

The Attenuation and Dispersion Analyses in Porous and Fractured Medium with Arbitrary Fracture Fill

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Abstract To study the effect of fracture fill on the elastic anisotropy of the rock and frequency-dependent attenuation and dispersion in fractured reservoirs, a model for porous and fractured medium is developed. In this model, the fractured medium is considered as a periodic system of alternating layers of two types: thick porous layers representing the background, and very thin and highly compliant porous layers representing fractures. By taking the simultaneous limits of zero thickness and zero normal stiffness of the thin layers, we obtain expressions for dispersion and attenuation of the P-waves. The results show that in the low-frequency limit the elastic properties of such a medium can be described by Gassmann equation with a composite fluid, while the P-wave speed is relatively high at high frequencies for two layers can be treated as ‘hydraulically isolated’. However, there appears to be a critical case where no dispersion is observed, which is caused by the balance of fractures compliance and fluid compressibility filling in them.

Keywords porous media, fracture, attenuation, dispersion

1 Introduction

Flow of the pore fluid by the passing wave is widely believed to be the main cause of attenuation and dispersion of elastic waves in porous rocks. In particular, flow that occurs due to spatial variations of rock or fluid properties on mesoscopic scale (larger than the pore size but smaller than the wavelength) is considered to be significant at seismic frequencies [1-4]. The magnitude of attenuation and dispersion caused by mesoscopic wave-induced flow is proportional to the squared contrast (variance) of spatial variations of rock or fluid properties. Thus attenuation and dispersion are only significant if the contrast of spatial variations is large.

In recent years, two situations with large contrast in rock/fluid properties have been identified: partial saturation and fractured rock. Partial saturation refers to the situation where a rock is saturated with a mixture of two immiscible fluids with large difference between their properties (say, liquid and gas). When an elastic wave propagates through such a rock, the patches of rock saturated with gas and liquid will deform differently, resulting in pressure gradients and fluid flow [5-8]. Fractured rock refers to a situation where a porous rock is permeated by open fractures. When a wave propagates through such a rock, fractures will deform to a greater extent than the porous background, resulting in fluid flow between pores and fractures [9-13]. These situations (partial saturation and fractures) are usually treated separately: analysis of wave propagation in a partially saturated rock usually ignores variations in elastic properties of the solid frame, while the porous rock permeated by fractures is usually assumed to be saturated with a uniform fluid. However, in some situations, particularly when a fluid such as water or carbon dioxide is injected into a tight hydrocarbon reservoir, fractures may be filled with a different fluid (with capillary forces

preventing fluid mixing). In this paper we consider the simplest situation of this kind: a porous rock saturated with one fluid and permeated by a single set of aligned planar fractures filled with another fluid. For such a medium, we derive a dispersion equation following a method originally proposed by Brajanovski et al. [12] for a porous fractured medium saturated with a single fluid.

The paper is organized as follows. First, we review the theory in case of a medium saturated with a single fluid. Then we extend the method to the situation where the porous background and fractures are saturated with different fluids and derive the corresponding dispersion equation, which yields expressions for dispersion and attenuation due to wave induced flow between pores and fractures. To explore the behavior of attenuation and dispersion, we explore various limiting cases and present several numerical examples. Finally, we discuss the physical nature of the results obtained.

2 Liquid saturated porous and fractured medium

Brajanovski et al. [12] developed a model for a porous medium with aligned fractures. The medium comprises a periodic (with spatial period H) stratified system of alternating layers: relatively thick layers of a background material (with a finite porosity ϕ_b) and relatively thin layers of a high-porosity material representing the fractures. This double porosity model is a limiting case of a periodically layered poroelastic medium studied by White et al. [1] and Norris [14]. Norris showed that for frequencies much smaller than Biot's characteristic frequency $\omega_B = \eta\phi/k\rho_f$, and also much smaller than the resonant frequency of the layering $\omega_R = V_p/H$, the compressional wave modulus of a periodically layered fluid-saturated porous medium composed of two constituents, b and c , can be written in the form:

$$\frac{1}{c_{33}} = \frac{h_b}{c_b} + \frac{h_c}{c_c} + \frac{\left(\frac{\alpha_b M_b}{c_b} - \frac{\alpha_c M_c}{c_c}\right)^2}{\sqrt{\frac{i\omega\eta_b M_b L_b H}{c_b k_b} \cot\left(\sqrt{\frac{i\omega\eta_b c_b h_b H}{k_b M_b L_b}}\right)} + \sqrt{\frac{i\omega\eta_c M_c L_c H}{k_c c_c} \cot\left(\sqrt{\frac{i\omega\eta_c c_c h_c H}{k_c M_c L_c}}\right)}}, \quad (1)$$

In Eq. (1), both constituents are assumed to be made of the same isotropic grain material with bulk modulus K_g , shear modulus μ_g and density ρ_g , but they have different solid frame parameters: porosity ϕ , permeability k , dry bulk modulus K , shear modulus μ and thickness fraction h . The layers b and c may be saturated with different fluids with bulk modulus K_f , density ρ_f and dynamic viscosity η , as indicated by adding 'b' and 'c' in the subscript. Parameter $C_j = L_j + \alpha_j^2 M_j$ denotes the fluid-saturated P-wave modulus of layer j given by Gassmann's equation [15], where $\alpha_j = 1 - \frac{K_j}{K_g}$ is Biot's effective stress coefficient, M_j is pore space modulus defined by $\frac{1}{M_j} = \frac{\alpha_j - \phi_j}{K_g} + \frac{\phi_j}{K_{fj}}$ and $L_j = K_j + 4\mu_j/3$ is the dry P-wave modulus of the layer j .

To construct a model for a porous medium permeated by parallel fractures, Brajanovski et al. [12] considered parameters with subscript b to represent the porous background, and parameters with subscript c to represent fractures (cracks). They then assumed fractures to be very thin and very

compliant layers and thus considered Eq. (1) in the limit $h_c \rightarrow 0$, $K_c \rightarrow 0$ and $\mu_c \rightarrow 0$ such that both K_c and μ_c (and hence $L_c \rightarrow 0$) are $O(h_c)$. By assuming that both pores and fractures are saturated with the same fluid with the viscosity $\eta_b = \eta_c = \eta$, Brajanovski et al [12] obtained the equation

$$\frac{1}{C_{33}} = \frac{1}{C_b} + \frac{\left(\frac{\alpha_b M_b - 1}{C_b}\right)^2}{\sqrt{\frac{i\omega\eta M_b L_b H}{C_b k_b} \cot\left(\sqrt{\frac{i\omega\eta C_b h_b H}{k_b M_b L_b}}\right) + \frac{1}{Z_N}}}, \quad (2)$$

Where $Z_N = \lim_{h_c \rightarrow 0} \frac{h_c}{L_c}$ is the normal excess fracture stiffness of the dry frame given by Schoenberg and Douma [16]. Implicit in the derivation of Eq. (2) was an assumption

$$K_f \gg h_c / Z_N, \quad (3)$$

When both pores and fractures are saturated with a liquid, Eq. (2) exhibits significant attenuation and velocity dispersion. However the model is limited to the case where the fluid is the same in both matrix pores and fractures, and there is an upper limit on the fluid compressibility (Eq. (3)). Below we develop a model that overcomes these limitations.

3 Arbitrary fluid in the fractures

The analysis in the previous section suggests that the effect of fracture fill on the overall modulus of the porous and fractured medium depends on how the fluid bulk modulus scales with h_c as $h_c \rightarrow 0$. To analyze this effect, we use the following parameterization

$$K_{fc} / K_g = B h_c, \quad (4)$$

where B is a dimensionless nonzero constant that defines the type of fluid in fractures, liquid, gas or intermediate. If B is large enough (e.g., if the fluid is a liquid), K_{fc} may satisfy the condition (3). In this case, taking the limit $h_c \rightarrow 0$ in Eq. (1), we obtain Eq. (2) with fluid properties K_f and η replaced by the corresponding values for the fluid in the pores K_{fb} and η_b . Conversely, $B = 0$ corresponds to dry fractures.

When $h_c \rightarrow 0$, we have $K_c \rightarrow 0$, and $\phi_c \rightarrow 1$, and thus $\alpha_c \rightarrow 1$, $M_c \rightarrow K_{fc}$, and $C_c \rightarrow L_c + K_{fc}$. Combining these results with the parameterizations (4), and considering $\cot(x) \approx 1/x$ for any complex x with $|x| \ll 1$ and $\cot(\sqrt{ix}) \rightarrow i$ for any real $x \gg 1$, Eq. (1) in the limit $h_c \rightarrow 0$ can be simplified as

$$\frac{1}{C_{33}} = \frac{1}{C_b} + \frac{Z_N}{1 + Z_N B K_g} + \frac{\left(\frac{\alpha_b M_b - \frac{Z_N B K_g}{1 + Z_N B K_g}}{C_b}\right)^2}{\sqrt{\frac{i\omega\eta_b M_b L_b H}{C_b k_b} \cot\left(\sqrt{\frac{i\omega\eta_b C_b H}{k_b M_b L_b}}\right) + \frac{B K_g}{1 + Z_N B K_g}}}, \quad (5)$$

Eq. (5) is the approximation of Eq. (1) for a porous and fractured medium with an arbitrary fracture fill.

Here, we have introduced a dimensionless constant B to define the type of fracture fluids, so that we can attain gas and liquid limiting cases. Additionally, we can define low and high frequencies with respect to fluid flow between fractures and background. Therefore, in the following section, we derive and analyze some limiting cases of fluids and frequencies.

4 Limiting cases

4.1 Fluid limits

4.1.1 Liquid in fractures

If the fluid in the fractures is liquid, then $B \gg 1$ and Eq. (5) gives

$$\frac{1}{c_{33}} = \frac{1}{c_b} + \frac{\left(\frac{\alpha_b M_b}{c_b} - 1\right)^2}{\sqrt{\frac{i\omega\eta_b M_b L_b H}{c_b k_b} \frac{1}{2} \cot\left(\sqrt{\frac{i\omega\eta_b c_b H}{k_b M_b L_b^2}}\right) + \frac{1}{Z_N}}}, \quad (6)$$

Note that Eq. (6) is exactly the same as Eq. (2). This shows that the result of Brajanovski et al [12] is valid not only if both pores and fractures are saturated with the same liquid, but also when the two liquids are different.

4.1.2 Dry or nearly dry fractures

When fractures are dry or nearly dry, Eq. (5) cannot, strictly speaking, be used because it was derived by assuming that B is nonzero. Instead, we take the limit $B \rightarrow 0$, directly in Eq (1). This gives

$$\frac{1}{c_{33}} = \frac{1}{c_b} + Z_N + \frac{\left(\frac{\alpha_b M_b}{c_b}\right)^2}{\sqrt{\frac{i\omega\eta_b M_b L_b H}{c_b k_b} \frac{1}{2} \cot\left(\sqrt{\frac{i\omega\eta_b c_b H}{k_b M_b L_b^2}}\right)}}, \quad (7)$$

Incidentally, exactly the same result is obtained by taking the limit $B \rightarrow 0$ in Eq. (5). This means that Eq. (5) is valid even in the limit of small B . Eq. (7) gives the P-wave modulus for a porous medium with dry or gas-filled fractures, and it is quite different from Eq. (5) for liquid case. To further analyze the reason for the difference, we derive the limiting cases of low and high frequencies next.

4.2. Frequency limits

4.2.1 Low frequencies

In the low-frequency limit $\omega \rightarrow 0$, the cotangent function in Eq. (5) can be replaced by the inverse of its argument. Thus, Eq. (5) is reduced to

$$\frac{1}{c_{33}} = \frac{1}{c_b} + \frac{1}{\frac{1}{Z_N} + BK_g} + \frac{\left(\frac{\alpha_b M_b}{c_b} - \frac{BK_g}{\frac{1}{Z_N} + BK_g}\right)^2}{\frac{M_b L_b}{c_b} + \frac{BK_g}{1 + Z_N BK_g}}, \quad (8)$$

In the low frequency limit, the fluid pressure should be fully equilibrated between pores and fractures. Thus in this limit the result must be consistent with the anisotropic Gassmann equations for a fractured medium saturated with a single composite fluid [17] with a bulk modulus defined by

$$\frac{1}{K_f^*} = \frac{S_{fb}}{K_{fb}} + \frac{S_{fc}}{K_{fc}}, \quad (9)$$

Eq. (9) is known as the Wood equation, and corresponds to so-called uniform saturation of the partial saturation theory [7, 8, 18]. So, if we replace K_b with K_f^* in Eq. (5), and then take the low

frequency limit, we can also get the same expression as given by Eq. (8).

To clarify the physical meaning of the low-frequency Eq. (8) we again consider liquid and dry (or gas) cases. For liquid-filled fractures (large B), we have

$$\frac{1}{c_{33}} = \frac{1}{c_b} + \frac{\left(\frac{\alpha_b M_b - 1}{c_b}\right)^2}{\frac{M_b L_b + 1}{c_b} + \frac{1}{Z_N}}, \quad (10)$$

while for $B \ll 1$

$$\frac{1}{c_{33}} = \frac{1}{L_b} + Z_N, \quad (11)$$

The liquid limit, Eq. (10), corresponds exactly to the low frequency limit of the result of Brajanovski et al [12], with only the bulk modulus of the liquid in the pores affecting the overall modulus. This result may be understood from Eq. (10), which shows that when the bulk moduli of the two fluids are comparable, the effect of the fracture fluid is negligible for its relatively small saturation. In turn, Eq. (11) is exactly the modulus of the dry medium [12, 16]. This is because when K_{fc} is very small (much smaller than $h_c K_{fb}$), Wood Eq. (10) for the effective fluid modulus reduces to

$$\frac{1}{K_f^*} = \frac{S_{fc}}{K_{fc}}, \quad (12)$$

and thus $K_f^* \rightarrow 0$, which means that the whole porous and fractured model can be considered as dry or nearly dry medium. Physically, this is the result of the fact that at low frequencies, the pore pressure is equilibrated between pores and fractures, so when the pressure in fractures is zero, this is also zero in the pores. This is the drained – or dry – limit.

4.2.2 High frequencies

In the high-frequency limit $\omega \rightarrow \infty$, the cotangent function in Eq. (5) can be replaced by i . So, we can get the expression of P-wave modulus at high frequencies

$$\frac{1}{c_{33}} = \frac{1}{c_b} + \frac{1}{\frac{1}{Z_N} + BK_g}, \quad (13)$$

In liquid and gas cases we have

$$\frac{1}{c_{33}} = \frac{1}{c_b}, \quad (14)$$

and

$$\frac{1}{c_{33}} = \frac{1}{c_b} + Z_N, \quad (15)$$

respectively.

Note that at high frequencies, the fluid pressure does not have time to equilibrate between pores and fractures, and thus they can be considered ‘hydraulically isolated’ [12, 17]. Thus the P-wave modulus in this limit corresponds to the modulus of a porous medium with isolated fractures. In the liquid case, the modulus given by Eq. (14) is the same as if there were no fractures. This is because liquid can stiffen the otherwise very compliant fractures so that P-wave velocities for waves propagating parallel and perpendicular to layering are both approximately equal to the modulus of the background medium [12, 16, 19]. Conversely, when fractures are dry, the P-wave modulus (15) is the same as for a medium with dry isolated fractures (cf Eq. (11)).

5 Numerical examples

Our main result, Eq. (5), shows that the P-wave modulus is complex-valued and frequency dependent regardless of fluid saturation of fractures. This means that waves will have attenuation and dispersion.

To explore these effects, we compute the complex phase slowness in the direction normal to the fractures $V_p^{-1} = \sqrt{\rho_b/C_{33}}$, where $\rho_b = (1 - \phi_b)\rho_g + \phi_b\rho_{fb}$ is mass density of the fluid-saturated background (the effect of fractures on the density can be ignored as their volume fraction is negligibly small). This complex phase slowness can be used to evaluate the frequency dependence of the P-wave phase velocity and attenuation for waves propagating perpendicular to fractures. The P-wave phase speed is the inverse of real part of the complex phase slowness, and the attenuation Q is given by half the ratio of the real part to the imaginary part of the complex phase slowness.

Now, we rewrite Eq. (5) as a function of normalized frequency Ω [12]

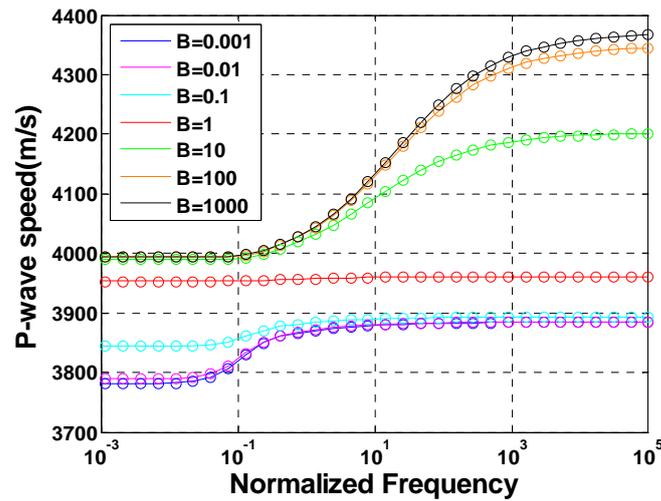
$$\frac{1}{C_{33}} = \frac{h_b}{c_b} + \frac{1}{\frac{L_b(1-\delta_N)}{\delta_N} + BK_g} + \frac{\left(\frac{\alpha_b M_b}{c_b} - \frac{BK_g}{\frac{L_b(1-\delta_N)}{\delta_N} + BK_g} \right)^2}{L_b \sqrt{i\Omega} \cot\left(\frac{h_b c_b \sqrt{i\Omega}}{M_b} \right) + \frac{BK_g}{1 + \frac{\delta_N}{L_b(1-\delta_N)} BK_g}}, \quad (16)$$

where $\Omega = \frac{\omega H^2 \eta_b M_b}{4 c_b k_b L_b}$ is the normalized frequency and $\delta_N = \frac{Z_N L_b}{1 + Z_N L_b}$ is a dimensionless (normalized) fracture weakness with values between 0 and 1 [16]. All our calculations are made for a water-saturated sandstone using quartz as the grain material ($K_g=37\text{GPa}$, $\rho_g=2.65 \times 10^3 \text{kgm}^{-3}$). The dependency of bulk and shear moduli of the background dry on porosity was assumed to follow the empirical model of Krief et al. [20].

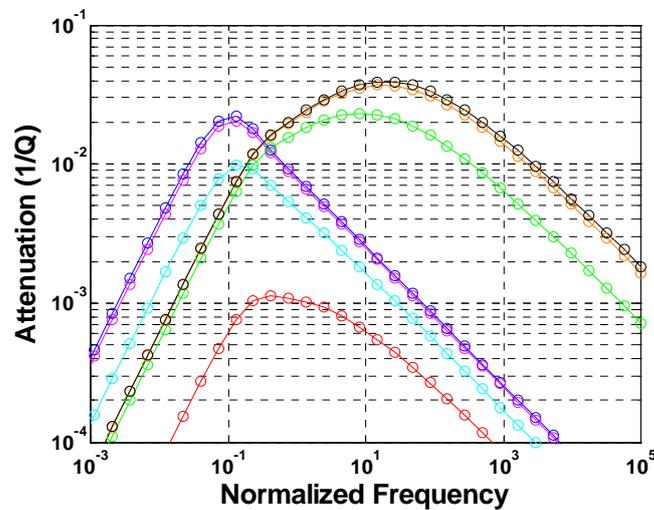
To explore the validity of our approximation, we compare the attenuation and dispersion results with the original Norris [14] model, Eq. (1). For the Norris model, we set fracture parameters to satisfy the assumptions of the approximation ($h_c=0.001$, $\phi_c=0.999$, $\delta_N=0.2$, $k_c=200\text{mD}$, $\eta_{fc}=18.1e^{-6}\text{Pa.s}$). Then, the P-wave speeds and inverse quality factor Q^{-1} are calculated for different values of B , and the results are shown on Fig. 1. Alternatively, we could have given an input value to K_{fc} and then computed B using Eq. (4). However, we prefer to evaluate the results for different values of the dimensionless constant B .

Fig. 1 shows dispersion and attenuation of P-waves propagating along the symmetry axis (normal to fracture plane) for different values of parameter B . Symbols show the values obtained by our approximation, Eq. (5), while the curves correspond to the Norris general solution, Eq. (1). We see that for the whole range of parameter B , the approximation matches the general model very accurately. Curves of dispersion and attenuation have a shape typical for a relaxation phenomenon. It is interesting that dispersion and attenuation is significant for both liquid-filled ($B=1000$) and dry ($B=0.001$) fractures, but is much lower for intermediate values of the parameter B . This somewhat

surprising observation can be explained as follows. When both pores and fractures are filled with liquids, the compression caused by the wave will compress the fractures to much greater extent than the background porous material (since fractures are much more compliant than pores), causing the fluid to flow from fractures into pores. Conversely, when the pores are saturated with a liquid and the fractures are dry (or filled with very compressible gas), the compression will cause the flow from pores into fractures (it will be easier to compress gas than deform the fractures). Thus, at some intermediate value of the fluid compressibility (or parameter B), there will be no flow at all, and hence no dispersion or attenuation.



(a)



(b)

Figure 1. Frequency dependency of P-wave velocity (a) and inverse quality factor Q^{-1} (b) computed using our approximation (17) and Norris model (1) for different values of the parameter B .

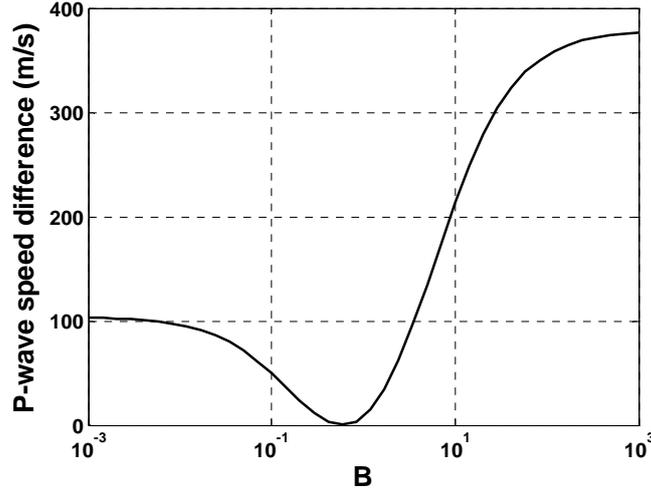


Figure 2. Dispersion magnitude (difference between high- and low-frequency velocities) as a function for parameter B . As B increases, the dispersion first decreases, reaches zero, and then increases again.

Fig. 2 shows the dispersion magnitude (difference between high- and low-frequency velocities) as a function for parameter B . As B increases, the dispersion first decreases, reaches zero, and then increases again. We also see that the dispersion is almost insensitive to B both for very small and very large values of B (corresponding to highly compressible gases and liquids, respectively), but quite sensitive to B for values of B in a range around the critical value where dispersion reaches zero. This critical value can be obtained by equating low- and high-frequency limits, Eqs (8) and (13). This gives

$$B^* = \frac{\alpha_b M_b}{c_b - \alpha_b M_b} \frac{1}{K_g} \frac{1}{Z_N} \quad (17)$$

The value B^* given by Eq. (17) corresponds to a critical fracture fluid modulus case, where there will be zero dispersion and attenuation in the general porous and fractured model. For the parameters used in the numerical example of Fig. 1, Eq. (5) gives $B^* \approx 0.59$. This value is quite close to 1.0, and thus we see very small dispersion and attenuation for $B = 1$.

6 Conclusions

We have developed a model for wave propagation in a porous medium with aligned fractures such that pores and fractures can be filled with different fluids. The model considers the fractured medium as a periodic system of alternating layers of two types: thick porous layers representing the background, and very thin and highly compliant porous layers representing fractures. The results show that in the low-frequency limit the elastic properties of such a medium can be described by Gassmann equation with a composite fluid, whose bulk modulus is a harmonic (Wood) average of the moduli of the two fluids. At higher frequencies, the model predicts significant dispersion and attenuation. The dispersion and attenuation are the highest when both pores and fractures are saturated with liquids. The dispersion and attenuation are also significant (but somewhat weaker) when the pores are filled with a liquid but fractures are dry or filled with a highly compressible gas. However, there is an intermediate case where no dispersion is observed. This can be explained by observing that when the medium is uniformly saturated with a liquid, wave-induced compression causes flow from fractures into pores due to high compliance of the fractures. Conversely, when

pores are filled with a liquid but fractures are filled with gas, flow will occur from pores into fractures due to high compressibility of gas. Thus an intermediate case exists where there is no flow and hence no dispersion or attenuation.

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