

# Numerical Analysis of $K_I$ of Semi-elliptical Surface Crack in X80 Steel Plate Strengthened with FRP under Tensile Load

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**Abstract:** Semi-elliptic surface crack is the common defect in metal structures. To ensure the structural integrity and security, the new material such as fiber reinforced polymer (FRP) was adopted for strengthening and repairing. In this paper, three dimensional semi-elliptic surface crack in X80 steel tension specimen strengthened with carbon fiber laminate (CFL) was studied, and numerical analysis was undertaken by Abaqus finite element software to study the stress intensity factor (SIF,  $K_I$ ) of the surface crack for CFL reinforcement effect. The results shown that, strengthening by CFL reduced the stress intensity factors of semi-elliptical surface crack significantly; Crack shape ratio  $a/c$  and crack relative depth  $a/B$  had a great effect on stress intensity factors of strengthened specimen; An expression for calculating stress intensity factors of semi-elliptic surface crack under strengthening condition was obtained by adopting MATLAB software.

**Keywords:** surface crack, fiber reinforced polymer (FRP), stress intensity factor (SIF), numerical analysis

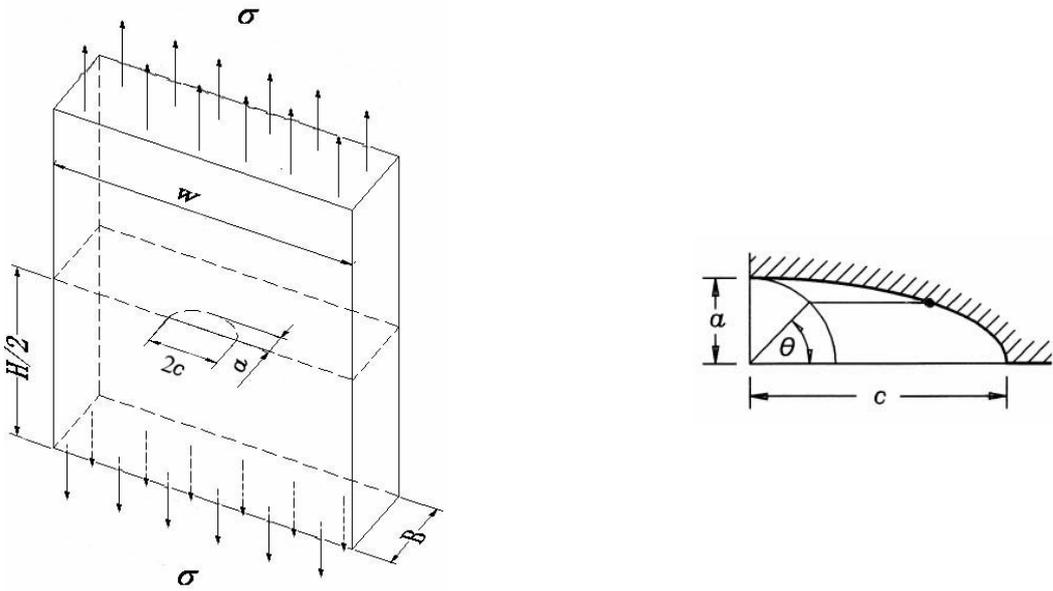
## 1. Introduction

With the development of society, the steel structure has been widely used in all kinds of engineering construction, such as oil and gas pipelines, bridges, marine oil platform and so on. In the process of long-term use, it will inevitably produce some defects in steel structure, such as surface crack, under the influence of environmental conditions and external load. Surface crack propagation will bring a lot of potential safety problems to engineering facilities. In recent years, scholars from home and abroad carried out a number of studies [1,2], and to the strengthening with crack repair technology have also made some progress [3-8]. Edberg et al [3] made some experimental studies to research the properties of I-font section steel beam strengthened with FRP using several different kinds of reinforcement scheme, the results show that the stiffness of the steel after strengthening increased by 23%, the ultimate bearing capacity increased by 42%. Nicholas G.Tsouvalis et al [4] conducted experiment and the finite element method to study the steel structures strengthened with composite. The results show that the bonded composite materials can effectively reduce crack propagation rate, the fatigue life of specimen increased by 74% or more. However, the research for stress intensity factor of the semi-elliptic surface crack in steel structure strengthened with FRP has not yet been reported. For this reason, in this paper, three dimensional semi-elliptic surface crack in X80 steel tension specimen strengthened with carbon fiber laminate (CFL) [9] was studied, and numerical analysis was undertaken by Abaqus finite element software to study the stress intensity factor (SIF,  $K_I$ ) of the surface crack for CFL reinforcement effect.

## 2. Finite element model

Research object in this paper was the 3D semi-elliptical surface crack in the center of the rectangle X80 steel plate. The steel plate was strengthened with carbon fiber laminate (CFL) in front side invented by this research group [9], and was subjected to uniform tension load of 100MPa on both ends, shown in Figure.1 and Figure.2. Where, the size of X80 steel plate was: length

$H=90\text{mm}$ , width  $W=70\text{mm}$ , thickness  $B=8\text{mm}$ , Young' modulus  $E_s=206\text{GPa}$  and Poisson's ratio  $\nu_s=0.3$ . The CFL has length  $H=90\text{mm}$ , width  $W=70\text{mm}$ , thickness  $t_f=0.2, 0.4, 0.6, 0.8, 1.0\text{mm}$ , Young' modulus  $E_f=230\text{GPa}$  and Poisson's ratio  $\nu_f=0.25$ . Surface crack shape ratio  $a/c=0.4, 0.6, 0.8$ , and relative depth  $a/B=0.1, 0.2, 0.3$ .



(a) Steel plate with surface crack (b) Parametric angle for the crack

Figure 1. Steel plate with surface crack under tensile load

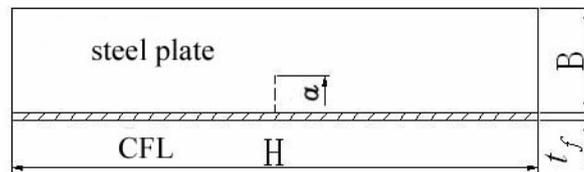


Figure 2. Steel plate strengthened with CFL

Finite element software Abaqus was used to establish the model for calculating the stress intensity factor of the semi-elliptical surface crack in different strengthening conditions. Due to symmetry of the analysis object, only one-quarter of the steel plate was analyzed. Two symmetrical surfaces of the model were restricted by symmetrical boundary conditions. In this analysis model, to avoid incompatible situations between CFL elements and crack front elements in front side strengthening, in the same time, to make the results comparable, steel plate and CFL were meshed using the three-dimensional elements. Around the crack tip area, a total of 11 layers elements were created. The first layer was wedge elements, and other 10 layers were hexahedral elements, there were totally 132 elements, shown in Figure. 3(a). Along the semi-elliptical curve, 20 equal parts were divided, which means there were 20 groups of elements and totally 2640 elements in the crack tip, shown in Figure. 3(b). The whole finite elements model was shown in Figure. 3(c), and there were totally 25932 elements.

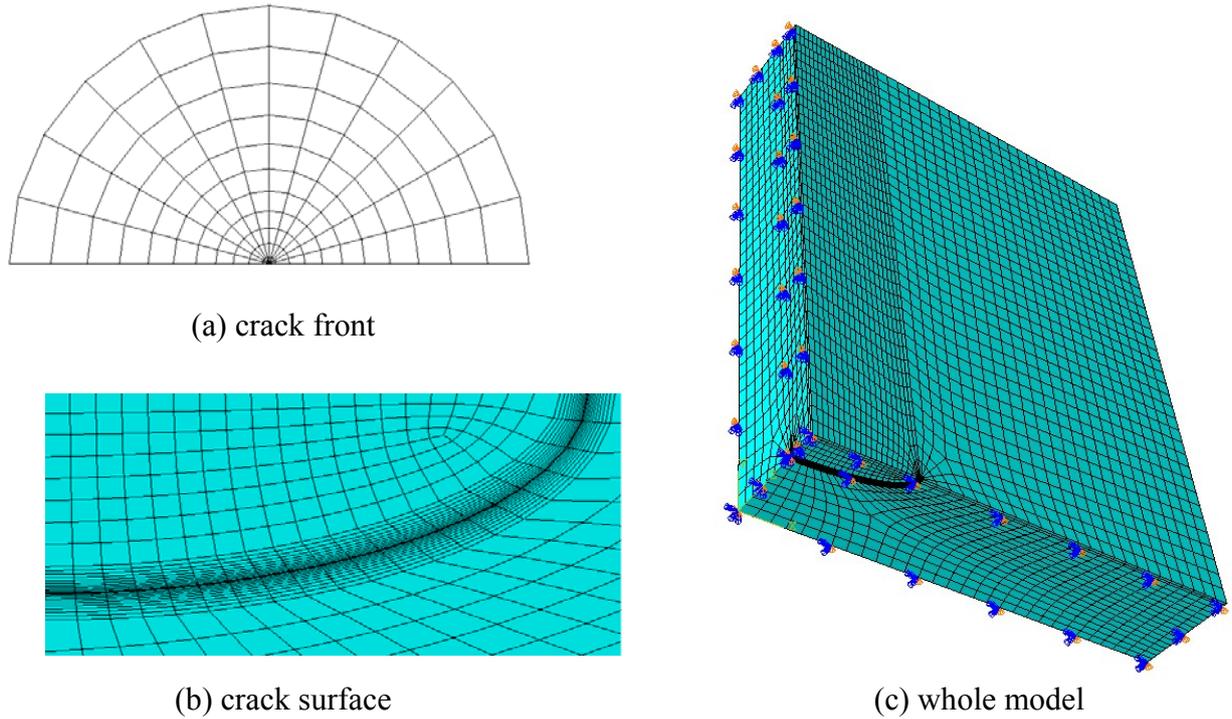


Figure 3. Meshes of the FEM model

### 3. SIF of 3D surface cracks

#### 3.1 SIF of crack in non-strengthened steel plate

Finite element analysis for stress intensity factor (SIF,  $K_I$ ) of 3D semi-elliptical surface crack in non-strengthened steel plate was compared with that obtained by the Newman-Raju equation to prove the effectiveness of the FEM method in this paper.

For finite element calculation, due to 11 layers of elements were meshed at the crack tip, the corresponding SIF obtained by contour J-integration at every point in front of the crack. Generally, the calculated values of the inside layer had a larger fluctuation, compared with that of the outer layer. In default situations, the results were outputted by the software Abaqus from the inner layer. Therefore, at least 6 SIFs were obtained at each point in front of the crack to insure the accuracy. The relatively stable value of the outer layer was selected as the final SIF.

SIF of three  $a/c$  ratios and three relative depth  $a/B$  of the surface crack were calculated using the above FEM method and Newman-Raju equation were shown in Figure.4. From this Figure, the finite element analysis results agreed well with that of the Newman-Raju equation in generally, the average relative error is less than 3%.

The minimum relative error was 0.38% when  $\theta = 90^\circ$ . There was a larger error near  $\theta = 0^\circ$  especially when the crack was shallower ( $a/B$  is smaller) and flatter ( $a/c$  is smaller). The maximum relative error was 7.8% when  $a/B=0.1$ ,  $a/c=0.4$  and  $\theta = 0^\circ$ . When the crack was shallower, flatter and the centrifugal angle was smaller ( $a/B$ ,  $a/c$ ,  $\theta$  was smaller), the relative error was larger. This was caused by the default conditions that the whole model was in plane strain state during the finite element analysis. However, it was in fact in plane stress state near the surface of the steel plate which caused greater relative error around  $\theta = 0^\circ$  and also caused larger relative error when  $a/B$  and  $a/c$  were smaller. The results show that the above method of calculating the SIF of 3D surface cracks in steel plates using Abaqus was effective and available.

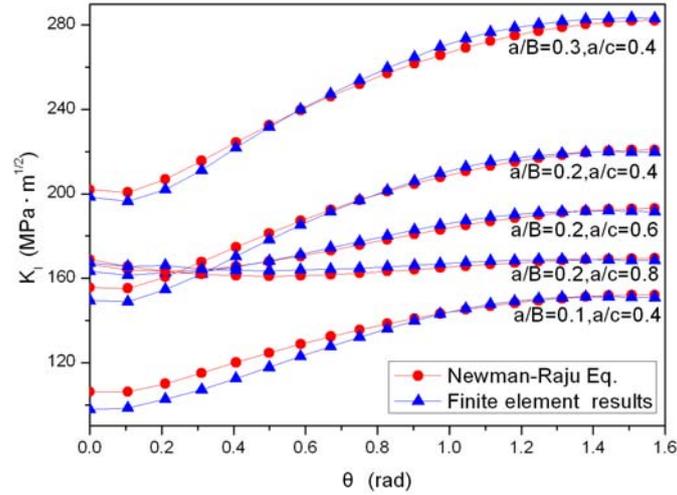


Figure 4. Computing values of  $K_I$  by FEM and Newman-Raju Eq.

### 3.2 SIF of crack in steel plate strengthened with FRP

In front surface strengthening (shown in Figure.2), the calculation values of SIF of 3D surface crack with  $t_f=0, 0.2, 0.4, 0.6, 0.8, 1.0$  when  $a/B=0.2$ ,  $a/c=0.6$  were listed in Figure.5. This Figure shown that the SIF generally decreased by 21% in front surface strengthening compared with that of non-strengthening ( $t_f=0$ ). As shown in Figure.5, the strengthening effect decreased as the angle changing from  $0^\circ$  to  $90^\circ$ , such as the reduction of stress intensity factor was the most (54.9%) when  $t_f = 0.2$  and  $\theta=0^\circ$ , and the stress intensity factor to reduce small (20.6%) when  $\theta=90^\circ$ . This is because that the displacement between the upper and the underside of the crack was restricted by the CFL bonded to the surfaces. Therefore, the closer distance to the surface of the steel was, the better effect of the reinforcement was obtained, where, the SIF on the surface of the steel was reduced most. Furthermore, the decreasing degree of SIF became smaller as the increasing of the thickness of CFL. That means overabundance of FRP caused material waste. The calculation values of SIF of the surface crack by FEM had the same properties when the shape and relative depth changed.

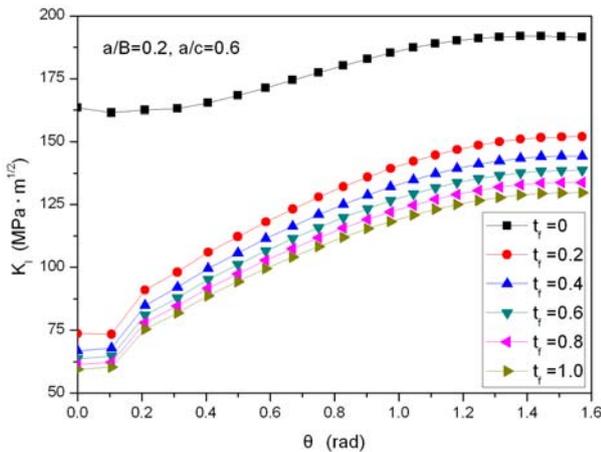


Figure 5. Computing values for  $K_I$

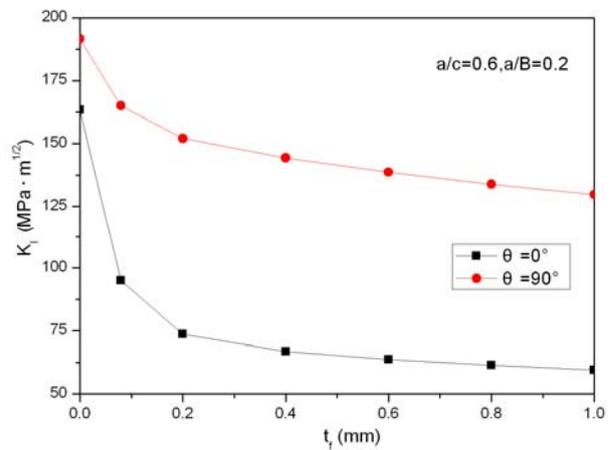


Figure 6. Relationship between  $K_I$  and  $t_f$

The strengthening effect was lowest when  $\theta=90^\circ$ . For safety (conservative) considerations, the series was chosen as the calculation reference values in strengthening design. Therefore, functions of SIF and different thickness of CFL when  $\theta=90^\circ$  were simulated as:

$$K_{I,\theta=90^\circ} = K_0 - 2.49t_f^{0.29} \quad (t_f \geq 0, \theta = 90^\circ) \quad (1)$$

Where,  $K_I$  was the stress intensity factor (SIF), which unit was  $\text{MPa}\cdot\text{m}^{1/2}$ ;  $t_f$  was the thickness of FRP which unit was mm;  $K_0$  was the stress intensity factor without strengthening ( $t_f=0$ ).

The SIF was gradually decreasing as the increasing of the thickness of CFL in front surface strengthening from Figure.6, and becoming flat after  $t_f \geq 0.6\text{mm}$ .

### 3.3 Effect of crack parameters on $K_I$

#### 3.3.1 Effect of $a/c$

To discuss the effect of crack shape ratio  $a/c$  on  $K_I$ , the relative depth  $a/B$  was supposed to be invariable in FEM analysis. The curves of  $K_I$  in front side strengthening were listed in Figure. 7 when  $t_f=0.2\text{mm}$ ,  $a/B=0.2$ ,  $a/c=0.4, 0.6, 0.8$ . The strengthening effect was better from Figure.7, while  $a/c$  was larger, the effect was weaker. For example, strengthening effect was 26.1% when  $\theta=90^\circ$  and  $a/c=0.4$ , and that was 20.6% and 16.1% when  $a/c=0.6$  and  $a/c=0.8$  respectively as shown in Figure.8. It can be considered as  $a/B$  was invariant,  $a/c$  was larger,  $c$  was lower, and the interaction region between FRP and surface crack was shorter, the function of FRP was smaller.

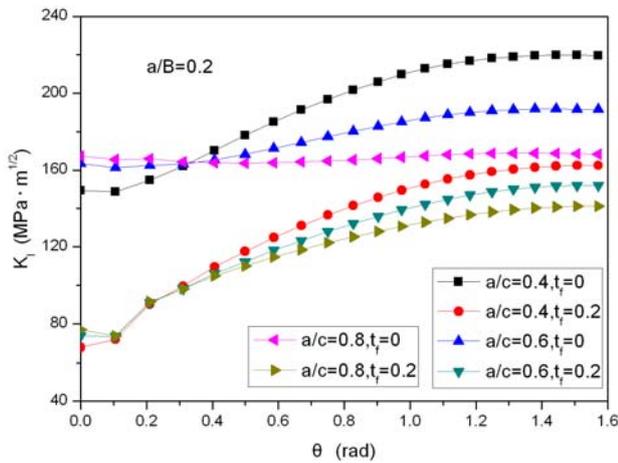


Figure 7. Effect of  $a/c$  on  $K_I$

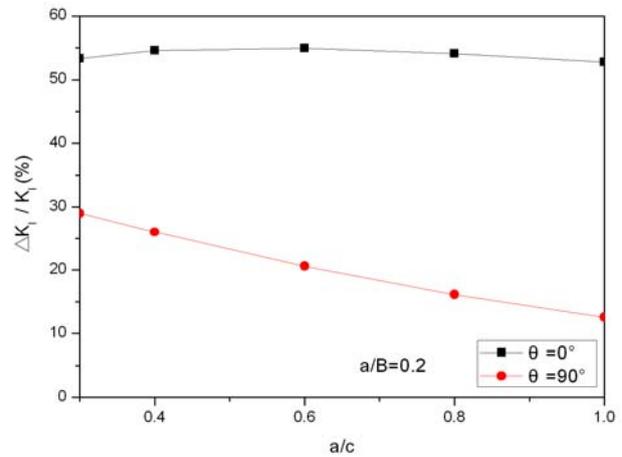


Figure 8. Effect of  $a/c$  on  $K_I$  for  $\theta=90^\circ$  and  $\theta=0^\circ$

#### 3.3.2 Effect of $a/B$

To discuss the effect of the relative depth  $a/B$  on  $K_I$ , the shape ratio  $a/c$  was supposed to be invariable for FEM analysis. The curves of  $K_I$  in front strengthening were listed in Figure. 9 when  $t_f=0.2$ ,  $a/c=0.4$ , and  $a/B$  was equal to 0.1, 0.2 and 0.3. The strengthening effect in front side strengthening was better, while  $a/B$  was larger, the effect was weaker. As shown in Figure.10, the reinforcement effect can be reduced when  $a/B < 0.3$  and  $\theta=90^\circ$ , but the reinforcement effect will increase when  $a/B > 0.3$  and  $\theta=90^\circ$ . For example, the effect of reinforcement was 32.1%, 26.1%, 24.8%, 25.9% and 28.1% when  $a/B$  was equal to 0.1, 0.2, 0.3, 0.4 and 0.5 respectively near  $\theta=90^\circ$ . It was because that the increasing of the crack depth caused further distance from CFL to the crack

tip, and the resistance to crack development was weaker when the CFL was bonded to the front side surface.

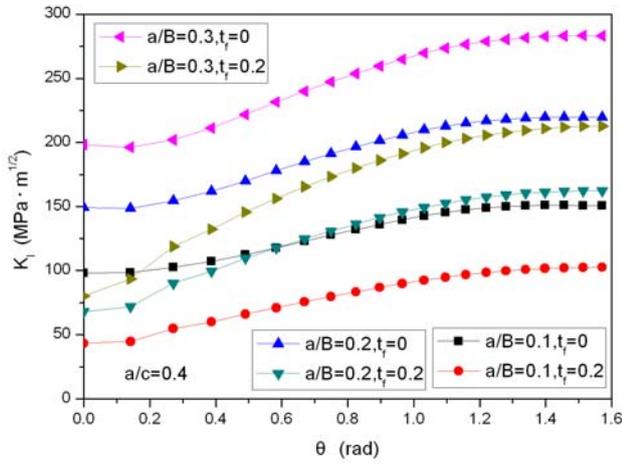


Figure 9. Effect of  $a/B$  on  $K_I$

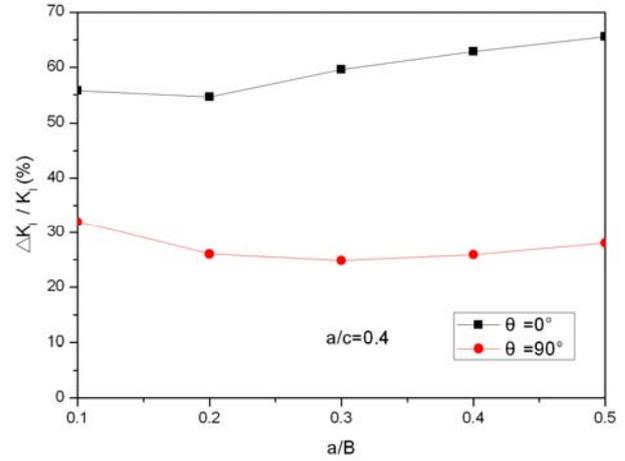


Figure 10. Effect of  $a/B$  on  $K_I$  for  $\theta=90^\circ$  and  $\theta=0^\circ$

#### 4. Expression for $K_I$

In order to facilitate the calculation of SIF for the 3D semi-elliptical surface crack in the center of the rectangle steel plate strengthened with FRP, and very convenient for engineering application, based on the analysis results of a large number of finite element calculation, the mathematical software Matlab was used to fit the calculation expression of SIF, and get the expression as follows:

$$K_I = \sigma \cdot F\left(\frac{a}{c}, \frac{a}{B}, t_f, \theta\right) \quad (2)$$

Where,  $\sigma$  is tensile stress acted on the ends of the steel plate, and

$$\begin{aligned} F\left(\frac{a}{c}, \frac{a}{B}, t_f, \theta\right) = & 0.4 + 0.23 \cdot p + q \cdot (4.96 - 10.09 \cdot t + 4.24 \cdot r) + t^2 \cdot (6.44 \cdot q - 0.39) + 0.44 \cdot r^2 \\ & + p \cdot q \cdot r \cdot (27.35 \cdot t - 6.82) + t \cdot r^2 \cdot (3.11 \cdot q - 0.82 \cdot p) - p \cdot q \cdot t \cdot r \cdot (26.09 \cdot t + 17.35 \cdot r) \\ & + t^2 \cdot r^2 \cdot (0.66 \cdot p - 6.4 \cdot q + 22.01 \cdot p \cdot q) \end{aligned} \quad (3)$$

Where,  $p = a/c$ ,  $q = a/B$ ,  $t = \ln(1 + t_f^{0.25})$ ,  $r = \sin \theta$ ,  $0 < a/c < 1.0$ ,  $0 < a/B < 1.0$ ,  $2c/W \leq 0.5$ ,  $0^\circ \leq \theta \leq 90^\circ$ .

Comparing the calculating values by equations (2) and (3) with the data from FEM, it is shown that the average error is within 3% and the fitting result is reasonable.

#### 5. Conclusions

Finite element software Abaqus was applied to study the stress intensity factors (SIF) of 3D semi-elliptical surface crack in rectangle steel plate strengthened with CFL, and the strengthening effects were discussed. The conclusions were as follows:

(1) Front side strengthening reduced the stress intensity factors of 3D semi-elliptical surface crack significantly, but the increasing amplitude of strengthening effects decreased gradually with increasing of thickness of CFL.

(2) Crack shape ratio  $a/c$  had a great effect on front side strengthening, and its effect decreased with the increasing of  $a/c$ .

(3) Crack relative depth  $a/B$  also had some effect on front side strengthening. The strengthening effect decreased with the increasing of  $a/B$ . The strengthening effect can be reduced when  $a/B < 0.3$  and  $\theta = 90^\circ$ , but the strengthening effect will increase when  $a/B > 0.3$  and  $\theta = 90^\circ$ .

(4) An expression for calculating the stress intensity factors of semi-elliptic surface crack under strengthening condition was proposed by adopting MATLAB software.

### Acknowledgments

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### References

- [1] X.B.Lin, R.A.Smith, Numerical analysis of fatigue growth of external surface cracks in pressurized cylinders. *Pres.Ves.& Piping*, 71(1997)293-300.
- [2] Hadi Khoramishad, Majid Reza Ayatollahi, Finite element analysis of a semi-elliptical external crack in a buried pipe. *Transactions of the Canadian Society for Mechanical Engineering*, 33(2009) 399-409.
- [3] Edberg W, Mertz D, Gillespie Jr J, Rehabilitation of Steel Beams Using Composite Materials. *Proceedings of the Materials Engineering Conference, Materials for the New Millenium*, New York, NY:ASCE, 1(1996)502-508.
- [4] Nicholas G.Tsouvalis, Lazarus S.Mirisiotis, Dimitris N.Dimou, Experimental and numerical study of the fatigue behaviour of composite patch reinforced cracked steel plates. *International Journal of Fatigue*, 31(2009)1613-1627
- [5] P.Y. Huang, XP. Zhou, C. Zhao, Some Key Mechanical Problems on FRP Sheet Applied in Civil Engineering. *Journal of South China University of Technology*, 30(2002):101-105.
- [6] X.L. Zhao, L. Zhang, State-of-the-art review on FRP strengthened steel structures. *Engineering Structures*, 29(2007) 1808-1823.
- [7] L.Briottet, R.Batisse, G.de Dinechin, et al, Recommendations on X80 steel for the design of hydrogen gas transmission pipelines. *International Journal of Hydrogen Energy*, 11(2012) 9423-9430.
- [8] Kent A. Harries, Andrew J. peck, Elizabeth J. Abraham, Enhancing stability of structural steel sections using FRP. *Thin-Walled Structures*, 47(2009) 1092-1101.
- [9] P.Y. Huang, J.C. Zeng, Application of Carbon Fiber Laminate. Chinese Patent No. ZL200410026742.8, China Patent Administration, Beijing, 2006