Applications of Fracture Theory to Instability Co-seismic Fault Problem

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Abstract: Co-seismic earthquake faults can be treated as cracks of fracture theory in the framework of continuum mechanics. It is well known that the instability of the faults leads to earthquakes, and the relevant problem can be explored through the fracture theory. There are many factors influencing the instability such as tectonic stresses, geology structure and so on. In this work, we discuss the effects of geometry, interaction between faults and the speed of nucleation and co-seismic process. For single fault, the closed formulations of fracture theory is simpler; for two collinear faults, the analytic solution is available; while for two parallel faults the semi-analysis is approximate with finite element/boundary element method. The theoretical prediction is compared with the observation data of Xingtai earthquake 1966, Hejian earthquake 1967, Tangshan-Luanxian earthquake 1976 and Wenchuan earthquake 2008.

Keywords Fault, Strong Earthquake, Block and Plate, Instability, Fracture theory

1. Introduction

In China, some strong earthquakes happened in plate rather than in boundary between plates. These strong earthquakes are called in plate strong earthquake, or land strong earthquakes. The typical examples are the Ms7.3 Xingtai earthquake on March 8, 1966, the Ms7.8 Tangshan earthquake on July 28, 1976, the Ms8.0 Wenchuang earthquake on May 8, 2008, etc., which resulted in serious destroy and killed a lot of people.

Why did the strong earthquakes happen in plate? To explain the phenomenon, geologists, geophysicists and seismologists in China put forward the block hypothesis of the land strong earthquakes [1-5]. They observed that these earthquakes appeared in the boundaries between blocks. In China land the blocks are very developed, some blocks have quite big sizes though they are smaller compared with plates. For example, the Wenchuang earthquake is in Wenchuang-Beichuang fault, which is located in the boundary between the Tibet Block and the South China Block that have huge sizes. The land strong earthquake presents some natures themselves different from those of earthquakes in boundary between plates.

In this work, we focus on the block characters and apply fracture theory doing some analysis, and preliminarily explain phenomena concerning Xingtai Earthquake, Tangshan earthquake and Wenchuang earthquake those happening in China land.

2. Physics modeling

The land strong earthquakes occurred along some faults in the boundary between blocks, a crack model concerning fault instability has been developing since the 1970s and numerous work has been done since then, e.g. refs. [1-6]. Thus studies may be beneficial for investigating the land strong earthquakes in China. Due to the dependence of the earthquakes with blocks, the complex system consisting of blocks and faults are studied. For this system a characteristic size is needed. For this purpose we consider the model shown in Fig.1. The configuration concerning a strike slip fault is only an example, if the applied shear stress in plane instead by a shear stress out of plane, then it represents an inverse fault. The boundary between blocks is a transitional region with thickness *2H*, and the fault with length *a*, we takes *a/H* as a characteristic size of the complex system. The tip of the fault is often the earthquake source, it has some intra-structures, one among them is the so-called the slip weakening zone (or breakdown zone), which can be described by a size *R*, so that have another characteristic size *R/H*.

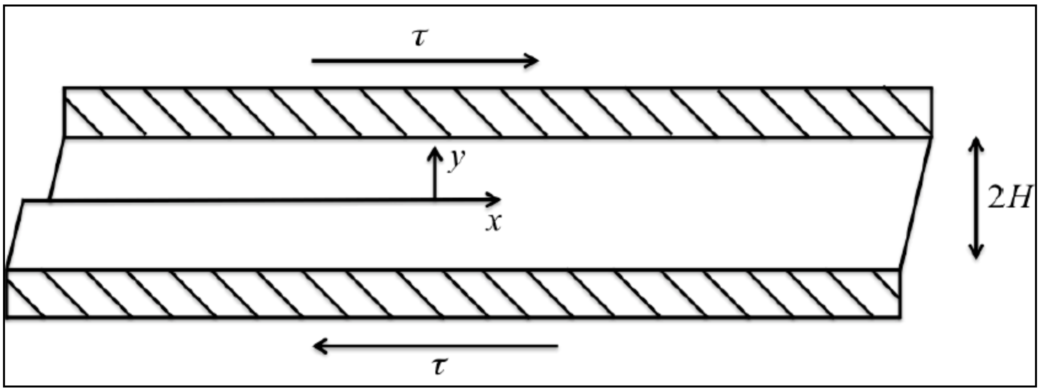

Fig. 1. The cracked strip model on fault

The tectonic shear stress τ acting at the external boundary of the body is removed, whereas the two faces of the fault are subjected to the stress τ over the length an extending back from the fault tip; the quantity a may simulate a finite fault. The fact that tectonic stress acts at fault plan has been proved by quite a lot of geological and geophysical observations, which comes from the interaction between blocks. Because of the roughness of rock materials, there is the frictional stress τ_f at the fault plane, so that the total stress applied at the fault faces should be ($\tau - \tau_f$).

There is a breakdown zone with length *R* near the fault tip mentioned above, whose magnitude is unknown so far, whereas it will be determined in the successive analysis. At the break down zone another stress drop (τ_b-τ_f) acts as well, in which τ_b denoting the breakdown limit of the rock material, should be a material constant. The introduction of the breakdown zone is consistent with the slip-weakening model adopted in the above-mentioned references, or say, it is a concrete application of the slip-weakening model. Fig. 2 shows the model schematically. According to the terminology of the fracture theory, (τ_b-τ_f) is also named the cohesive stress.

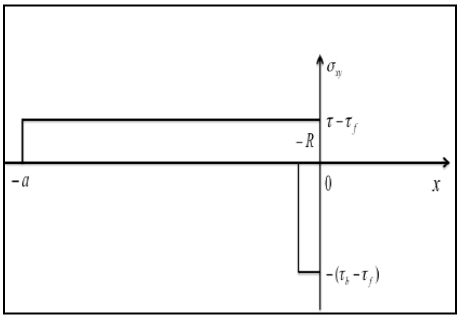

Fig.2 Breakdown zone and the stress distribution in the neighborhood of the fault tip

Reference [6] used this model, but the analysis there is preliminary only for the static case. The present analysis develops the study of Ref [6] in a quite wide range, i.e. it not only studies the static problem, but also the dynamic problem.

2.1 Fault extension initiation

The initiation of fault extension is a static problem. Assume that the fault body is anisotropic homogeneous elastic body. We define Airy stress function as $U(x, y)$, and then the strike slip process of the fault is governed by equation

$$\nabla^2\nabla^2 U = 0 \tag{1}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ denotes the two-dimensional Laplace operator, and the stress component, σ_{ij} , can be written as

$$\sigma_{xx} = \frac{\partial^2 U}{\partial^2 y^2} \quad \sigma_{yy} = \frac{\partial^2 U}{\partial^2 x^2} \quad \sigma_{xy} = \frac{\partial^2 U}{\partial^2 xy} \tag{2}$$

The problem of inverse fault is governed by

$$\nabla^2 u_z = 0 \tag{3}$$

The problem of the fault extension initiation induced by stress drop (τ_b - τ_f) (the tectonic stress) is described by the following boundary condition:

$$\left.\begin{aligned} &y = \pm H, -\infty < x < \infty : \sigma_{yy} = \sigma_{xy} = 0, \\ &y = \pm\infty, -H < x < H : \sigma_{xx} = \sigma_{xy} = 0, \\ &y = \pm 0, -\infty < x < a : \sigma_{yy} = \sigma_{xy} = 0, \\ &y = \pm 0, -a < x < 0 : \sigma_{yy} = 0, \sigma_{xy} = -\left(\tau - \tau_f\right) \end{aligned}\right\} \tag{4}$$

The most effective tool for solving boundary value problem (1), (4) is complex analysis, i.e., take the solution

$$U(x, y) = Re\left[\bar{z}\phi_1(z) + \int \psi_1(z)dz\right] \tag{5}$$

where $\phi_1(z)$ and $\psi_1(z)$ represents any analytic functions of complex variable $z = x + iy, i = \sqrt{-1}$. The author suggested a conformal mapping approach to solve the boundary value problem (4) of eq. (1), i.e. by taking the function

$$z=\omega\left(\zeta\right)=\frac{H}{\pi}\ln\left[1+\left(\frac{1+\zeta}{1-\zeta}\right)^{2}\right] \tag{6}$$

and mapping the region of the physical plane (z=x+iy plane) onto the interior of the unit circle γ in the $\zeta\,(=\zeta+i\eta)$ plane, the solution in closed form is found[9-11]

$$\phi'\left(\zeta\right)=\frac{Hp}{2\pi^{2}i}\left\{\frac{1}{1-\zeta}\ln\left(\sigma-1\right)-\frac{1+\zeta}{\left(1-\zeta\right)\left(1+\zeta^{2}\right)}\ln\left(\sigma-\zeta\right)-\frac{\zeta}{2\left(1+\zeta^{2}\right)}\ln\left(1+\sigma^{2}\right)+\frac{i}{2\left(1+\zeta^{2}\right)}\ln\left(\frac{\sigma-i}{\sigma+i}\right)\right\}_{\sigma_{-a}}^{\overline{\sigma_{-a}}} \tag{7}$$

in which

$$\left.\begin{aligned}\sigma_{-a}&=\frac{-e^{\pi a/H}+2i\sqrt{1-e^{-\pi a/H}}}{2-e^{-\pi a/H}}\\ \overline{\sigma_{-a}}&=\frac{-e^{\pi a/H}-2i\sqrt{1-e^{-\pi a/H}}}{2-e^{-\pi a/H}}\end{aligned}\right\} \tag{8}$$

and

$$\phi(\zeta)=\phi_{1}(z)=\phi_{1}[\omega(\zeta)],\psi(\zeta)=\psi_{1}(z)=\psi_{1}[\omega(\zeta)] \tag{9}$$

The corresponding stress intensity factor is determined as

$$K_{\tau-\tau_f}^{static}=\frac{\sqrt{2}\left(\tau-\tau_f\right)\sqrt{H}}{2\pi}\ln\left(\frac{2\exp(\pi a/H)-1+2\exp(\pi a/H)\sqrt{1-\exp(-\pi a/H)}}{2\exp(\pi a/H)-1-2\exp(\pi a/H)\sqrt{1-\exp(-\pi a/H)}}\right) \tag{10}$$

The initiation of fault growth induced by stress drop (τ_b - τ_f) (the cohesive stress) is interpreted by the boundary condition as follows:

$$\left.\begin{aligned}&y=\pm H,-\infty<x<\infty:\sigma_{yy}=\sigma_{xy}=0,\\&y=\pm\infty,-H<x<H:\sigma_{xx}=\sigma_{xy}=0,\\&y=\pm0,-\infty<x<-R:\sigma_{yy}=\sigma_{xy}=0,\\&y=\pm0,-R<x<0:\sigma_{yy}=0,\sigma_{xy}=\tau_{B}-\tau_{f}\end{aligned}\right\} \tag{11}$$

Boundary value problem (11) of eq. (1) can be solved similarly as above. And the corresponding stress intensity factor is determined as

$$K_{\tau_b-\tau_f}^{static}=-\frac{\sqrt{2}\left(\tau_b-\tau_f\right)\sqrt{H}}{2\pi}\ln\left(\frac{2\exp(\pi a/H)-1+2\exp(\pi a/H)\sqrt{1-\exp(-\pi a/H)}}{2\exp(\pi a/H)-1-2\exp(\pi a/H)\sqrt{1-\exp(-\pi a/H)}}\right) \tag{12}$$

Because the stresses over the breakdown zone are refer to (11), it means that there is no stress singularity at the fault tip, i.e. the total stress intensity factor must vanish:

$$K_{\text{total}}^{\text{static}}=K_{\tau-\tau_f}^{\text{static}}+K_{\tau_b-\tau_f}^{\text{static}}=0 \tag{13}$$

Substituting (5) and (12) into (13) determines the size R of the breakdown zone. Actually, $R/a\ll 1$; this and (13) offer a very simple expression for R. For explicitly, we here denote it by R^{static}, i.e.

$$R^{\text{static}} = \frac{\pi}{8}\left(\frac{K_{\tau-\tau_f}{}^{\text{static}}}{\tau_b - \tau_f}\right) \tag{14}$$

where $K_{\tau-\tau_f}^{\text{static}}$ is given by (11). At the end of the breakdown zone, i.e. at y=0, x=-R, the slip of the fault is

$$u_x(-R,+0) - u_x(-R,-0) \equiv \delta^{\text{static}} = \frac{\left(K_{\tau-\tau_f}^{\text{static}}\right)}{E^{'}(\tau_b - \tau_f)} \tag{15}$$

in which

$$E^{'} = \begin{cases} E & (\text{plane stress state}) \\ E/(1-\nu^2) & (\text{plane strain state}) \end{cases} \tag{16}$$

E and v represent Young's modulus and Poisson's ratio of the rock material, respectively. The physical quantity (15) may be taken as a control parameter in checking the fault instability. According to the fracture theory, there is a critical value of δ^{static} denoted by $\delta_{\text{c}}^{\text{static}}$, which should be a material constant, while

$$\delta^{\text{static}} = \delta_{\text{c}}^{\text{static}} \tag{17}$$

It can be considered as a criterion for judging the instability. By the criterion one can determine the critical shear stress τ_{c} of the initiation of fault growth. Inserting (11) into (15) and then into (17) yields

$$\tau_{\text{c}} - \tau_{\text{f}} = \frac{2\pi}{\sqrt{2H}}\sqrt{E^{'}(\tau_b - \tau_{\text{f}})\delta^{\text{static}}} \,/\ln\left(\frac{2\exp(\pi a/H) - 1 + 2\exp(\pi a/H)\sqrt{1-\exp(-\pi a/H)}}{2\exp(\pi a/H) - 1 - 2\exp(\pi a/H)\sqrt{1-\exp(-\pi a/H)}}\right) \tag{18}$$

The initiation will occur as $\tau > \tau_b$. At H/a →∞, it follows from (10) that

$$(K_{\tau-\tau_f}{}^{\text{static}})_{H/a\to\infty} = \left(\tau_b - \tau_f\right)\sqrt{8a/\pi} \tag{19}$$

Which is identical with the well-known exact solution[12,13]. In this way, as H/a →0, it follows from (10) that

$$(K_{\tau-\tau_f}{}^{\text{static}})_{H/a\to 0} = \sqrt{2}\left(\tau_b - \tau_f\right)a/\sqrt{H} \tag{20}$$

This is a completely new result. Ref. [6] shows that one can find a similar result by using J-integral, in which it contains an unknown constant which could not be determined by the method itself. According to formula (10) the constant=$\sqrt{2}$. Formulae (19) and (20) give an examination of the correctness of the above derivation.

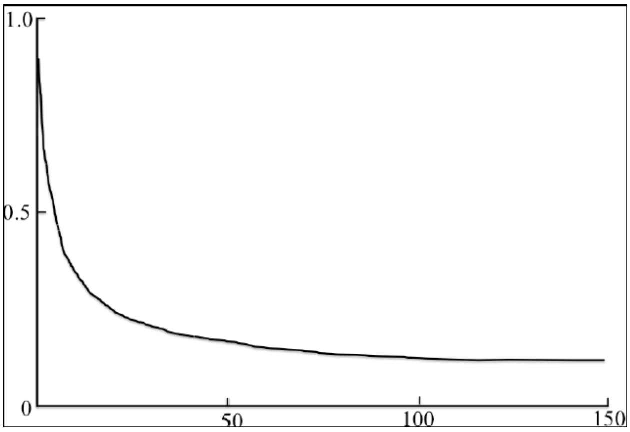

Fig.3.The effect of size of fault and the low-stress drop

Substituting (19) into the left-hand side of (17) determines the critical shear stress corresponding to the infinite fault body, and marked by $(\tau_c - \tau_f)_{H/a\to\infty}$, the ratio $(\tau_c - \tau_f)/(\tau_c - \tau_f)_{H/a\to\infty}$ is the normalized critical shear stress, the variation of which versus the characteristic size *a/H* of fault body describes the effect of fault body size on the fault instability, as description in Fig. 3. This result is identical with that obtained in ref. [2], and gives a quantitative description of the low-stress drop observed in earthquake source.

2.2 Fast propagation of fault

After the initiation of fault growth, it may experience a stable but further unstable growth, or it may become unstable growth directly after the initiation. Before and after the unstable propagation, there is another possibility that the propagation is arrested. Since the velocity of the unstable growth may reach the order of magnitude of elastic wave velocity, it may lead to an earthquake. We here discuss the case in which earthquake rather than arrest may be caused.

As pointed out just now, the state corresponding to the fast fault propagation is quite different from that corresponding to static case. It is necessary to carry out a fully dynamic analysis, which is governed by the following equations:

$$\nabla^2\phi = \frac{1}{c_1^2}\frac{\partial^2\phi}{\partial t^2}, \nabla^2\psi = \frac{1}{c_2^2}\frac{\partial^2\psi}{\partial t^2} \tag{21}$$

for strike slip fault; while for inverse fault

$$\nabla^2 u_z = \frac{1}{c_2^2}\frac{\partial^2 u_z}{\partial t^2} \tag{22}$$

where c_1 and c_2 represent the speeds of longitudinal and transverse waves, respectively. The dynamics analysis reveals the low stress drop effect again.

3. Interaction between co-linear faults

The principle discussed in the previous section can be used to the interaction between co-linear faults. The Ms7.2 Xingtai earthquake happened on March 8, 1966 in North China Block, immediately after the event, Prof. Li Shi-Guang, pointed out the earthquake transmission was going along the north-east direction [5] ， and the fact of the happening of Ms6.3 Hejian earthquake on March 2, 1967 proved the prediction of Prof. Li Shi-Guang. This shows the interaction between earthquake faults.

3.1 The static fracture analysis

We take the physical model for the interaction between co-linear faults shown in Fig.4

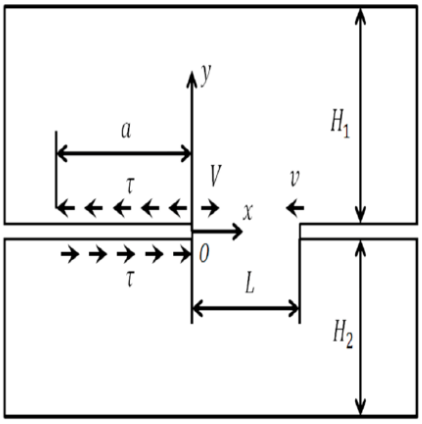

Fig.4 Model for co-linear faults in North China Block

The static fracture analysis on the interaction between co-linear strike slip faults can be based on the equation (1) and (2), but the boundary conditions are given as follows

$$\left.\begin{array}{l} y=H_1,-H_2,-\infty<x<+\infty:\sigma_{yy}=\sigma_{xy}=0 \\ y=0,-a<x<0:\sigma_{yy}=0\,,\ \sigma_{xy}=-\tau \\ x<-a:\sigma_{yy}=0\,,\ \sigma_{xy}=0 \\ x>L:\sigma_{yy}=0\,,\sigma_{xy}=0 \\ H_1>y>-H_2\ ,\ x=\pm\infty:\sigma_{xx}=0\,,\sigma_{xy}=0 \end{array}\right\} \tag{23}$$

The complex analysis is still effective for the present problem, but we should use the following conformal mapping

$$U(x,y)=Re\left[\bar{z}\phi_1(z)+\int\psi_1(z)dz\right] \tag{24}$$

$$z=\omega(\zeta)=\frac{H}{\pi}\ln\frac{1+\alpha\left(\dfrac{1-\zeta}{1+\zeta}\right)^2}{1+\gamma\alpha\left(\dfrac{1-\zeta}{1+\zeta}\right)^2} \tag{25}$$

$$\zeta=\xi+i\eta \qquad \alpha=\frac{1-e^{-\pi a/H}}{1-e^{-\pi(a+L)/H}} \qquad \gamma=e^{-\pi L/H} \tag{26}$$

and find the solution

$$\phi'(\zeta)=\frac{2\tau H}{\pi^2}\left[\frac{\sqrt{\gamma\alpha}M}{(1+\zeta)^2+\gamma\alpha(1-\zeta)^2}-\frac{\sqrt{\alpha}A}{(1+\zeta)^2+\alpha(1-\zeta)^2}\right]-\frac{2\tau H}{\pi^2}\frac{i\alpha(1-\gamma)(1-\zeta^2)}{[(1+\zeta)^2+\gamma\alpha(1-\zeta)^2][(1+\zeta)^2+\alpha(1-\zeta)^2]}\times\ln\frac{i-\zeta}{1-i\zeta} \tag{27}$$

where

$$\phi(\zeta)=\phi_1(z)=\phi_1[\omega(\zeta)],\psi(\zeta)=\psi_1(z)=\psi_1[\omega(\zeta)] \tag{28}$$

and

$$A=\ln(1+\sqrt{\alpha})/(1-\sqrt{\alpha}) \qquad M=\ln[1+\sqrt{\gamma\alpha}/(1-\sqrt{\gamma\alpha})] \tag{29}$$

so that

$$K\equiv K_{\mathrm{I}}-iK_{\mathrm{II}}=\lim_{\zeta\to\zeta_1}2\sqrt{\pi}\,\frac{\phi'(\zeta)}{\sqrt{\omega''(\zeta)}} \tag{30}$$

$$\zeta=\omega^{-1}(z)=\left[\sqrt{\alpha}-\sqrt{\frac{e^{\pi z/H}-1}{1-\gamma e^{\pi z/H}}}\right]\Big/\left[\sqrt{\alpha}+\sqrt{\frac{e^{\pi z/H}-1}{1-\gamma e^{\pi z/H}}}\right] \tag{31}$$

$$K_{\mathrm{II}}^{(0,0)}=\frac{\sqrt{2H}\tau}{\pi\sqrt{1-\gamma}}[\ln\frac{1+\sqrt{\alpha}}{1-\sqrt{\alpha}}-\sqrt{\gamma}\ln\frac{1+\sqrt{\gamma\alpha}}{1-\sqrt{\gamma\alpha}}] \tag{32}$$

$$K_{\mathrm{II}}^{(\mathrm{L},0)}=\frac{\sqrt{2H}\tau}{\pi\sqrt{1-\gamma}}[\sqrt{\gamma}\ln\frac{1+\sqrt{\alpha}}{1-\sqrt{\alpha}}-\ln\frac{1+\sqrt{\gamma\alpha}}{1-\sqrt{\gamma\alpha}}] \tag{33}$$

The fracture criterion

$$K_{\mathrm{II}}^{(\mathrm{L},0)}=K_{\mathrm{IIC}} \tag{34}$$

Is used, in which $K_{\mathrm{IIC}}\approx K_{\mathrm{IC}}=2.81MPa\bullet m^{1/2}$

3.1.1 Earthquake example 1: *Interaction between Xingtai fault and Hejian fault*

The Xingtai fault and Hejian fault with $a=55km, L\approx 400km, H\approx 300km$ and Ms7.1 Xingtai earthquake in N37.5^0, E115.1^0 happened on March 8, 1966 with the stress drop $\tau=42$ bar. Substituting $a=55km, L\approx 400km, H\approx 300km$ into (34) can determine the critic stress τ_c which is $\tau_c=3.4$ bar. Because $\tau_c=3.4\,\text{bar}\le\tau=42$ bar, this means the Xingtai earthquake can induce the Hejian fault to occur earthquake. In fact, the Ms 6.3Hejian earthquake happened on March 2, 1967.

3.1.2 Earthquake example 2: *Interaction between Tangshan fault and Luanxian fault*

The Tangshan fault and Luanxian fault with $a=115km, L\approx 10km, H\approx 150km$ and Ms 7.8 Tangshan earthquake in N38.5^0, E118.7^0 happened on July 28, 1976 with the stress drop $\tau=8$ bar. Substituting $a=115km, L\approx 10km, H\approx 150km$ into (25) can determine the critic stress τ_c that is $\tau_c=0.35$ bar. Because $\tau_c=0.35\,\text{bar}\le\tau=8$ bar, this means the Tangshan earthquake can induce the Luanxian fault to occur earthquake. In fact, the Ms7.1 Luanxian earthquake happened on July 29, 1976 (only 15 hours after the main shock of Tangshan earthquake).

3.2 The dynamic fracture analysis

The dynamic fracture analysis is also carried out in which the wave equations (21~22) are used, and the initial and boundary value conditions are as follows

$$t=0: u_x=u_y=0\,,\ \frac{\partial u_x}{\partial t}=\frac{\partial u_y}{\partial t}=0 \tag{35}$$

$$\left.\begin{array}{l}\sqrt{x^2+y^2}\to\infty:\sigma_{ij}=0\\ y=0\,,-\infty<x<-a:\sigma_{yy}=\sigma_{xy}=0\\ -a<x<0:\sigma_{xy}=-\tau\,,\sigma_{yy}=0\\ L<x<+\infty:\sigma_{yy}=\sigma_{xy}=0\\ -H_2<x<H_1:x=\pm\infty:\sigma_{xx}=\sigma_{xy}=0\end{array}\right\} \tag{36}$$

We developed a complex analysis method, in which the approximate conformal mapping

$$z_1/\alpha_1\,,z_2/\alpha_2=\omega(\zeta)=\frac{H}{\pi}\ln\frac{1+\alpha\left(\frac{1-\zeta}{1+\zeta}\right)^2}{1+\gamma\alpha\left(\frac{1-\zeta}{1+\zeta}\right)^2} \tag{37}$$

Which is used with parameters $\alpha=\frac{1-e^{-\pi a/H}}{1-e^{-\pi(a+L)/H}}$, $\gamma=e^{-\pi L/H}$ and determine the approximate dynamic stress intensity factors $\alpha=\frac{1-e^{-\pi a/H}}{1-e^{-\pi(a+L)/H}}$, $\gamma=e^{-\pi L/H}$

$$\left.\begin{array}{l}K_{\mathrm{II}}^{(0,0)}(t)=\frac{\sqrt{2H}\tau}{\pi\sqrt{1-\Gamma}}[\ln\frac{1+\sqrt{A}}{1-\sqrt{A}}-\sqrt{\Gamma}\ln\frac{1+\sqrt{\Gamma A}}{1-\sqrt{\Gamma A}}]\\ K_{\mathrm{II}}^{(\mathrm{L},0)}(t)=\frac{\sqrt{2H}\tau}{\pi\sqrt{1-\Gamma}}[\sqrt{\Gamma}\ln\frac{1+\sqrt{A}}{1-\sqrt{A}}-\ln\frac{1+\sqrt{\Gamma A}}{1-\sqrt{\Gamma A}}]\end{array}\right\} \tag{38}$$

With $A=\frac{1-e^{-\pi a/\alpha_1 H}}{1-e^{-\pi(a+L)/\alpha_1 H}}$, $\Gamma=e^{-\pi L/\alpha_1 H}$,$\alpha_1=\sqrt{1-V^2/c_1^2}$, in which V is the fault propagating speed, and c_1 the longitudinal wave speed. We use the fracture criterion for fault propagation

$$K_{\mathrm{II}}^{(\mathrm{L},0)}(t)=K_{\mathrm{IIC}} \tag{39}$$

and $K_{\mathrm{IIC}}\approx K_{\mathrm{IC}}=2.81MPa\bullet m^{-1/2}$.

3.2.1 Earthquake example 1 *Interaction between Xingtai fault and Hejian fault*

The Xingtai fault and Hejian fault with *a=55km/s, L≈400km/s, H≈300km* and 1966 Ms 7.1 Xingtai earthquake (N37.5°, E115.1°) with the stress drop $\tau=42$ bar and $c_1=6\mathrm{km/s}$, $V=3$ km/s. Substituting the earthquake geometry parameters and $c_1=6\mathrm{km/s}$ $V=3$ km/s into (39) can determine the critic stress τ_c which can induce the Hejian earthquake is $\tau_c^{dyn}=0.39$ bar。 Because $\tau_c^{dyn}=0.39$ bar$\le\tau=42$ bar, this means the Xingtai earthquake can induce the Hejian fault to occur earthquake. In fact, the Ms 6.3Hejian earthquake happened on March 2,1967.

3.2.2 Earthquake example 2 Interaction between Tangshan fault and Luanxian fault

The Tangshan fault and Luanxian fault with *a=111km/s, L≈115km/s, H≈150km* and 1976 Ms7.8 Tangshan earthquake (N38.5°, E118.7°) with the stress drop $\tau=8$ bar, and c_1*=6km/*s,$V=2.7$ km/s.

Substituting the earthquake fault geometry parameters and $c_1 = 3\text{km/s}$, $V = 2.7$ km/s into (39) can determine the critic stress τ_c which can induce the Luanxian earthquake is $\tau_c = 0.024$ bar. Because $\tau_c = 0.024\,\text{bar} \le \tau = 8$ bar, this means the Tangshan earthquake can induce the Luanxian fault to occur earthquake. In fact, the Ms7.1 Luanxian earthquake happened on July 29, 1976 (only 15 hours after the main shock of Tangshan earthquake).

4. Interaction between parallel faults

The zone of 2008 Ms8.0 Wenchuan earthquake and the simplified mathematical model are shown in Fig.5.The zone is located in the Longmenshan earthquake region that is the boundary between Tibet Block and South China Block. The line AB in Fig 5 represents Maoxian fault, which does not appear earthquake during the Wenchuan earthquake event 0f 2008, and can be treated as a free boundary. The line A_2B_2 represents the main shock fault, the Yingxiu-Beichuan fault, the epicenter is the Yingxiu Town, and line A_1B_1 the Pengxian-Guangxian fault, where the strong earthquake happened induced by the main shock, and line CD the boundary of South China Block, which can be seen as a fixed boundary. The main shock and induced shock are mainly the inverse motion rather than strike slip motion. This complicated configuration is not available for analytic solving. We have done the numerical analysis by finite element for initiation of fault growth only; the fast fault propagation has not been completed yet. At meantime, for the main shock fault (fault A_2B_2) an approximate solution is given as below

$$K_{\tau-\tau_f}^{static}\Big|_{yingxiu} = \frac{\sqrt{2}\left(\tau_1-\tau_f\right)\sqrt{H_0}}{2\pi}\ln\left(\frac{2\exp(\pi a_{20}/H_0)-1+2\exp(\pi a_{20}/H_0)\sqrt{1-\exp(-\pi a_{20}/H_0)}}{2\exp(\pi a_{20}/H_0)-1-2\exp(\pi a_{20}/H_0)\sqrt{1-\exp(-\pi a_{20}/H_0)}}\right) \tag{40}$$

The analytic solution for fault A_1B_1 in general cannot be obtained, we introduce an approximate formula [11]

$$K_{\tau-\tau_f}^{static}\Big|_{pengxian} = \frac{2\left(\tau_1-\tau_f\right)}{\pi}\sqrt{H_0\tanh\frac{\pi a_{20}}{H_0}}\arcsin\left(\frac{\sinh\frac{\pi(a_2-a_{20})}{H_0}}{\sinh\frac{\pi a_{20}}{H_0}}\right) \tag{41}$$

For the dynamic analytic solution cannot be obtained even if for approximate study. The comparison between numerical and approximate analytic solutions, we find that

$$\frac{K_{\tau-\tau_f}^{static}\Big|_{yingxiu}}{K_{\tau-\tau_f}^{static}\Big|_{pengxian}} \approx 3.0 \sim 8.0 \tag{42}$$

This means the accuracy of the approximate analytic solution and numerical solution is in the same order of magnitude. This also shows the main shock induces the shock of Pengxian-Guanxian. The shock is propagates in elastic wave speed, after happening of the main shock, the induced shock occurred immediately, because the distance between the two faults is only 10 to 20 kilometers.

5. Conclusion and discussion

The block hypothesis on the strong earthquake in plate in China land is explained with fracture theory, the single fault analysis explained the low stress drop phenomena, in the co-linear faults analysis the theoretical prediction is in good agreement to the observation results for Xingtai-Hejianfault interaction and Tangshan-Luanxia fault interaction. However the analysis on parallel faults is as just a preliminary work, which must be down the further analysis.

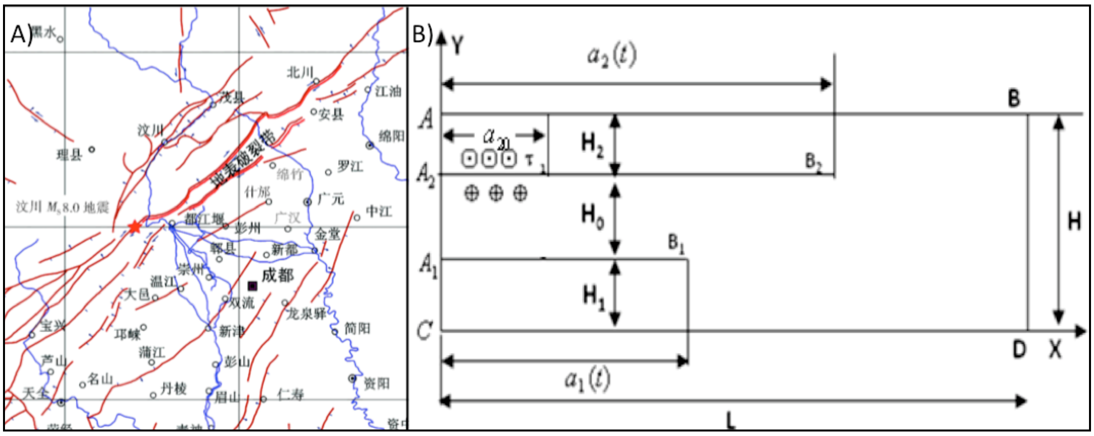

Fig.5 Yinxiu fault and Pengxian fault in Wenchuan Earthquake

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