CANONICAL APPROACH TO FATIGUE CRACK GROWTH

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Discarding the crack closure concepts completely, the authors have developed recently new theoretical concepts, and classified the entire area of fatigue crack growth behavior. In this paper, the canonical aspect of this theory is briefly discussed by extending the theory to account for the observed anomalous growth behavior of short cracks that has hither to been attributed to crack closure. These concepts are universal and have relevance to metals, alloys, composites, ceramics and polymers and do not depend on the presence or absence of crack closure.

INTRODUCTION

Crack closure concepts have pervaded the fatigue literature for the past two decades[1,2] and in spite of thousands of papers and several symposia, the concepts have not been critically examined. We have shown recently[3-4] using the dislocation theory that (a) plasticity originated for the crack can not contribute to its closure, and (b) closure due to asperities are only local and their contribution is minimal.

In a companion paper[5], we have discussed an alternate theory to explain experimental results. This theory is based on the fact that fatigue is fundamentally a two parametric problem and hence there are two thresholds, one in terms of $\Delta K$ and other in terms of $K_{\text{max}}$, that need to be satisfied simultaneously for a crack to grow. Since the two parametric requirement is

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very fundamental to fatigue, the concepts should be equally valid whether crack closure exists or not, and even whether crack exists or not, that is, even under S-N fatigue. In this paper we extend the theory to the growth of short cracks.

**SHORT CRACK GROWTH**

Behavior of short cracks has been considered as anomalous[6], since it differs from that of long cracks. Fig. 1 compares the behavior of short cracks and long cracks. The contrasting behavior of the two cracks can be summarized as follows: (a) There is a uniqueness associated with long cracks but not with short cracks, (b) Short cracks grow at stress intensities below long crack thresholds, (c) For the same applied driving forces, AK, crack growth rates are not the same (lack of similitude) between long and short cracks and even among different initial short cracks, (d) Shorts cracks can accelerate and decelerate with increasing driving forces, (e) For very short cracks or low driving forces deceleration could lead to complete crack arrest.

There are three types of explanations offered to account the anomalous behavior of short cracks: (1) Long crack growth behavior is intrinsic and a correction term is required for short cracks[7]. (2) Short crack behavior is intrinsic and represents the material behavior free from crack closure. (3) While large cracks suffer from high degree of crack closure, short cracks suffer from large degree of plasticity in comparison to its crack size. Consensus is that the crack closure or lack of it is the main cause for the anomalous behavior, and in essence the growth of short cracks is more intrinsic than that of long cracks.

Since we have ruled out closure as significant, an alternate explanation is required. We note that the anomalous behavior represented in Fig. 1 is universally shown by all short cracks irrespective of the origin of the short cracks that is whether they are from notches, or from preexisting long cracks, or they are physically or mechanically small or short cracks. We believe that in all cases considered the short cracks grow in the preexisting stress fields, what we call internal stress fields. With this background, we offer the following explanation to account for the observed anomalous behavior of short cracks: (a) The behavior of long cracks is fundamental. (b) Short cracks grow in the presence of internal stress gradients plus external driving forces. (c) Ob-
served lack of similitude is nothing to do with short cracks but with the variability of the internal stresses. (d) Short cracks also grow at the same thresholds ($\Delta K^\text{th}$ and $K^\text{max}_\text{th}$) as long cracks. (e) In computing the total driving forces for any crack (small or large) both applied and internal stresses need to be considered. (f) There are two thresholds that must be satisfied for any fatigue crack (short or long) to grow. (g) The total driving forces $\Delta K = (\Delta K_{ap} + \Delta K_{in})$ and $K^\text{max} = (K_{ap} + K_{in})$ must exceed the two corresponding threshold values for crack growth to occur. (h) To a first approximation internal stresses are non-cyclic in nature i.e. they do not change during loading and unloading. Therefore critical threshold values, $\Delta K^\text{th}$, are the same for both short and long cracks. (i) Internal stresses, therefore, mostly augment $K$ value ($K^\text{max}_\text{th}$ and $K^\text{min}_\text{th}$ since $\Delta K$ is not changed). (j) Short cracks therefore experience higher effective $R$ value in the presence of internal stress field even though the remote $R$ value is low. (k) Limiting behavior of short cracks therefore converges to high $R$ behavior of long cracks. (l) When during the growth, because of stress gradient, the total $K^\text{max}$ is less than the critical threshold, $K^\text{max}$, crack arrest occurs leading to non-propagating short cracks. (m) When both internal and applied driving forces are considered then short crack growth behavior is identical to the growth of long cracks, and to a first approximation the fundamental threshold curves are identical for both.

Fig. 2 shows schematically the variation of internal and applied stresses with crack length increment, $\Delta l$. With increase in crack length internal stresses decrease as the crack grows out of the preexisting stress field. For constant load amplitude, applied $\Delta K$ and $K^\text{max}$ increase with crack increment. Depending on the internal stress gradient, the total stress is below the threshold $K^\text{max}$ then a growing crack gets arrested. On the other hand, if the minimum is above the threshold then crack growth decelerates and then accelerates as Fig. 1 indicates.

Fig. 3 compares the behavior of long and short cracks. We note that upper bound crack growth rates of short cracks (that is when the short cracks experience their maximum internal stresses) correspond to high $R$ ratio data of long cracks. The results are in agreement with the contention that internal stresses affect only $K^\text{max}$ and $K^\text{min}$ and not $\Delta K$, and hence they increase the $R$ value. With crack length increment at constant $\Delta K$, internal stress contribution decreases and the growth rate
drops quickly as shown by arrows. If the total stresses are below the long crack thresholds then crack arrest occurs at $\Delta K = 10\text{MPa}\sqrt{\text{m}}$ ($R=1$) while no arrest for $\Delta K = 16\text{MPa}\sqrt{\text{m}}$. In summary, anomalous behavior of short cracks arises not because of any fundamental differences between short and long cracks but due to the presence of internal stress gradients that augment the applied stresses in providing the driving forces. Effect of overloads on fatigue crack growth can also be attributed to changing internal stresses.

**SUMMARY**

We have developed new concepts that are canonical in nature since the behavior of short crack to long crack, effects of overloads and under loads, crack growth in inert or aggressive environments, crack growth in metals, alloys, ceramics, plastics, monolithics and composites can all be accounted within the framework of the theory without invoking extraneous factors such as crack closure, lack of similitude, or limitation of LEFM etc.

**REFERENCES**


Figure 1. Comparison of the behavior of short and long cracks.

Figure 2. Schematic showing variation of internal and applied stresses with crack length increment.
Figure 3. In the limit, short crack growth behavior converges to that of long cracks at high $R$ ratio.