## Scale Effect of the Tensile Strength of Aligned-Flax-Fiber Reinforced Composite

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**Abstract** Application of plant-derived natural fibers as the reinforcement of polymer-matrix composites is driven by economic and environmental reasons. In order to fully exploit the reinforcing effect of the fibers, they have to be aligned in the composite material. When using such UD composites in load-bearing applications, the dependence of strength on the material volume subjected to loading, i.e. the scale effect of strength, should be taken into account in design. UD-reinforced flax fiber/epoxy matrix composites, produced from prepregs, were tested in tension in fiber direction in order to elucidate the scale effect of the tensile strength. The strength distribution at a fixed specimen size and the dependence of the mean strength on the size were shown to agree reasonably well with the Weibull strength statistics, corroborating the previous results for flax-fabric-reinforced composites. A probabilistic model of the strength in tension along the reinforcement direction, using fiber strength distribution, interfacial shear strength and morphology parameters, was applied to theoretically evaluate the magnitude of scale effect.

Keywords flax fibers, polymer composites, strength, Weibull distribution

#### **1. Introduction**

Application of plant-derived natural fibers as the reinforcement of polymer-matrix composites is driven by economic and environmental reasons. In order to fully exploit the reinforcing effect of the fibers, they have to be aligned in the composite material. For bast fibers, this is achieved either by using traditional textile technologies to produce aligned fiber yarns (with a twist level optimized for processing) or by manufacturing prepregs with unidirectional (UD) bast fiber orientation. When using such UD composites in load-bearing applications, the dependence of strength on the material volume subjected to loading, i.e. the scale effect of strength, should be taken into account in design. For brittle composites, the scale effect closely follows the weakest-link statistics [1] reflected by the Weibull strength distribution:

$$P(\sigma) = 1 - \exp\left[-\frac{V}{V_0} \left(\frac{\sigma}{\beta}\right)^{\alpha}\right]$$
(1)

where V is the specimen volume subjected to stress  $\sigma$  and  $\alpha$  and  $\beta$  denote Weibull shape and scale parameters, respectively. Aligned flax fiber composites with textile reinforcement have been shown to exhibit scale effect of tensile strength consistent with Eq. (1) when subjected to tensile loading along the fibers [2].

Probabilistic models of fracture of UD continuous-fiber reinforced composites provide expressions of Eq. (1) parameters in terms of fiber, interface and matrix properties, see e.g. [3-5]. Flax fibers, although being relatively long, are discontinuous; moreover, typical reinforcement of a UD composite contains both elementary and technical fibers, defects in the latter triggering fracture of the composite [6]. Therefore, direct application of the probabilistic strength models derived for

uniform continuous fiber composites to bast fiber composites is likely to yield an upper estimate of their strength [7]. The current study concerns experimental evaluation of the scale effect of tensile strength of UD flax-fiber composites made of commercially available prepregs. Probabilistic strength model [4] is applied for approximate evaluation of strength.

# 2. Experimental

#### 2.1. Material

A composite plate was produced from LINEO flax/epoxy prepreg FLAXPLY<sup>©</sup>. Six UD prepreg plies of the areal density of 150 g/m<sup>2</sup> were aligned, stacked and cured in a thermopress for one hour at ~1 atm pressure and 130 °C temperature. Four groups of specimens, with widths w = 10, 20, 40, and 70 mm, were cut out of the plate along the fiber direction so that the gauge length to width ratio for all specimens amounted to 5.

#### 2.2. Tests

Fiberglass tabs were glued onto the specimen ends. The specimens were tested for strength by applying a stroke-controlled tensile loading in the fiber direction. The loading rate was varied according to specimen gauge length so that the nominal strain rate for all the specimens tested amounted to 0.75 %/min. The strength was evaluated from the failure load employing the average width of a specimen and the average thickness of the plate

### 3. Model

#### 3.1. Weibull scale effect

For the rectangular geometry considered, the specimen volume subjected to load is a product of its width, gauge length, and thickness V = wLt (resp. unit volume  $V_0 = w_0L_0t_0$ ). Due to uniform

thickness of the specimens, we select  $t_0 = t$ , hence Eq. (1) takes the form:

$$P(\sigma) = 1 - \exp\left[-\frac{wL}{w_0 L_0} \left(\frac{\sigma}{\beta}\right)^{\alpha}\right]$$
(2)

It follows from Eq. (2) that the mean strength  $\langle \sigma \rangle$  depends on specimen dimensions as follows:

$$\langle \sigma \rangle = \beta \left( \frac{wL}{w_0 L_0} \right)^{-\frac{1}{\alpha}} \Gamma \left( 1 + \frac{1}{\alpha} \right)$$
(3)

Hence, having determined the strength distribution Eq. (2) parameters from test results at one gauge length, the dependence of mean strength on specimen size can be evaluated according to Eq. (3) if the Weibull scale effect of brittle fracture applies.

#### 3.2. Strength model

A probabilistic model of the tensile strength of UD continuous fiber reinforced composites has been elaborated in [4] for fibers with the Weibull two-parameter strength distribution

$$P(\sigma) = 1 - \exp\left[-\frac{l}{l_0} \left(\frac{\sigma}{\beta_f}\right)^{\alpha_f}\right]$$
(4)

where *l* stands for fiber length,  $l_0$  is a unit length, and the distribution shape and scale parameters are designated as  $\alpha_f$  and  $\beta_f$ , respectively.

In the following we briefly recapitulate the principal relations of the model [4]. Distribution of the average (over cross-section of the composite normal to the fibers) stress taken by the fibers at the failure of a UD composite is given by

$$P(\sigma_f) = 1 - \exp\left[-\left(\frac{\sigma_f}{\widetilde{\sigma}}\right)^{\widetilde{\rho}}\right]$$
(5)

The scale,  $\tilde{\sigma}$ , and shape,  $\tilde{\rho}$ , parameters of the Weibull distribution Eq. (5) are evaluated as follows

$$\widetilde{\sigma} = \sigma_c b_{m,n} \tag{6}$$

and

$$\widetilde{\rho} = b_{m,n} / a_{m,n} \tag{7}$$

where  $a_{m,n}$  and  $b_{m,n}$  are expressed by following relations

$$a_{m,n} = \frac{\gamma_{n_l}^{**}}{\sqrt{2\ln(m \cdot n)}} \tag{8}$$

and

$$b_{m,n} = \mu_{n_l}^* + \gamma_{n_l}^{**} \left( \frac{\ln(\ln(m \cdot n)) + \ln(4\pi)}{\sqrt{8\ln(m \cdot n)}} - \sqrt{2\ln(m \cdot n)} \right)$$
(9)

The characteristic stress entering Eq. (6) is given by

$$\sigma_{c} = \left(\frac{\beta_{f}^{\alpha_{f}} \tau l_{0}}{r}\right)^{\frac{1}{1+\alpha_{f}}}$$
(10)

where *r* is fiber radius and  $\tau$  designates the interfacial shear strength (IFSS) between fibers and matrix. In Eqs. (8) and (9),  $\mu_{n_l}^*$  and  $\gamma_{n_l}^{**}$  are theoretical mean and standard deviation of strength of a bundle of  $n_l$  fibers with  $n_l$  - the number of fibers in a critical element. The latter is given by

$$n_l = 403\alpha_f^{-1.28} \tag{11}$$

for  $2 \le \alpha_f \le 10$ . Length of the critical element  $\delta_l$  is

$$\delta_l = 0.4 \,\delta_c \tag{12}$$

where  $\delta_c$  is the characteristic length given by

$$\delta_c = \frac{r\sigma_c}{\tau} \tag{13}$$

In Eqs. (8) and (9),  $m = L/\delta_l$  and  $n = n_f/n_l$ , where *L* is the length of composite subjected to load and  $n_f$  - number of reinforcing fibers the composite contains.

#### 4. Results and discussion

The parameters of the strength distribution were determined from the strength data of 50-mm-gage-length specimens, shown in Fig. 1, by the maximum likelihood method, as  $\alpha = 22.8$  and  $\beta = 404$  MPa (at  $w_0 = L_0 = 1$  mm). The empirical fiber fracture probabilities, *P*, have been evaluated via the median rank of the measured strength values using the approximation P = (i - 0.3)/(n + 0.4), where *i* is the *i*-th number in ascendingly ordered strength data of the sample and *n* is the number of specimens.

Flax reinforcement is inherently heterogeneous since it contains not only elementary flax fibers but also technical fibers (i.e. naturally adhering elementary fiber bundles) of various sizes; an additional characteristic length is introduced by the presence of transverse stitching fibers in a prepreg. For heterogeneous quasi-brittle materials, strength distribution has been shown to change gradually from normal to Weibull with increasing size and brittleness of the specimen or structure [8]. At intermediate sizes, the strength distribution possesses a Weibull left tail switching to normal distribution for high strengths. A kink can be discerned in Fig. 1 suggesting the presence of such a transition. It appears of interest to establish the size of a representative volume element for strength in flax-fiber composites and its relation to reinforcement structure, thus potentially enabling a more accurate modeling of the scale effect of strength.

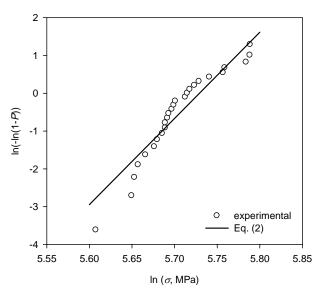


Fig. 1. Tensile strength distribution of specimens with width w = 10 mm in Weibull co-ordinates.

Comparison of the experimental dependence of the mean strength on specimen size and the prediction by Eq. (3) is shown in Fig. 2. It is seen that both, strength scatter at a fixed specimen size, Fig. 1, and strength-size scaling, Fig. 2, agree reasonably well with the Weibull statistics. Note that the sale effect of strength for a UD flax-fabric-reinforced composite has also been shown to follow the scaling of Eq. (2), although with a somewhat smaller Weibull shape parameter  $\alpha$  of ca. 18 [2]. The higher scatter of strength in the case of textile UD reinforcement is likely to result from the additional geometrical variability associated with yarn alignment in the composite.

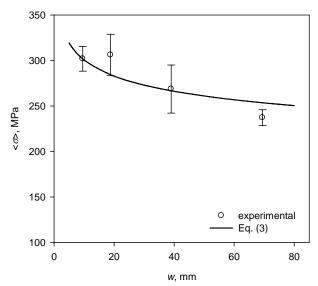


Fig. 2. Dependence of the mean tensile strength  $\langle \sigma \rangle$  on specimen width w

Application of the probabilistic strength models is hampered in this case by lack of information regarding flax fibers used in prepreg manufacture and their adhesion to the epoxy matrix. Nevertheless, for the purposes of qualitative comparison, strength distribution Eq. (5) parameters were estimated for several types of flax fibers assuming a plausible IFSS value. IFSS of bast fibers and epoxy matrix is reported to range from ca. 10 to 33 MPa [9-11]. For a conservative estimate,

IFSS of 10 MPa was assumed. Strength distribution Eq. (4) parameters, evaluated from test results of elementary flax fibers at 10 mm gauge length, are presented in Table 1 (at  $l_0 = 1$  mm). Fiber diameter 16  $\mu$ m was used in calculations.

		2
Reference	Shape parameter $\alpha_{\rm f}$	Scale parameter $\beta_{\rm f}$ , MPa
[12]	3	2080
[13]	1.8	2033
[错误!未找到引用源。]	2.7	1913

Table 1. Strength distribution Eq. (4) parameters of elementary flax fibers

The parameters of composite strength distribution, Eq. (5), were evaluated for composite specimens with 50 mm gauge length and 10 mm width by Eqs. (6) and (7) using the fiber and interface parameters listed above. The results are presented in Table 2. Note that the product of fiber volume fraction and the predicted scale parameter is given in Table 2 for ease of comparison with the experimental scale parameter.

ruble 2. Strength distribution Eq. (5) parameters of OD composite with 50 min length		
Fiber strength from	Shape parameter $\widetilde{ ho}$	Reduced scale parameter
reference		$v_{_f}\widetilde{\sigma}$ , MPa
[12]	107	502
[13]	117	508
[错误!未找到引用源。]	110	511

Table 2. Strength distribution Eq. (5) parameters of UD composite with 50 mm length

It is seen that the predicted shape parameter, characterizing strength scatter, is about 5 times higher than the experimentally determined value. This is likely to stem from such factors as fiber misalignment, heterogeneity, and clustering not allowed for in the theoretical model and increasing the variability of strength. Note also that the predicted scale parameter values exceed the experimental one, i.e. the model provides an upper limit of the tensile strength of flax fiber reinforced composite.

# 5. Conclusions

Scale effect of the tensile strength of UD flax/epoxy composite has been characterized experimentally and shown to comply with strength-specimen size scaling implied by the Weibull two-parameter distribution of strength. Hence strength distribution parameters, determined using only specimens of a fixed size, can be used to predict the variation of strength with volume of the composite subjected to load. A probabilistic model of composite strength has been shown to underestimate the scatter in strength and overestimate mean strength. This is likely to be related to the heterogeneity inherent in natural fiber composites and not reflected in the models derived for synthetic continuous fiber composites.

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