Models for Microcracks Extension and Damage Evolution Based on Acoustic Emissions

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Abstract In the paper, aiming to build models of microcracks extension and damage evolution in the phase of microcracks evolution based on number series of microdefects nucleation, we firstly built a relation formula between accumulated number series of microdefects nucleation and microcracks size, which we called the crack growth model(CGM), by means of the fractal statistical method. Then, we set up a damage evolution model based on growing microcrack size which is determined by microdefects nucleation number series, which we called the damage evolution model(DEM).The statistical analysis on the microcracks evolution in a gradually fractured rock plate shows that the predictive crack size growth by CGM and damage evolution by DEM are in good accord with the measured values. Based on the fact that a microdefect nucleation is accompanied with a acoustic emission(AE), the two models maybe become good methods to predict microcrack growth and damage evolution by use of AE series.

Keywords fractal; microcrack; rough surface; microdefect; damage evolution; acoustic emission

1. Introduction

The research[1,2] shows that in the early stage of damage evolution, the evolution mechanism of microcrack system is microdefects random nucleation and growth in limited size. Microdefect nucleation means the embryo defects in inhomogeneous material extends by tension strain and grows into a microdefect which is the smallest size in microcrack system. In the macro stress field, microdefects nucleate at the inhomogeneous positions which are uniformly distributed in the materials. The microdefect size can be as small as a grain size. Then with the distribution of microdefects nucleation approaching saturation, microdefects nucleation take place closely next to the existing cracks under controlling of the macro shear stress field, which is called as microcrack growth. The microcracks in the larger deformation bands take the superiority of prior growth and in return they can stir up more microdefects nucleation at their two ends and grow into larger-scale microcracks. So the size distribution in the microcrack system is non-uniform. The connection of larger-sized microcracks brings about macro crack nucleation, which marks the end of microcracks system evolution .

In the following, based on the fact that microdefects nucleate in clustering arrange without overlapping and grow into microcrack, we establish the microcrack growth model based on ANMDN series.

2. The number of microdefects in single microcrack

As is well-known, the crack surface is rough, according to the fractal theory, the fractal area $A_T(\delta)$ could be calculated as[3]

$$A_T(\delta) = A_T \delta^{2-D_s} \qquad (1)$$

In which A_T is the apparent area; D_s is the fractal dimension of the rough surface; δ is the measure size, its minimum is the grain size δ_c (Xie,1994) [4].So, on the surface of a penny-shaped crack with apparent diameter c, the number of grains n_d is calculated as

$$n_d \frac{\pi}{4}\left(\frac{\delta}{c}\right)^2 = \frac{\pi}{4}\left(\frac{c}{c}\right)^2 (\frac{\delta}{c})^{2-D_s} \quad (2)$$

Reduction to

$$n_d(c) = \left(\frac{\delta}{c}\right)^{-D_s} \quad (3)$$

For a penetrating crack in a two-dimensional plate, $D_s = d_f$. Here d_f is the fractal dimension of the crack line. The fracture mechanism of the fractal crack line was known as intergranular fracture, transgranular fracture, and coupling fracture of them. And the fractal dimensions d_f can be obtained by theoretical calculation (Xie,1988) [5]. For example, the crystallite size of marble is $\delta_c = 10^{-2} cm$, in tensile condition, the three kinds of fractal dimensions in theory are $d_f = 1.26$,1.365 and 1.30, and the measured values are $d_f = 1.18$,1.31,1.29. For a embedding crack in three dimensional condition, D_s is valued approximately as[6,7]

$$1 + \max(d_f^x,\ d_f^y) = D_s < d_f^x + d_f^y \quad (4)$$

In which, d_f^x and d_f^y represents respectively the mean value of fractal dimensions on fracture lines in x-direction and y-direction. So, if $d_f = 1.3$, then $D_s = 1 + d_f = 2.3$.

3The microcracks growth model based on ANMDN

At a certain time t, the parameters of the microcracks system are marked as

$$\Omega(t) = \left[c_{\min} \le c \le c_{\max},\ D_c,\ N(\delta),\ \ N_0(c_{\max}),\ \ N(c),\ N_d\right] \quad (5)$$

Here $c_{\min}$ represents the minimum microcrack size, in theory, can be valued as crystal grain size; $c_{\max}$ represents at time t the maximum crack size; D_c represents the fractal dimension of crack size scale-frequency distribution; $N(\delta)$ represents the total number of cracks with size no less than δ; $N(c)$ represents the total number of cracks with size no less than c; $N_0(c_{\max})$ represents the total number of cracks with maximum size $c_{\max}$; N_d represents the accumulated number of microdefects nucleation(ANMDN).

According to that the crack size-frequency distribution is fractal(Peng & Xie,1989[8]; Xie & Gao,1991[9]), at time t, we have the equation

$$N(c) = N_0(c_{\max})\left(\frac{c}{c_{\max}}\right)^{-D_c} \quad (6)$$

So in $\Omega(t)$, the number of microcracks with size in $c \sim dc + c$ is

$$N(c-dc)-N(c)=N_0(c_{\max})\left[\left(\frac{c-dc}{c_{\max}}\right)^{-D_c}-\left(\frac{c}{c_{\max}}\right)^{-D_c}\right]$$
$$=\frac{N_0(c_{\max})}{c_{\max}{}^{-D_c}}[(c-dc)^{-D_c}-(c)^{-D_c}] \tag{7}$$

Use Tailor series expansion

$$(c-dc)^{-D_c}=c^{-D_c}+(-D_c)c^{-D_c-1}(-dc)+\frac{1}{2}(-D_c)(-D_c-1)c^{-D_c-2}(-dc)^2+\Lambda$$
$$\approx c^{-D_c}+(-D_c)c^{-D_c-1}(-dc) \tag{8}$$

Substituting Eq.(8) into Eq.(7) we have

$$N(c-dc)-N(c)=\frac{N_0(c_{\max})}{c_{\max}{}^{-D_c}}D_c c^{-D_c-1}dc \tag{9}$$

Substituting in Eq.（3）we have the total number of microdefects nucleation that compose all the micro cracks with size in $c \sim dc+c$ in $\Omega(t)$

$$[N(c-dc)-N(c)]n_d(c)=\frac{N_0(c_{\max})}{c_{\max}{}^{-D_c}}D_c c^{-D_c-1}\left(\frac{\delta}{c}\right)^{-D_s}dc \tag{10}$$

Further, we have

$$[N(c-dc)-N(c)]n_d(c)=\frac{N_0(c_{\max})D_c\delta^{-D_S}}{c_{\max}{}^{-D_c}}c^{D_s-D_c-1}dc \tag{11}$$

So, the total number of all the micro defects nucleation in $\Omega(t)$ is

$$N_d(t)=\int_{\delta_c}^{c_{\max}}[N(c-dc)-N(c)]n_d(c)=\int_{\delta_c}^{c_{\max}}\frac{N_0(c_{\max})D_c\delta^{-D_S}}{c_{\max}{}^{-D_c}}c^{D_s-D_c-1}dc \tag{12}$$

Integration of Eq.(12) is

$$N_d(t)=\frac{N_0(c_{\max})D_c\delta^{-D_S}}{(D_s-D_c)}\left(\frac{c_{\max}{}^{D_s-D_c}-\delta^{D_s-D_c}}{c_{\max}{}^{-D_c}}\right) \tag{13}$$

From Eq.(6), if at time t, the total number of all the micro cracks $N(\delta)$ is known in $\Omega(t)$, then, $N_0(c_{\max})$ can be determined

$$N_0(c_{\max})=N(\delta)\left(\frac{\delta}{c_{\max}}\right)^{D_c} \tag{14}$$

Substituting Eq.(14) into Eq.(13), we have relation $N_d \sim c_{\max}$

$$N_d(t)=\frac{N(\delta)D_c}{(D_s-D_c)}\left[1\text{-}\left(\frac{c_{max}}{\delta}\right)^{D_s-D_c}\right] \tag{15}$$

From which, the maximum crack size should be expressed as

$$c_{max}=\delta\left(1-\frac{N_d(t)(D_s-D_c)}{N(\delta_c)D_c}\right)^{-(D_s-D_c)} \tag{16}$$

In which the parameters D_c , $N(\delta_c)$, N_d need to be obtained from the AE series.

4 Damage evolution model based on ANMDN series

The purpose of studying the microcracks system is to answer how the microcracks growth in material will affect the macroscopic mechanical properties. In statistical meso-scopic damage mechanics[1], a series of damage functions are used to describe the macro-effects caused by microcracks. The m-th order damage function is defined as（Bai & Xia,1991[10],Xia,1995[1]）

$$D_m(t,\sigma) = \int_0^\infty n(c,t;\sigma)c^m dc \tag{17}$$

Here c is the linear size of microcrack; $n(c,t;\sigma)$ represents in macro stress field σ , at time t , the number density of microcracks with size c . The zero-th order damage function, D_0 represents the total number effect of microcracks; The1st order damage function, D_1 represents the total size effect of microcracks; The second order damage function, D_2 represents the reduced load bearing area and D_3 is proportional to the volume occupied by the microcracks.

In the stage of microcracks evolution, the size of the microcracks is small and their spatial distribution is uniform and sparse[2], which is approximately thought to be suitable for mean field model[1], therefore the damage effects by microcracks on macro mechanical properties are isotropous. So in three dimensional conditions, we define the damage variable as

$$D = K\frac{D_3}{V} \tag{18}$$

In which V represents macro element volume with linear size L ; K is correction factor. Adopting the form of Eq.(17), we have

$$D = K\frac{\int_0^\infty n(c,t;\sigma)c^3 dc}{V} \tag{19}$$

In which $0 < D < 1$. Adopting the form of Eq.(9), $n(c,t;\sigma)dc$ takes the form of

$$n(c,t;\sigma)dc = \frac{N_0(c_{\max})}{c_{\max}^{-D_c}} D_c c^{-D_c-1} dc \tag{20}$$

Substituting Eq.(20) in Eq.(19), we have

$$D = K\frac{1}{V}\int_{c_{\max}}^{\delta} \frac{N_0(c_{\max})}{c_{\max}^{-D_c}} D_c c^{2-D_c} dc \tag{21}$$

Use $V = L^3$, and integrating, then

$$D = K\frac{N_0(c_{\max})D_c}{(3-D_c)}\left[\left(\frac{\delta}{L}\right)^3 \cdot \left(\frac{\delta}{c_{\max}}\right)^{-D_c} - \left(\frac{c_{\max}}{L}\right)^3\right] \tag{22}$$

Substituting in Eq.（14）and put it in order, we have

$$D = K\frac{N(\delta)D_c}{(3-D_c)}\left[1-\left(\frac{c_{\max}}{\delta}\right)^{3-D_c}\right]\left(\frac{\delta}{L}\right)^3 \qquad (23)$$

In which $c_{\max}$ is determined by the total number of microdefects, so, substituting in Eq.（16）,we have

$$D = K\frac{N(\delta)D_c}{(3-D_c)}\left[1-\left(1+\frac{N_d(t)(D_s-D_c)}{N(\delta)D_c}\right)^{-(D_s-D_c)(3-D_c)}\right]\left(\frac{\delta}{L}\right)^3 \qquad (24)$$

Which is the damage evolution equation. The parameters $N(\delta)$, $N_d(t)$ and $D_c(t)$ are needed to be obtained from AE series.

In two dimensional condition, the material volume is written as $V = hL^2$, in which h represents the thickness of the plate. Substitute $V = hL^2$ in Eq.(21),we have another form of damage model

$$D = K\frac{N(\delta)D_c}{(3-D_c)}\left[1-\left(1+\frac{N_d(t)(D_s-D_c)}{N(\delta)D_c}\right)^{-(D_s-D_c)(3-D_c)}\right]\left(\frac{\delta}{L}\right)^2\cdot\frac{\delta}{h} \qquad (25)$$

5 Test

Now, use the AE data in literature[28] to test the validity of the model proposed in the paper.

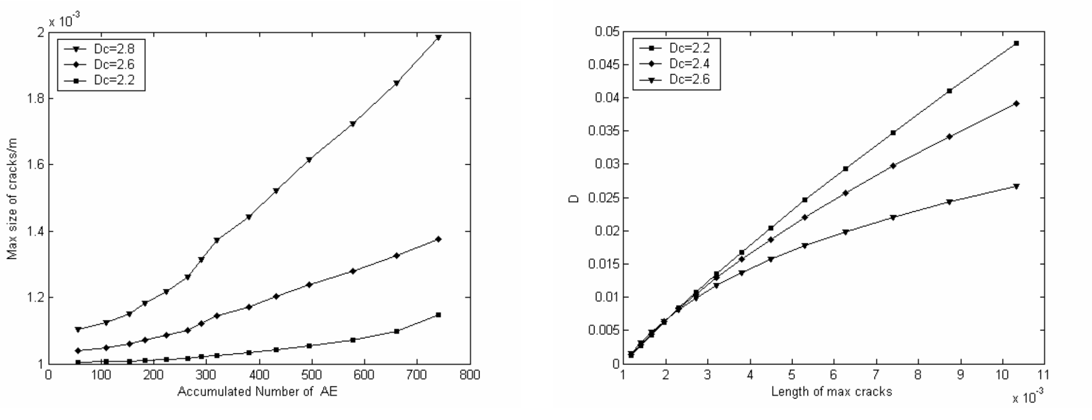
$L = 50mm$, $\delta_c = 10^{-2}cm$, $D_s = 2.3$ The series of AE number per 400 seconds is

$$N_{AE}(t) = \{57\ \ 52\ \ 45\ \ 29\ \ 41\ \ 41\ \ 25\ \ 30\ \ 60\ \ 52\ \ 63\ \ 83\ \ 83\ \ 80\}、$$

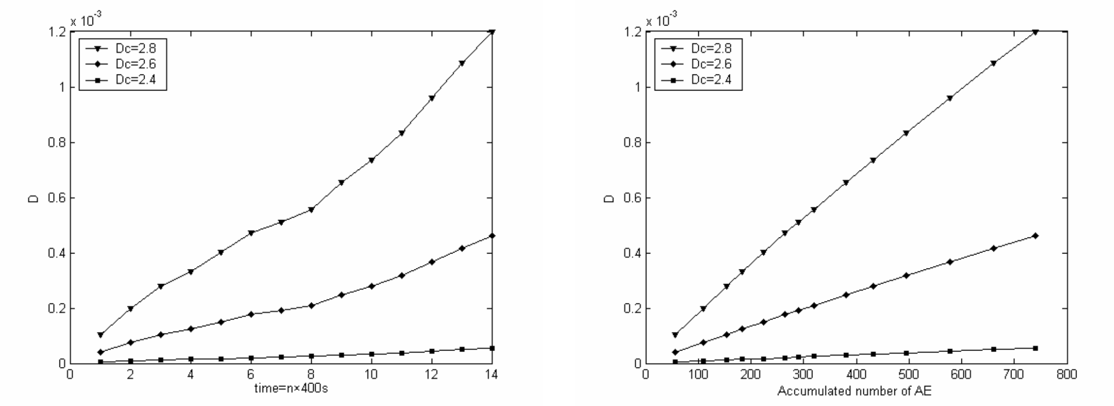
Here we assume the total micro cracks number increases as a rule of

$$N(\delta_c, t) = N_\delta^i(t) = N_{AE}^i(t) = N_{AE}(t)/exp(n/5) \qquad (26)$$

The evolution of the size of the maximum cracks and damage of sample according to the paper shows as figure 1.



（a）　　　　　　　　　　　　（b）



（c）　　　　　　　　　　　　（d）

Fig.4 The preliminary test of the model

6 Conclusions

In the paper, based on the idea of the microcrack formed by microdefects nucleation gathered in line without overlapping, and the fractal statistics relations between the number of microdefects nucleation and microcrack size, we established the microcrack growth model and elasticity damage model based on accumulated microdefects nucleation number. They are needed to be proved effective in predicting microcrack size growth and damage evolution using AE accounts series. In practical use, some problems needed to be resolved: (1) How to get $N(\delta,t)$, the total number of cracks with size no less than δ. A likely workable way is to detach the isolated nucleation AE accounts from the accumulated AE accounts, the isolated nucleation number represents $N(\delta,t)$;(2) How to get the microcrack size scale-freqiency fractal dimension $D_c(t)$ from the AE series. The fractal dimension of seismic magnitude-frequency distribution in AE series has been researched [11-13], the next work is to establish the relationship of the two kinds of fractal dimensions; (3) How to identify the microcrack evolution stage from the AE series. It needs to identify the macro Nucleation from the AE series, so the AE characters of macro Nucleation should be specially researched; (4) in engineering catastrophe monitoring, AE location is necessary to determine L, the size of the rock element. In the end, we need to note that the damage model in the paper is multiscaled with the rock element size L, microdefect size δ and macro crack nucleation size.

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