Multiple Delamination of Laminated Structures

Martin G. Andrews[*], Roberta Massabò[**], Brian N. Cox[***]
[*]Northwestern University, Department of Civil and Environmental Engineering, IL, USA
[**]University of Genova, Department of Structural and Geotechnical Engineering, Genova, Italy
[***]Rockwell Scientific, California, USA

ABSTRACT

In this paper the elastic interaction of multiple delaminations in laminated structures subjected to out of plane loading is studied. This has been accomplished by employing beam theory approximations of elasticity. For a cantilever beam, the shielding or amplification of the energy release rate of the cracks has been determined. Important short and long range interactions between the cracks, depending on their transverse spacing, have been shown. The results are similar to those found for the interaction of cracks in infinite bodies, but with strong modification of certain characteristics due to geometry. The shielding and amplification effects strongly influence the macrostructural behavior of the beam, leading to local instabilities, hyper-strength phenomena and crack arrest. The finite element method has been used to validate the results.

1. INTRODUCTION

Laminates are widely used in man-made structures due to their superior strength to weight ratio and mechanical properties. Industries such as aeronautical, aerospace, naval and civil depend on the remarkable properties of these materials in order to create innovative and reliable structural systems. Laminates are also found in biological structures such as seashells and insect cuticle. The strength and stiffness of these structural systems can be significantly reduced and the collapse mechanisms changed by the presence of internal damage in the form of interlaminar flaws. Damage can take place during manufacturing or growth and when the structure is in service. Under different load and geometric conditions, these delaminations may tend to localize into one crack or diffuse into a region of multiple cracks, drastically altering the macrostructural response of the structure. Under service loading conditions the structure may prematurely fail due to the catastrophic propagation of the delaminations. The understanding of this failure mechanism is therefore an important consideration when working with these material systems. The quasi-static and dynamic response of structures with a single delamination has been studied extensively, accounting also for cohesive and bridging mechanisms acting along the delamination faces (see [1-2] for some work of the authors). This paper deals with the problem of multiple delamination.

In a structure with multiple delaminations, an important phenomenon to consider is the interaction between the delaminations. An example of this, from a different class of problems that have been studied over the last years, is the interaction of a main crack with a field of micro-cracks in an infinite medium, which can simulate damage in materials such as concrete or coarse-grain ceramics [3-4]. This type of problem has introduced the concept of crack tip shielding and amplification, referring to the tenancy of the micro cracks to either decrease or increase the stress intensification at the main crack tip. The buckling response of axially loaded laminated structures with multiple delaminations has been studied by many authors to investigate the problem of Compression After Impact [5-6]. These studies have highlighted the presence of contact along the delamination faces during post buckling deformation. The problem of the multiple delamination of plates subject to out of plane loading has been studied in [7]. In this paper a simplified model to avoid crack-face contact is formulated and the problem of the stability of a system of equal-length equally-spaced delaminations to length perturbations is studied. The main conclusion is that, under these conditions, the system of cracks tends to grow self-similarly. The problem of the multiple delamination of plates subject to transverse load has been tackled recently by the authors [8]. The investigation relaxes the assumptions of [7] and focuses on the elastic interaction of the cracks.

In this paper, the problem of the quasi-static response of a cantilever beam with multiple through-width edge delaminations is considered. The elastic interaction of the multiple delaminations is studied along with the effect of the geometry on this interaction. Shielding and amplification effects are quantified and the quasi-static propagation of a system of cracks is studied. The results are verified with the finite element method.

2. THEORETICAL MODEL

The problem under consideration is a beam or plate, with n through-width cracks of arbitrary length and transverse spacing. The material is assumed to be homogeneous, isotropic, and linear elastic. Fig. 1a shows an exemplary cantilever beam with multiple edge cracks, subjected to a static, concentrated force P at its end.

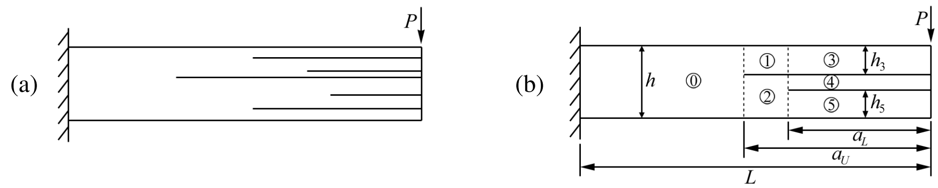


Figure 1 – Cantilever beams with multiple delaminations

The system is divided up into beam segments, as shown in Fig. 1b for a system of two cracks. In the general case shown in Fig. 1(a), contact may occur between delaminated beam segments. Non-frictional contact has been assumed and accounted for in three different ways. The first is to allow the beams to interpenetrate (*unconstrained model*). The second is to assume that the deflections of the beams in the cracked region are the same, thus preventing interpenetration (*constrained model*). The third is to approximate contact with distributed linear elastic springs that resist interpenetration (*spring-contact model*). The stiffness of the springs depends on the transverse stiffness of the two contacting beam segments. The *unconstrained* and *constrained* models are upper and lower bounds of this last model that better reproduces the actual response. In order to determine the solution, Euler-Bernoulli beam theory has been applied to the system of beam segments which interact due to the assumed contact approximation. It is assumed that at each crack tip the beam segments have equal rotations (e.g., $\phi_1=\phi_0=\phi_2$ at a_U). When using the spring-contact model the length of the contact regions are unknown a priori and therefore the solution is non-linear.

The simplest configuration of the beam in Fig. 1a is when all of the cracks are of the same length a and are assumed to propagate simultaneously. In this case the beam segments in the cracked region all have the same deflections, thus there is no contact between them. The energy release rate for the extension of one of the cracks, $\mathcal{G}_i$, has been easily determined from the change in total potential energy of the system. Equations (1) show $\mathcal{G}_i$ corresponding to n equally spaced cracks (1.a) and to n arbitrarily spaced cracks (1.b):

$$\text{(a)} \quad \mathcal{G}_i = \frac{6P^2a^2}{Eh^3}(n+2) \qquad\qquad \text{(b)} \quad \mathcal{G}_i = \frac{1}{2n}\frac{P^2a^2}{EI_0}\left(I_0 \Big/ \sum_{n+1} I_k - 1\right) \tag{1}$$

where E is the Young's modulus, h is the height of the beam, I_k (k = 1..n+1) is the moment of inertia of the beams cross sections in the cracked region and I_0 is the moment of inertia of the intact beam. Results show that as the number of equally spaced, equal length cracks increases so does $\mathcal{G}_i$ (diffuse damage), whereas, if the cracks are concentrated into a small layer (localized damage) then as the number of cracks increases, $\mathcal{G}_i$ decreases.

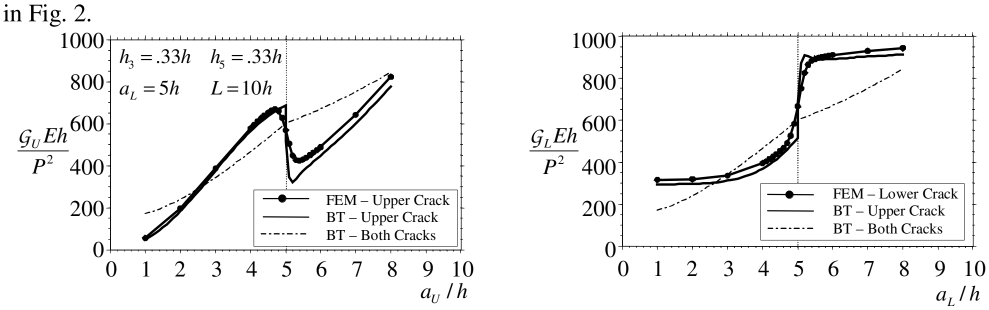
When the cracks are equally spaced it can be shown that the energy release rate for their simultaneous propagation, given by eq. (1.a), is higher than the energy release rate corresponding to the propagation of one of the cracks of the system. However, when they are not equally spaced, simultaneously propagation (eq. (1.b)) does not always lead to higher energy release rate and the response depends on the transverse spacing of the cracks. Similarly, when the cracks are of different length, simultaneous crack propagation is typically not the case. In order to study the non-simultaneous propagation of the crack system and investigate cracks of different length, the more general crack configuration shown in Fig. 1b has been considered. The analysis is limited to a beam with two cracks that, as will be seen, offers a number of interesting solutions.

When the upper crack is longer, $a_U > a_L$, *contact* arises between the beam segments in the cracked region. Non-frictional contact is included by one of the three contact approximations discussed above. When the lower crack is longer, $a_L > a_U$, there is *opening* between beam segments 4 and 5. No contact approximation is required. The energy release rates for the upper, $\mathcal{G}_U$, and the lower, $\mathcal{G}_L$, cracks have been calculated using the J-integral.

When the cracks have similar lengths, beam segment 1 (see Fig. 1b) becomes stocky and the validity of Euler-Bernoulli beam theory is questionable. The effect of shear deformation on $\mathcal{G}$ has been investigated by applying Timoshenko beam theory and a finite element method. The finite element method has been applied to solve the 2-D elasticity problem utilizing the commercial software ANSYS 5.5. The mesh consisted of plane stress isoparametric triangular elements. The stress singularities at the crack tips have been modeled with a rosette of quarter point elements. Contact along the crack faces have been simulated using gap elements that prevent interpenetration of the beams with stiff linear springs. The energy release rate has been calculated using two methods, which yielded the same results (the J-integral and the displacement correlation technique, which has been used to determine the mode I and II stress intensity factors and derive the total energy release rate).

3. ENERGY RELEASE RATES

The energy release rates for a system of two equally spaced cracks of different lengths are shown in Fig. 2.



(a) Upper Crack (b) Lower Crack

Figure 2 - Normalized Energy Release Rates

In the figure the lower crack is kept at a fixed length a_L/h=5 and the upper crack length is varied from a_U/h=0 to a_U/h=L/h. The figure compares the beam theory spring-contact solution and the FEM solution. Fig. 2a shows the energy release rate of the upper crack as its length varies. Fig. 2b shows the energy release rate for the growth of the lower crack on varying the length of the upper crack. The dash-dot curve, shown in both figures, is the energy release rate of each crack when they grow simultaneously for different lengths of the upper crack. Using these diagrams, the crack with the maximum energy release rate can be determined. If a crack growth criterion based on the total energy release rate is used, this result defines the crack growth conditions of the system. In the example shown, for instance, the upper crack will grow when its length is between 2.7h and

5h; when its length is below 2.7h and above 5h the lower crack will grow. In this example the condition for the simultaneous propagation of the two cracks is met only for $a_U = 5h$ and when the curves cross each other. In Section 5 a map will be introduced which allows for the determination of the maximum energy release rate for the system of two cracks, for all crack lengths and distributions. Comparisons of the three contact approximations are shown in Fig. 4.

A notable behavior from figure 2 is the distinct jump in $\mathcal{G}$ when the two cracks approach the same length. In the beam theory solution, this jump is instantaneous resulting in a discontinuity. In the FEM solution the curves are continuous; however the jump is sharp and similar to that of beam theory.

The relative error on $\mathcal{G}$ between the beam theory and the FEM solutions is below 5% when the upper crack is long enough ($a_U/h > 2$) and the cracks are not of similar length. When the two crack tips are closer together ($4.2 < a_U/h < 7$), the error increases due to the discontinuity in $\mathcal{G}$ predicted by the beam theory solution and to some of the assumptions of the model. The assumption of the model that mainly contributes to this error is that of zero relative root rotations at the crack tips that leads to an overestimation of the contact pressures along the crack faces. This contribution to the error is highest when the cracks are of similar length ($5.2 < a_U/h < 6$).

The analysis of the same problem utilizing Timoshenko beam theory shows that for sufficiently long cracks of similar length the shear deformations of beam segment 1 have negligible effect on the macrostructural response and $\mathcal{G}_i$. Including shear deformation improves the centerline deflection profile when contact occurs, however there is still an overestimation of the contact pressures due to the effect of root rotations that are neglected in the model.

4. AMPLIFICATION AND SHIELDING

A crack in a system of cracks can amplify or shield the energy release rates of the other cracks or has no influence on them. In order to investigate these effects the energy release rate of lower crack, $\mathcal{G}_L$, in the system of Fig. 1b has been compared with the energy release rate of the same crack when the upper crack is not present, $\mathcal{G}_{Lo}$. The results show that when the lower crack is longer than the upper crack, $\mathcal{G}_L$ is always amplified ($\mathcal{G}_L/\mathcal{G}_{Lo} > 1$). The magnitude of the amplification depends on the transverse positions and lengths of the cracks. When the lower crack is shorter, the energy release rate is either amplified or shielded depending on the transverse position on the cracks. Fig. 3 shows a map of this behavior. The map shows that if the point corresponding to the transverse position of the two cracks falls in the upper region (c), $\mathcal{G}_L$ will be shielded by the upper crack for all crack lengths. The magnitude of the shielding effect depends on the lengths of the cracks. If the point falls in the lower region (a), then $\mathcal{G}_L$ is always amplified by the upper crack. If the point falls in the middle region (b) then $\mathcal{G}_L$ is either shielded or amplified, depending on the lengths of the two cracks.

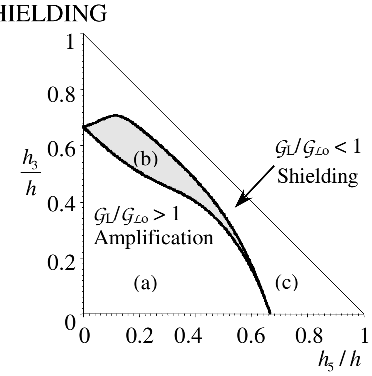


Figure 3 – Map of amplification and shielding

An example from region (a), amplification of $\mathcal{G}_L$, is shown in Fig. 4a. This case corresponds to $h_3/h = 0.25$ and $h_5/h = 0.25$. The figure shows $\mathcal{G}_L$ normalized with respect to $\mathcal{G}_{Lo}$ on varying the length of a_L with the length of the upper crack fixed at $a_U/h=5$. When the lower crack is shorter than the upper crack, there is an amplification of $\mathcal{G}_L$. When the lower crack is longer, $\mathcal{G}_L$ is only slightly amplified. An example of shielding of $\mathcal{G}_L$, a point in region (c), is shown in Fig. 4b. This case corresponds to $h_3/h = 0.2$ and $h_5/h = 0.6$, with the length of the upper crack fixed at $a_U/h=5$. In this case there a significant shielding effect on $\mathcal{G}_L$.

It is worth mentioning that the effect of the interaction of the cracks is strong also on the mode ratio at the crack tip, which is modified by the presence of other cracks.

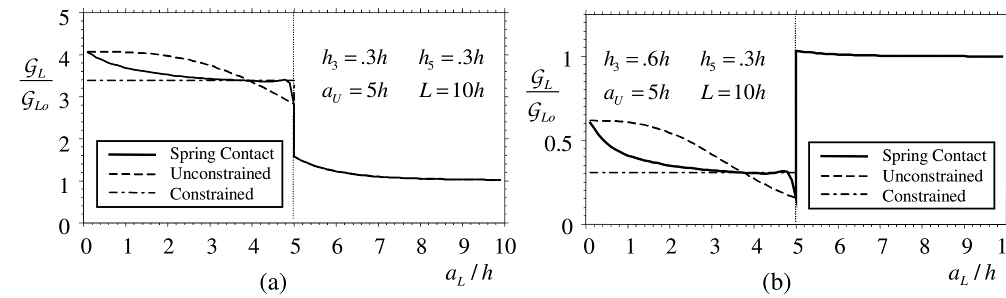


Figure 4 – Examples of Shielding and Amplification

5. MACROSTRUCTURAL RESPONSE

To investigate the macrostructural response of the beam, the quasi static propagation of the system of two cracks has been studied. A crack has been assumed to propagate when its energy release rate equals the fracture energy of the material, $\mathcal{G}_{cr}$. The map in Fig. 5 defines regions of different energy release rate for the two cracks. It has been constructed using the *unconstrained contact* model. A more complicated map, depending on the length of the cracks, can be constructed using the spring-contact model.

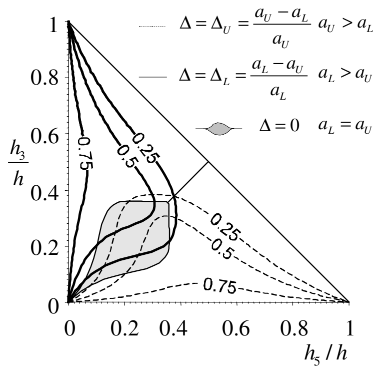


Figure 5 - Map of Energy Release Rates of the crack system, Unconstrained Model

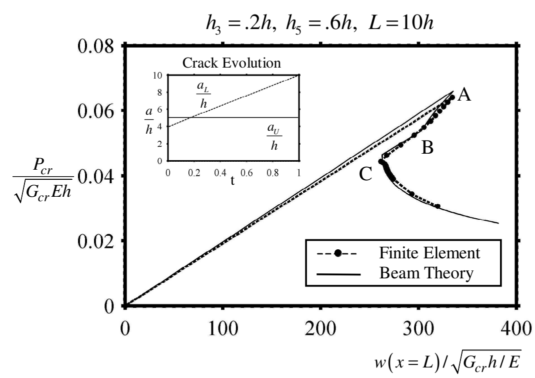


Figure 6 - Example Load Deflection Diagram

The map depends on the transverse position of the two cracks and incorporates contours of equal $\mathcal{G}$ of both cracks. The contours correspond to the relative length of the cracks Δ_U, when $a_U > a_L$ (solid curves), and to Δ_L, when $a_L > a_U$ (dotted curves). The relative lengths are defined in Fig. 5. Points to the left or above the contour corresponding to the relative length of the two cracks correspond to transverse distributions of the cracks for which the upper crack has the higher energy release rate and will propagate when it reaches critical value. Points below or to the right of the contour correspond to transverse distributions of the cracks for which the lower crack has the higher energy release rate. The shaded region in the figure refers to cracks of the same length, $\Delta_U = \Delta_L = 0$, and define configurations for which the energy release rate of each crack propagating simultaneously is maximum. While the contour curves depend on the contact model used, the shaded region is independent. The evolution of the cracks can be followed in this diagram by updating the contour from that corresponding to the current Δ, to the configuration reached by the last crack growth event.

An example of the load versus deflection response of the delaminated plate is shown in Fig. 6. The initial configuration of the system is a_U/h = 5, a_L/h = 4. The crack system has been assumed to grow under crack length control, thus allowing the virtual crack branches, i.e. snap-back and snap-through, to be followed. The inset in the figure shows the evolution of the lengths of the two cracks during loading. In this case the lower crack begins to propagate unstably at point A, which can be verified from Fig. 5 (Δ_U = .25). When the lower crack reaches the length of the upper crack at B, there is a corresponding negative jump in critical load due to positive (amplification) discontinuity in the energy release rate. After the drop in load at C, the lower crack continues to propagate until failure of the structure. The beam theory curve is compared in the diagram with the FE curve showing good agreement.

The behavior of the plate is strongly affected by the initial lengths and transverse locations of the two cracks. Other geometric configurations can for instance lead to load versus deflection behaviors that show local strain hardening, leading to hyper-strength phenomena, or where propagation of one crack will induce the propagation of the other (crack pull-along).

6. CONCLUSIONS

This study has focused on the characterization of the fracture behavior of a transversely loaded cantilever beam with multiple delaminations. It has been shown that there are significant short and long range interaction effects between the delaminations. These effects can be shielding or amplification of the energy release rate of the cracks and are controlled by the geometry of the system. A discontinuity in the energy release rate is observed when the cracks reach the same length. This discontinuity leads to distinct macrostructural effects during crack propagation, such as local strain hardening, crack arrest and crack pull-along. The results of the beam theory model have been verified with a 2-D finite element method of analysis and show excellent agreement for sufficiently long cracks. Additional effects, such as friction in the regions of crack face contact and the effect of cohesive and bridging mechanisms acting along the cracks are under investigation.

ACKNOWLEDGMENTS: MGA and RM supported by Northwestern University, BNC supported by ARO Research Contract DAAD19-01-C-0083.

REFERENCES

[1] Massabò, R., and Cox, B.N., (1999), Concepts for Bridged Mode II Delamination Cracks, *J. Mech. Phys. Solids*, **47**(6), 1265-1300.
[2] Sridhar, N., Massabò, R., Cox, B.N., and Beyerlein, I., (2002), Delamination Dynamics in Through-Thickness Reinforced Laminates with Application to DCB Specimen, *Int. J. Fracture,* **118**, 119-144.
[3] Brencich, A. and Carpinteri, A., (1996), Interaction of a Main Crack with Ordered Distributions of Microcracks: A Numerical Technique by Displacement Discontinuity Boundary Elements, *Int. J. Fracture*, **76**, 373-389.
[4] Kachanov , M. (1985), On Crack - Microcrack Interactions , *Int. J. Fracture*, **30**(4), 65-72.
[5] Larsson, PL., (1991), On Multiple Delamination Buckling and Growth in Composite Plates, *Int. J. Solids Structures*, **27**(13), 1623-1637.
[6] Suemasu, H. (1993), Postbuckling Behaviors of Composite Panels with Multiple Delaminations, *J. of Compos. Mater*., **27**(11), 1077-1096
[7] Suemasu, H., Majima, O. (1996), Multiple Delaminations and Their Severity in Circular Axisymmetric Plates Subjected to Transverse Loading, *J. of Compos. Mater*., **30**(4), 441-463
[8] Andrews, M. G., Massabò, R. and Cox, B. N., (2004), Elastic Interaction of Multiple Delaminations in Laminated Structures, *XXI ICTAM*, August 2004, Warsaw, Poland, in press.