approximation of the uncertain behaviour or structure of materials.

Here, it is suggested to use the mathematical methods of the theory of fuzzy sets to model the destruction of materials. These methods make it possible to take into account the stochasticity and uncertainty of material behaviour as well as the influence of the material heterogeneity on the strength and damage evolution.

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FUZZY MODEL OF DAMAGE

Conditions of local failure of any real material are uncertain. So, a region of elastic behaviour of a material, which is limited by this condition, can be considered as a fuzzy set, i.e. a set with noncertain border, the degree of membership in which is varied from 0 to 1 (see Kauffmann (1)). The complement of this set is a fuzzy set of state of a destructed material. The membership function γ of a material into this fuzzy set characterizes the degree of closeness of the body to failure. In the continuum damage mechanics, the damage parameter is considered often as a measure of closeness of loaded body to failure (Lemaitre (2)). This meaning of the damage parameter corresponds evidently to this membership function, which we just defined, and that is why one can use the short term "fuzzy damage parameter" instead of "membership function of the material into the fuzzy set of failed state". The function γ can be considered as some kind of generalization of the probability of fracture, but it characterizes not the possibility of transition from one state to another (failed state of material), but the present state or kind of behaviour of the material. Generally speaking, the function γ can be related with any continuously growing from monolithic to failed state value, or determined with the use of people experience and knowledge as it is customary in applications of the fuzzy sets . Yet, if one notes that the change of the function γ proceeds only in the direction from γ=0 to 1 (it is evident from thermodynamics and the physical nature of fracture), one can consider the value γ as a degree of irreversibility of the evolution of the material. That makes it possible to relate γ with the accumulated entropy per unit volume of material.

FUZZY DESCRIPTION OF THE HETEROGENEITY OF MATERIAL

The uncertainty of damage and fracture is caused not only by the uncertainty of destruction as such (stochasticity of microfracture), but also by the heterogeneity of the material. The data about the physical properties and local strength of material are uncertain, and it is caused by the availability of voids, inclusions, grains, etc. A homogeneous body with strength σ, for example, can be taken as an element of the set of materials of given strength σ with the membership degree 1. Yet, if one considers real materials, this set presents a fuzzy set: the strength of material depends on size, conditions of loading, dislocations distribution and movement, etc.; it turns out that the strength of the body is a averaged strength, and this membership degree is less than 1. The greater the difference between local properties of material and its total strength, the less the membership degree of this material into the fuzzy set of materials with given strength. So, one can define the membership function

$\eta(\sigma)$, which characterizes the closeness of behaviour (strength) of the material to a homogeneous material. The heterogeneity of the material can be characterized by by the statistical entropy of local properties (strength) of the material as well. The relation between the function $\eta(\sigma)$ and the statistical entropy looks as follows (Kauffmann (1)):

$$H = -\eta(\sigma)\ln\eta(\sigma) - [1-\eta(\sigma)]\ln[1-\eta(\sigma)] \quad (1)$$

where H - the statistical entropy of local properties (strength) of the material, σ - the strength of the material. The greater is the heterogeneity of the material, the less is the value $\eta(\sigma)$, and the greater the statistical entropy of the material properties.

FUZZY DAMAGE PARAMETER FOR HETEROGENEOUS MATERIALS

It is clear that the function γ is determined by the local strength of material. Thus, this membership function is a conditional one (Kauffmann(1)), and depends on the heterogeneity of the local strength of the material, which is characterized by the function η. In this case, the fuzzy damage parameter is determined by the following formula:

$$\gamma = 1 - MIN[1-\gamma(|\sigma), \eta(\sigma)] \quad (2)$$

where $\gamma(|\sigma)$ is a conditional fuzzy damage parameter for given strength of material. Eq.(2) gives the relation between the heterogeneity of material properties and the damaged state in the material.

Consider the fuzzy damage parameter for a body with a crack. Such body can be presented as a body from two components (or phases, Mishnaevsky Jr (3)). The value γ for a first phase (i.e. crack) is relatively large, and does not depend on applied stress. This value for the rest of material is an increasing function of local stress. So, one can apply the results obtained for multicomponent system, to a body with a crack. For the system which consists on several independently loaded elements (phases) (the crack and the rest of the body do not interact directly, only through the stress field), the fuzzy damage parameter can be calculated as follows: $\gamma = 1 - MAX[1-\gamma_1, 1-\gamma_2(|\sigma)]$, where γ_1 and γ_2 are the values of the fuzzy damage parameter for the crack and the rest of body, respectively. Taking into account the formula (2) and the unequality $\gamma_1 > \gamma_2$, one can conclude that the fuzzy damage parameter of cracked body is equal to this parameter for the most stressed part of the body, i.e. for the fracture zone in the vicinity of the crack tip.

DAMAGE EVOLUTION IN HETEROGENEOUS MATERIAL

Suppose that the loaded material is homogeneous initially, but it becomes more heterogeneous due to the damage evolution. In this case, only the damage distribution determines the heterogeneity of body, which is characterized by the function η. So, this function can be considered as a membership function of the complement of the fuzzy set of failed state of the material. One can write: $\eta(\sigma_i) = 1 - \gamma(|\sigma_i)$. This formula describes the heterogenization of local properties of a loaded material due to damage accumulation. Substituting it into eq.(2), one can obtain the damage evolution law for the local fuzzy damage parameter in the following form:

$$\gamma_{t+\Delta t} = 1 - MAX[1 - \gamma_t(|\sigma_i)], \tag{3}$$

where t and t+Δt mean two successive timesteps, σ_i - local strength which is changed due to the damage formation. Eq.(3) presents the damage evolution law in the recursive form for initially homogeneous material.

COMPUTATIONS AND DISCUSSION

Fig.1 shows the fuzzy damage parameter versus time relation obtained numerically with the use of eqs.(2) and (3). The time is given in timesteps, in correspondance with the recursive formula (3). The damage parameter γ was taken to be constant initially and equal to 0.001. The material was supposed to consist on 20 components, but the value η for one of the components is equal to 0.999 (it means that the material is practically homogeneous). The increase in the fuzzy damage parameter for each timestep was calculated by formula (3). It is seen from Fig.1 that the fuzzy damage parameter increases with time (at constant load). The curve shown on Fig.1 consists on three parts: first, the damage parameter grows almost linearly with time, but the rate of damage growth is rather small; second, the rate of growth of the damage parameter becomes sufficiently greater, and, third, the rate of damage growth decreases and approaches to zero, when the damage parameter approaches to 1. The transition from 1st to 2nd stage occurs at γ is about 0.1 ; the transition from 2nd to 3rd stage occurs at γ is about 0.85. The first stage of damage evolution corresponds to the independent formation of microcracks in the material. This stage is finished when the microcracks coalesce and form cracks, which begin to grow authocatalytically (Mishnaevsky Jr(3)). Then, the cracks grow with growing velocity, and it corresponds to the second stage, which is finished by dividing of the body into parts. The third stage corresponds to the destruction of a cracked body, up to crushing. The fact that the damage growth rate at third stage is sufficiently less than that at 1st or 2nd stages, is confirmed by the well-known experimental observation that the energy consumption in crushing of a material is much more than at the formation of initial cracks (Mishnaevsky Jr (4)). The energy needed to crack formation due to the damage coalescence is greater than the specific energy of crack growth as well ; this theoretical result obtained on the basis of the fractal model of fracture (Mishnaevsky Jr (5)) corresponds to our conclusion, that the damage growth rate at initial stage of destruction is much less than at 2nd stage.

CONCLUSIONS

It is shown that the methods of the theory of fuzzy sets can be effectively used in modelling damage and fracture of materials. As differentiated from the probabilistic methods, the methods of fuzzy set theory make it possible to allow not only two states of a body (i.e. elastic one and failure), but also all intermediate states as well as the states of material between cracked and crushed ones; these methods allow also to study the influence of the material heterogeneity on its properties. On the basis of the developed model, it is shown that the dependence of the fuzzy damage parameter in loaded body versus time is determined by the degree of destruction of the loaded material; the rate of damage evolution in a low- and high- damaged material (i.e. when the damage parameter is less than 0.1 or greater than 0.85) is much less than that in a medium- damaged material.

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SYMBOLS USED

H = statistical entropy of the local strength distribution

$\gamma, \gamma_1, \gamma_2$ = membership function of the body into the fuzzy set of failed state of material (fuzzy damage parameter), and the values of γ for a crack and non-cracked material, respectively

$\eta(\sigma)$ = membership function of the body into the fuzzy set of a materials with strength σ

σ = local strength of material, or some component of the material

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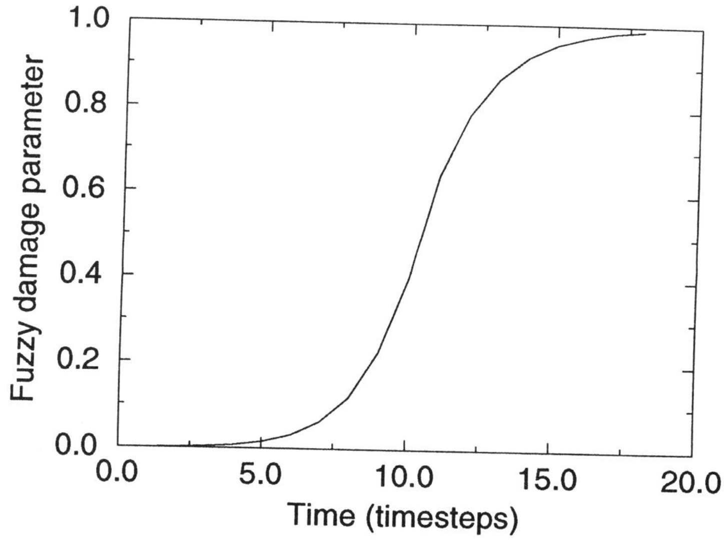

Figure 1 Plot of fuzzy damage parameter versus time