ELASTIC-PLASTIC FRACTURE MECHANICS ANALYSES OF
BEND SPECIMENS WITH SHALLOW AND DEEP CRACKS

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The effect of crack depth to specimen width ratio on elastic-plastic resistance curves and fracture toughness of a HSLA steel was studied. The experiments were performed on various single edge notch bend specimens (SEN) with different crack depth to specimen width ratios (a/W). The unloading compliance and the DC potential drop methods to determine the crack growth during the test were used.

INTRODUCTION

The notch depth of standard fracture specimen has a significant influence on measured fracture toughness results and should be carefully considered when laboratory test results obtained from deep cracked specimens are related to the service behaviour of cracked components.

Resistance Curves are curves where a certain parameter, in our case the J-integral or CTOD, is plotted over the crack extension Δa. The steep, straight line originating from zero corresponds to the process of crack tip blunting and the second flatter curve describes the stable crack extension (Fig. 3). At the intersection of the two lines, the process of stable crack is initiated (J0, CTOD0), but the values J0.2BL and CTOD0.2BL are usually used to determine an engineering measure of initiation of crack growth (1).

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MATERIALS AND SPECIMENS

The material selected for this investigation was a high strength low alloy (HSLA) steel. This steel is an economically interesting material for the manufacture of complex load bearing structures. Therefore the way how it behaves in the presence of shallow surface cracks, which cannot be avoided in various weld joints, is very important.

Table 1 indicates its chemical composition and mechanical properties. The engineering stress-strain curve obtained from a standard 6 mm diameter longitudinal tensile test conducted at room temperature and a slow loading rate is shown in Fig. 1. Because of the addition of micro-alloyed elements this steel has very high yield point with a yield stress to ultimate tensile strength ratio of 0.92 and low strain hardening exponent of 0.06. The same figure is also showing an estimation of sufficient accuracy for the strain hardening exponent evaluation. If modulus of elasticity (E), yield stress (R_p0.2), ultimate tensile strength (R_m) and elongation at maximum load (e_u) are known, the hardening exponent can be determined by:

\[
\log \frac{R_m (1 + e_u)}{R_p0.2} = \frac{\log \frac{R_m (1 + e_u)}{e_{0.2}}}{n}
\]

(1)

<table>
<thead>
<tr>
<th>TABLE 1 - Chemical Composition (weight percent) and mechanical Properties of HSLA Steel tested</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>0.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Modulus of elasticity</th>
<th>Yield Strength 0.2 percent offset</th>
<th>Ultimate Tensile Strength</th>
<th>Ultimate strain at fracture</th>
<th>Elongation at fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (MPa)</td>
<td>R_p0.2 (MPa)</td>
<td>R_m (MPa)</td>
<td>e_u (%)</td>
<td>A (%)</td>
</tr>
<tr>
<td>~198 000</td>
<td>718</td>
<td>778</td>
<td>7.45</td>
<td>22.4</td>
</tr>
</tbody>
</table>
with
\[ e_{0.2} = \frac{P_{0.2}}{E} + 0.002 \]

Once the hardening exponent is determined, the fictive yield stress follows from:
\[ \sigma_0 = R_{\sigma0.2} \cdot \frac{n}{n-1} \cdot \frac{E}{R_{\sigma0.2}} \cdot \frac{E e_{0.2}}{R_{\sigma0.2}} \]

(2)

Standard three-point-bend (SENB – single edge notch bend) specimens with rectangular cross-section of 18x36 mm were extracted in the longitudinal orientation with trough-thickness notches and then fatigue-cracked to crack depth ratios of approximately 0.1, 0.2, 0.3, 0.5 and 0.8 and they were all instrumented so that the crack growth could be correlated with the J-integral and CTOD.

**EXPERIMENTAL PROCEDURES**

In the first part of the experimental program the unloading compliance technique was used to determine the J-integral and the CTOD values at the onset of ductile tearing. Prefatigued specimens were monotonically loaded and during the test partial unloadings up to 25% of the actual load were performed to measure the specimen compliance. After marking the extent of ductile crack growth (secondary fatigue) and breaking the specimen, initial crack length and crack growth \( \Delta a \) were measured on nine places along the thickness. The average crack length was compared to the calculated values according to the compliance or DC potential drop method. The difference should be less than 10%.

The value of J-integral was estimated at each unloading point \( (i) \) from the area under the load versus load-line displacement record \( (A_{pi}) \) using the following expressions (1), (2):

\[ J = J_{el} + J_{pl} \]
\[ J_{el} = \text{elastic component of } J \]
\[ J_{pl} = \text{plastic component of } J \]

\[ J_{el}(y) = \frac{\kappa^2_m(1-\nu^2)}{E} \]

(3)

(4)
\[ J_p^{(i)} = \left[ J^{(i-1)} - \frac{A^{(i-1)}}{b^{(i)}} \left( \frac{1}{b^{(i)}} \right)^2 \right] \frac{1 - \left( \frac{\gamma_p}{\gamma_0} \right) (a^{(i)} - a^{(i-1)})}{1 - \left( \frac{\gamma_p}{\gamma_0} \right) (a^{(i)} - a^{(i-1)})} \]  

where \( K_t \) is the linear elastic stress intensity factor, \( V \) is Poisson's ratio, \( B_N \) is specimen net thickness and \( b \) is the remaining ligament \((b = W - a)\).

The value of plastic eta factor \( \eta_p \) is well established for perfectly plastic materials. For deeply cracked three-point bend specimens \((a/W > 0.5)\), current standards use \( \eta_p = 2 \) and \( \gamma_p = 1 \). For the shallow crack 3PB specimens, \( \eta_p \) is dependent on the \( a/W \) ratio and \( \gamma_p \) is related to \( \eta_p \) by \((3)\)

\[ \gamma_p = \frac{\eta_p - 1}{b/W} \frac{1}{\eta_p} \frac{d\eta_p}{d(a/W)} \]  

Sumpter \((4)\) derived the \( \eta_p \) equation from limit load analyses of SENB specimens. He has proposed the following relation expressed by a polynomial as

\[ \eta_p = 0.32 + 12 \left( \frac{a}{W} \right) - 49.5 \left( \frac{a}{W} \right)^2 + 99.8 \left( \frac{a}{W} \right)^3 \quad \text{for} \quad \frac{a}{W} < 0.282 \]  

\[ \eta_p = 2 \quad \text{for} \quad \frac{a}{W} > 0.282 \]

Table \(2\) presents the \( \eta_p \) and \( \gamma_p \) values for shallow crack case with no material hardening.

<table>
<thead>
<tr>
<th>( a/W )</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.282</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_p )</td>
<td>0.808</td>
<td>1.125</td>
<td>1.343</td>
<td>1.538</td>
<td>1.786</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>( \gamma_p )</td>
<td>-9.352</td>
<td>-3.951</td>
<td>-2.116</td>
<td>-1.633</td>
<td>-1.719</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Most CTOD based studies have used the BS 5762 \((5)\) CTOD-CMOD correlation to analyze the behaviour of shallow crack specimens. CTOD is regarded as the sum of small scale yielding and a fully plastic contribution.
\[
\text{CTOD} = \varepsilon_{\text{av}} + \delta_{\text{p}} = \frac{K_1^2(1-v^2)}{2ER_{\text{P.0.2}}} + \frac{r_p b_0}{r_p b_0 + a_0 + z} V_{\text{pl}} \quad (8)
\]

\(V_{\text{pl}}\) is the plastic contribution to the CMOD (Fig. 2), \(z\) is the knife edge height and \(r_p\) characterizes the plastic rotation factor, defined as the distance from the crack tip to the hinge point divided by the ligament length. The standard value of \(r_p\) for deep cracked specimens is 0.4 (5).

When the crack grows the center of rotation moves forward, but the location at which the CTOD has to be taken remains at the original fatigue crack tip. Therefore Eq. 8 is modified to take into account the crack growth \(\Delta a\):

\[
\text{CTOD}^m = \frac{K_1^2(1-v^2)}{2ER_{\text{P.0.2}}} + \frac{r_p b + a - a_0}{r_p b + a + z} V_{\text{pl}} \quad (9)
\]

**Crack length determination.** Crack length and crack extension were calculated using the specimen compliance \(C_{(j)}\) measured during a partial unloading of specimen (1). (2). (Fig. 2):

\[
\Delta a = a_j - a_0 \quad (10)
\]

\[
a_j = W (0.999748 - 3.9504 U_X + 2.9821 U_X^2 - 3.21408 U_X^3 + 51.51564 U_X^4 - 113.031 U_X^5) \quad (11)
\]

where \(U_X\) is given by:

\[
U_X = \left[ \frac{4E_{\text{E},\omega} W C_{(j)}}{S} + 1 \right]^{-1} \quad (12)
\]

\(S\) is specimen span and \(B_0\) is effective thickness for compliance based crack extension measurements: \(B_E = B - \left( B - B_N \right)^2 / B\).

The DC potential drop method uses Johnson’s equation for the crack length determination (6):

\[
a = \frac{2W}{\pi} \arccos \left( \frac{\cos \frac{\pi Y}{2W}}{\cosh \left[ \frac{U}{U_0} \arccosh \left( \cosh \frac{\pi Y}{2W} / \cos \frac{\pi a_j}{2W} \right) \right] \right) \quad (13)
\]
where \( U_0 \) and \( a_0 \) denote the initial values of potential and crack length, \( U \) and \( a \) are the actual values of both quantities and \( y \) is half gage span over which \( U \) is measured.

**Blunting line.** The slope of blunting line was determined from tensile test results (Eq. 1 and 2) using the EGF (ESIS) procedure (7):

\[
J_{\text{Bl}} = \frac{2 \sigma_g \Delta a}{0.4 d_n}
\quad (14)
\]

\[
\text{CTOD}_{\text{Bl}} = \frac{2 \sigma_g}{0.8 d_n R_{\text{Bl}}}
\quad (15)
\]

The factor \( d_n \) was evaluated from:

\[
d_n = \frac{1.155 E}{\pi \sigma_g (1+n)(1-v^2)} \left[ \frac{2 \sigma_g}{\sqrt{\frac{1+v}{E}} \frac{1+n}{n/(1+n)}} \right]^{(1+n)}
\quad (16)
\]

**\( \delta_5 \) Measurements** In the second part of the investigation the DC potential drop method was applied on shallow cracked SENB specimens with a crack depth ratio of 0.1 and deep cracked specimens with a crack depth ratio of 0.5. Now, CTOD was directly measured on the specimen’s side surface at the original fatigue crack tip by a special clip gage over a gage span of 5 mm. Therefore it was termed \( \delta_5 \) by GKSS research center (Geestacht, Germany) which developed this testing procedure (8).

**DISCUSSION**

Since the \( \eta_p \) factor is reduced rapidly for crack less than 0.282 a/W (Table 2), it seems that J-integral should be reduced dramatically, too. However, when the Sumpter \( \eta_p \) is used in Eq. 5, only a slight reduction in the J-R values results as shown in Fig. 3 for shallow crack specimen with a/W = 0.20. The reason for this is that the \( \gamma_p \) factor (Eq. 6) is between -4 to -2 (Table 2) rather than +1, the value used in ASTM E 1152 calculation. The second term is now greater than 1, which results in an elevation of the calculated J, especially after a considerable ductile crack extension has occurred.
Fig. 4 shows the complete set of J - R curves calculated the Sumpter \( r_p \) equations (Eq. 7) and the incremental method of Eq. 3 to 5. These results show that J-R curves of shallow cracked specimens are elevated in comparison with the J-R curves of the more deeply cracked standard three-point bend specimens.

A major difficulty for CTOD testing is assessing the plastic rotation factor for specimens with \( a/W \) ratio less than 0.2. Naturally, Eq. 8 and Eq. 9 will yield correct results only when the "right" values of \( r_p \) are inserted. \( \delta_5 \) measurements can be used to determine the plastic rotation factor \( r_p \) experimentally in two ways:

- by the double clip gage method

\[
\begin{align*}
\Delta_{2p} &= \frac{\delta_{5p}(a+z) - V_{pl}(a-a_0)}{b(V_{pl} - \delta_{5p})} \\
\end{align*}
\]  
(17)

- by substituting CTOD values in the BS formula by \( \delta_5 \)

\[
\begin{align*}
r_p &= \frac{(a_0 + z)(\delta_5 - \delta_{asy})}{b_0(V_{pl} - \delta_5 + \delta_{asy})} \quad \text{from CTOD} = \delta_5 \\
\end{align*}
\]  
(18)

\[
\begin{align*}
r_p &= \frac{(a + z)(\delta_5 - \delta_{asy}) - V_{pl}(a-a_0)}{b(V_{pl} - \delta_5 + \delta_{asy})} \quad \text{from CTOD} = \delta_5 \\
\end{align*}
\]  
(19)

The results of this \( r_p \) factor evaluation are presented in Fig. 5 for shallow crack specimen with \( a/W = 0.11 \). Equation 17 yields a slightly higher \( r_p \) value than Eq. 19 because total \( \delta_5 \) values were inserted instead of plastic component \( \delta_{5pl} \). Equation 18 does not give very reasonable results, probably because of the crack growth \( \Delta a \) is not taken into account. Figure 6 shows that the BS estimation procedure overpredicts the experimental \( \delta_5 \) values. With adjusting the plastic rotation factor \( r_p = 0.20 \) to 0.25 for shallow cracked specimens, we get a very good agreement between the two values.

The use of deep or shallow cracked fracture specimens should be carefully considered with their respective application areas. The use of laboratory test results obtained from deep notched specimens will certainly be conservative for the design of cracked components, where crack-like defects are in the form of shallow surface cracks.

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CONCLUSIONS

The effect of crack depth to specimen width ratio \(a/W\) on elastic plastic resistance curves and fracture toughness of HSLA steel was studied. The plastic rotation factor \(r_p\) was experimentally determined using the CTOD equation of the BS standard in combination with \(\delta_5\) clip gage measurements. The analysis of the experimental results lead to the following conclusions:

- The HSLA steel presented in this paper has shown a very good toughness level.
- The boundary value of crack depth ratio between shallow crack behaviour and deep crack behaviour seems to lie between 0.1 and 0.3 (Fig. 4).
- The \(J\)-integral and CTOD values at the initiation of ductile tearing, and especially their engineering approximations \(J_{0.2m}\) and \(\text{CTOD}_{0.2m}\), increase with the use of shallow cracked specimens.
- The \(\delta_5\) measurements are consistent with the calculated CTOD values according to the BS 5762 standard for shallow cracked specimens if the \(r_p\) of 0.20 to 0.30 is used. The standard value 0.4 (5) is apparently not adequate for all materials and crack lengths. Therefore it is very important to have a reasonably accurate estimate of \(r_p\).
- The application of \(\delta_5\) measurements on the shallow and deep cracked bend specimens offers a simple and quick toughness estimation technique without the need for knowing the material yield strength and any plastic rotation factor corrections.

REFERENCES


(8) "On the Experimental Determination of CTOD Based R-Curves", Workshop on CTOD Methodology, Geesthacht, Germany, 1985.

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**Figure 1** The engineering and the true stress-strain curve

**Figure 2** Load versus crack mouth opening displ. records
Figure 3 Influence of \( \eta_p \) factor on J-R curve for \( a/W = 0.20 \)

Figure 4 J-R curves for SENB specimens with different \( a/W \)

Figure 5 Plastic rotation factor for specimen with \( a/W = 0.11 \)

Figure 6 Influence of \( \eta_p \) factor on CTOD-R curves for \( a/W = 0.11 \)